AFT Arrow™

Verification Cases

AFT Arrow Version 9

Compressible Pipe Flow Modeling

Published: April 28, 2023

Dynamic solutions for a fluid world ™

Contents

AFT Arrow Verification Overview

There are a number of aspects to the verification process employed by Applied Flow Technology to ensure that AFT Arrow provides accurate solutions to compressible pipe flow systems. These are discussed in Verification Methodology. A listing of all of the verified models is given in Summary of Verification Models. The verification models are taken from numerous References.

References

1. John D. Anderson, Modern Compressible Flow, 1982, McGraw-Hill.
2. Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988.
3. Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, Third Edition, John Wiley & Sons, 1985.
4. William S. Janna, Introduction to Fluid Mechanics, PWS Publishers, Belmont, CA 1983.
5. Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, Seventh Edition, Professional Publications, Belmont, CA, 1984.
6. Mohinder L. Nayyar, Piping Handbook, Sixth Edition, McGraw-Hill, New York, 1992.
7. Robert H. Perry and Don W. Green Editors, Author James. N. Tilton, Perry's Chemical Engineer's Handbook, Seventh Edition.
8. Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993

Verification Reference - Anderson Title Page

List of All Verification Models

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

MODERN COMPRESSIBLE FLOW

With Historical Perspective

John D. Anderson, Jr.

Professor of Aerospace Engineering
University of Maryland, College Park

McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

Verification Reference - Crane Title Page

List of All Verification Models

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FLOW OF FLUIDS

THROUGH

VALVES, FITTINGS, AND PIPE

By the Engineering Department

CRANE®

CRANE CO.
104 N. Chicago St.
Joliet, IL 60434

Technical Paper No. 410

PRINTED IN U.S.A.

(Twenty Fifth Printing—1991)

Verification Reference - Fox and McDonald Title Page

List of All Verification Models

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INTRODUCTION TO FLUID MECHANICS

Third Edition

ROBERT W. FOX
ALAN T. McDONALD

School of Mechanical Engineering
Purdue University

JOHN WILEY & SONS

New York · Chichester · Brisbane · Toronto · Singapore

Verification Reference - Lindeburg Title Page

List of All Verification Models

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MECHANICAL ENGINEERING REVIEW MANUAL

Seventh Edition

A complete review course for the P.E. examination for Mechanical Engineers

Michael R. Lindeburg, P.E.

Professional Publications, Inc.

Belmont, CA 94002

Verification Reference - Perrys Title Page

List of All Verification Models

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PERRY'S CHEMICAL ENGINEERS' HANDBOOK

SEVENTH EDITION

Prepared by a staff of specialists under the editorial direction of

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Robert H. Perry

Editor
Don W. Green
Deane E. Ackers Professor of Chemical and Petroleum Engineering, University of Kansas

Associate Editor
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Verification Reference - Janna Title Page

List of All Verification Models

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Introduction to Fluid Mechanics

WILLIAM S. JANNA
University of New Orleans

PWS Engineering Boston, Massachusetts

Verification Reference - Saad Title Page

List of All Verification Models

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Compressible Fluid Flow

Second Edition

Michel A. Saad
Professor of Mechanical Engineering
Santa Clara University, California

PRENTICE HALL
Englewood Cliffs, New Jersey 07632

Verification Reference - Nayyar Title Page

List of All Verification Models

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PIPING HANDBOOK

Mohinder L. Nayyar, P.E.

ASME Fellow

The sixth edition of this Handbook was edited by

Mohindar L. Nayyar, P.E.

The fifth edition of this Handbook was edited by

Reno C. King, B.M.E, M.M.E., D.Sc., P.E.

Professor of Mechanical Engineering and Assistant Dean,
School of Engineering and Science, New York University
Registered Professional Engineer

The first four editions of this Handbook were edited by

Sabin Crocker, M.E.

Fellow, ASME: Registered Professional Engineer

Seventh Edition

MCGRAW-HILL

New York San Francisco Washington, D.C. Auckland Bogotá
Caracas Lisbon London Madrid Mexico City Milan
Montreal New Delhi San Juan Singapore
Sydney Tokyo Toronto

Verification Methodology

The *AFT Arrow* software is a compressible pipe flow analysis product intended to be used by trained engineers. As a technical software package, issues of quality and reliability of the technical data generated by the software are important. The following description summarizes the steps taken by Applied Flow Technology to ensure high quality in the technical data.

1. Comparisons with open literature examples

Open literature examples for compressible flow in pipe networks are very hard to find. However, examples for single pipe systems are more common. *AFT Arrow* has been compared against a number of single pipe examples with good agreement.

2. Software checks results to ensure mass and energy balance

AFT Arrow's network solution method is based on a popular iterative method to solve pipe network systems. The method is known as the Newton/Raphson method. The methods that are used in *AFT Arrow* iterate on the governing equations to obtain a balanced mass and energy in the system. After a solution is obtained, a final check is made by the software whereby the mass flow and energy flow into each node is checked for balance. If a balance is not found, the user is warned in the output. This ensures that the results generated by the software agree with the applicable fundamental equations.

See the AFT Arrow Help site for more information.

3. Hand/spreadsheet checks of the solutions confirm agreement with fundamental equations

AFT Arrow uses a marching method to solve the fundamental equations of compressible flow for each pipe. The results can be exported from *AFT Arrow* and checked by hand calculations or spreadsheet to evaluate agreement with the original equations. This has been performed by AFT and agreement has been demonstrated repeatedly.

See the AFT Arrow Help site for more information.

4. *AFT Arrow* offers three independent solution methods which can be cross-checked

AFT Arrow offers three independent solution methods. Each is optimized for a particular application, but in many cases all three methods can be used to solve the same system. In such cases, agreement between the three methods gives confidence that accurate solutions have been obtained.

See the AFT Arrow Help site for more information.

5. *AFT Arrow* predictions agree with *AFT Fathom* predictions for incompressible flow

For incompressible and moderately compressible gas systems, *AFT Arrow* results can be compared against those generated by *AFT Fathom*. In such cases agreement between *AFT Arrow* and *AFT Fathom* has been demonstrated repeatedly.

6. Software has been used in industry since September, 1995 demonstrating repeated agreement with other methods and data

AFT Arrow became available in September, 1995, and is currently being used by companies in the following industries: chemical, petrochemical, power generation, architectural, ship construction, aerospace, and pharmaceutical. Since its release, *AFT Arrow* has been applied to numerous gas systems with various working fluids and has repeatedly demonstrated agreement with data and other analysis methods. In addition, Applied Flow Technology issues maintenance releases of the software periodically to improve performance and correct any problems that may have been discovered.

Summary of Verification Models

All published compressible flow calculations of which AFT is aware are for single pipes. Because of the simplicity of the single pipe verification models, no views of the models are included in this documentation. Comparison of AFT Arrow predictions to the published calculation results is included herein for twenty-one cases from 8 sources. Below is a summary of the cases.

Case	Fluid	Reference	Sonic
Case 1	Air	Anderson	No
Case 2	Methane	Saad	Yes
Case 3	Steam	Crane	No
Case 4	Air	Crane	No
Case 5	Natural Gas	Crane	No
Case 6	Steam	Crane	Yes
Case 7	Air	Crane	No
Case 8	Air	Fox and McDonald	Yes
Case 9	Unspecified (γ = 1.4)	Lindeburg	No
Case 10	Methane	Lindeburg	No
Case 11	Air	Saad	No
Case 12	Air	Saad	Yes
Case 13	Natural Gas	Saad	Yes/No
Case 14	Air	Tilton	Yes
Case 15	Air	Janna	No
Case 16	Air	Nayyar	Yes
Case 17	Air	Nayyar	Yes
Case 18	Air	Nayyar	Yes
Case 19	Air	Nayyar	Yes
Case 20	Air	Nayyar	Yes
Case 21	Air	Nayyar	No

Verification Case 1

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify1.ARO

REFERENCE: John D. Anderson, Modern Compressible Flow, 1982, McGraw-Hill, page 76, example 3.4

GAS: Air

ASSUMPTIONS: 1) Adiabatic, 2) Perfect gas

RESULTS:

Parameter	Anderson	AFT Arrow
M_2 – Mach number at exit	0.475	0.473
P_2– Static pressure at exit (atm)	0.624	0.624
T_2– Static temperature at exit (deg. K)	265.8	265.9
P_{o2}– Stagnation pressure at exit (atm)	0.728	0.728

DISCUSSION:

As specified, inlet conditions are known and outlet conditions need to be determined. With the known inlet conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the exit.

The problem states that the inlet Mach number is 0.3, P1 = 1 atm, T1 = 273 K. From the ideal gas law, density, sonic speed and mass flow rate are:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{1\ atm}{(0.2868\frac{kJ}{kg\ K})(273\ K)} = 1.293\ kg/m^3$$

$$\alpha_1 = \sqrt{\gamma RT_1} = \sqrt{1.4(287\frac{J}{kg\ K})273\ K} = 331.2\frac{m}{s}\ (sonic\ velocity)$$

$$\dot{m} = \rho_1 V_1 A = \rho_1 (M_1 \alpha_1) A = (1.293\frac{kg}{m^3})(0.3)(331.2\frac{m}{s})(\frac{\pi}{4}0.15^2 m^2) = 2.27\frac{kg}{s}$$

Note that the friction factor in Anderson is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

List of All Verification Models

Verification Case 1 Problem Statement

Verification Case 1

John D. Anderson, Modern Compressible Flow, 1982, McGraw-Hill, page 76, example 3.4

Anderson Title Page

76 MODERN COMPRESSIBLE FLOW: WITH HISTORICAL PERSPECTIVE

Example 3.4 Consider the flow of air through a pipe of inside diameter = 0.15 m and length = 30 m. The inlet flow conditions are $M_1 = 0.3$, $p_1 = 1$ atm, and $T_1 = 273$ K. Assuming $f = \text{const} = 0.005$, calculate the flow conditions at the exit, M_2, p_2, T_2, and p_{o_2}.

SOLUTION From Table A.1: For $M_1 = 0.3$, $p_{o_1}/p_1 = 1.064$. Thus

$$p_{o_1} = 1.064(1 \text{ atm}) = 1.064 \text{ atm}$$

From Table A.4: For $M_1 = 0.3$, $4\bar{f}L_1^*/D = 5.299$, $p_1/p^* = 3.619$, $T_1/T^* = 1.179$, and $p_{o_1}/p^* = 2.035$. Since $L = 30 \text{ m} = L_1^* - L_2^*$, then $L_2^* = L_1^* - L$ and

$$\frac{4\bar{f}L_2^*}{D} = \frac{4\bar{f}L_1^*}{D} - \frac{4\bar{f}L}{D} = 5.2993 - \frac{(4)(0.005)(30)}{0.15} = 1.2993$$

From Table A.4: For $4\bar{f}L^*/D = 1.2993$, $\boxed{M_2 = 0.475}$, $T_2/T^* = 1.148$, $p_2/p^* = 2.258$, and $p_{o_2}/p_o^* = 1.392$. Hence

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = 2.258\frac{1}{3.169}(1 \text{ atm}) = \boxed{0.624 \text{ atm}}$$

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 1.148\frac{1}{1.179}273 = \boxed{265.8 \text{ K}}$$

$$p_{o_2} = \frac{p_{o_2}}{p^*}\frac{p^*}{p_{o_1}}p_{o_1} = 1.392\frac{1}{2.035}1.064 = \boxed{0.728 \text{ atm}}$$

Certain physical trends reflected by the numbers obtained from such solutions are summarized here:

1. For *supersonic* inlet flow, i.e., $M_1 > 1$, the effect of friction on the downstream flow is such that
   a. Mach number decreases, $M_2 < M_1$
   b. Pressure increases, $p_2 > p_1$
   c. Temperature increases, $T_2 > T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity decreases, $u_2 < u_1$
2. For *subsonic* inlet flow, i.e., $M_1 < 1$, the effect of friction on the downstream flow is such that
   a. Mach number increases, $M_2 > M_1$
   b. Pressure decreases, $p_2 < p_1$
   c. Temperature decreases, $T_2 < T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity increases, $u_2 > u_1$

From the above, note that friction always drives the Mach number towards 1, decelerating a supersonic flow and accelerating a subsonic flow. This is em-

Verification Case 2

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify2.ARO

REFERENCE: Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Page 215, example 5.3

GAS: Methane

ASSUMPTIONS: 1) Adiabatic, 2) Perfect gas

RESULTS:

Parameter	Saad	AFT Arrow
Maximum pipe length (m)	4,024	4,031
P_2– Static pressure at exit (kPa)	48.2	48.3
V_2– Velocity at exit (m/s)	433.17	431.82
T_2– Static temperature at exit (deg. K)	278.43	278.15

DISCUSSION:

As specified, inlet conditions are known and the outlet conditions are sonic. The pipe length that yields sonic flow is the objective. With the known inlet conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the exit.

The problem states that the inlet velocity V1 is 30 m/s, P1 = 0.8 MPa, T1 = 320 K. From the ideal gas law, density, sonic speed and mass flow rate are:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{0.8\ MPa}{(0.5179\ \frac{kJ}{kg\ K})(320\ K)} = 4.823\ kg/m^3$$

$$\dot{m} = \rho_1 V_1 A = (4.823\ \frac{kg}{m^3})(30\ \frac{m}{s})(\frac{\pi}{4}\ 0.3^2\ m^2) = 10.2265\ \frac{kg}{s}$$

AFT Arrow does not solve for pipe length. To obtain the maximum pipe length, different lengths must be guessed with lengths that exceed sonic flow discarded.

Note that the friction factor in Saad is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

It should also be noted that, from time to time, AFT finds it is necessary to modify the Solver used by Arrow to improve application performance, or for other reasons. These modifications to the Solver may cause slight changes to the appropriate pipe lengths determined by Arrow.

List of All Verification Models

Verification Case 2 Problem Statement

Verification Case 2

Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Page 215, example 5.3

Saad Title Page

(d) The change in entropy is:

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$= 1000 \ln \frac{293}{312.76} - 287 \ln \frac{150}{306}$$

$$= -65.26 + 204.65 = 139.39 \text{ J/kg K}$$

but

$$\dot{m} = \rho_2 A V_2 = \left(\frac{p_2}{RT_2}\right) A V_2$$

$$= \left(\frac{1.5 \times 10^5}{287 \times 293}\right) \left[\frac{\pi}{4}(0.3)^2\right] (235.79) = 29.74 \text{ kg/s}$$

Therefore:

$$\Delta \dot{S} = (139.39)(29.74) = 4145.77 \text{ J/K·s}$$

In the above examples the friction factor was assumed constant. But for a duct of a certain relative roughness the value of f is a function of Reynolds number Re as depicted by the Moody diagram. In a constant-area duct, Re in turn depends on the velocity, density, and viscosity, which change as the fluid flows in the duct. But from continuity the product ρV is constant, so that the only variable in Re is the viscosity and, unless viscosity changes drastically, the variations in f are small. An additional factor to be considered is that most engineering applications involve turbulent flow, where f depends on the relative roughness of the duct but is essentially insensitive to the magnitude of Re. The following example illustrates these effects.

Example 5.3

Methane ($\gamma = 1.3$, $R = 0.5184$ kJ/kg K) flows adiabatically in a 0.3 m commercial steel pipe. At the inlet the pressure $p_1 = 0.8$ MPa, the temperature $T_1 = 320$ K (viscosity $= 0.011 \times 10^{-3}$ kg/m-s), and the velocity $V_1 = 30$ m/s. Find:

(a) The maximum possible length of the pipe.

(b) The pressure and velocity at the exit of the pipe.

Solution

(a) The conditions at the exit are sonic ($M = 1$):

$$\rho_1 = \frac{p_1}{RT_1} = \frac{800}{0.5184(320)} = 4.823 \text{ kg/m}^3$$

$$M_1 = \frac{V_1}{c_1} = \frac{30}{\sqrt{(1.3)(518.4)(320)}} = \frac{30}{464.386} = 0.0646$$

$$\mathrm{Re}_1 = \frac{\rho_1 V_1 D}{\mu_1} = \frac{(4.823)(30)(0.3)}{0.011 \times 10^{-3}} = 3.946 \times 10^6$$

and

$$\frac{\epsilon}{D} = 0.00015 \quad \text{so that} \quad f_1 = 0.00333$$

If this value of f_1 is considered constant, then:

$$\frac{4\bar{f}L_1^*}{D} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma + 1}{2\gamma} \ln \frac{(\gamma + 1)M_1^2}{2\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}$$

$$= \frac{1 - 0.00417}{0.00543} + \frac{2.3}{2.6} \ln \frac{(2.3)(0.00417)}{2(1.000626)}$$

$$= 183.39 - 4.72 = 178.67$$

The maximum possible length of the duct is:

$$L_1^* = \frac{(178.67 \times 0.3)}{4(0.00333)} = 4024 \text{ m}$$

Noting that $\rho_1 V_1 = \rho^* V^*$, the Reynolds number at the exit is:

$$\mathrm{Re}^* = \frac{\rho^* V^* D}{\mu^*} = \frac{\rho_1 V_1 D}{\mu^*} = \mathrm{Re}_1 \left(\frac{\mu_1}{\mu^*}\right)$$

The temperature at the exit is given by:

$$T^* = T_1 \frac{2\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{\gamma + 1}$$

$$= 320 \left[\frac{2(1.000626)}{2.3}\right] = 278.43 \text{ K}$$

at $T^* = 278.43$ K, $\mu^* = 0.0104 \times 10^{-3}$ kg/m·s, so that:

$$\mathrm{Re}^* = 3.946 \times 10^6 \left(\frac{0.011 \times 10^{-3}}{0.0104 \times 10^{-3}}\right) = 4.174 \times 10^6$$

and $f^* = 0.0033$, which is close to f_1. For an average value $\bar{f} = 0.003315$, $L_1^* = 4042$ m.

(b) $$\frac{p^*}{p_1} = M_1 \sqrt{\frac{2\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{\gamma + 1}} = 0.0603$$

so that $p^* = 48.2$ kPa. The velocity at the exit is:

$$V^* = \sqrt{\gamma R T^*} = \sqrt{(1.3)(518.4)(278.43)} = 433.17 \text{ m/s}$$

Verification Case 3

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify3.ARO

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-6, example 4-10

GAS: Steam

ASSUMPTIONS: Example does not specify the heat transfer conditions, so it was assumed adiabatic in this model.

RESULTS:

Parameter	Crane	AFT Arrow
Static pressure drop (psi)	40.1	41.6

DISCUSSION:

Crane does not make a distinction between static and stagnation pressure, and it appears that static pressure is usually assumed. Therefore, the inlet pressure of 600 psig was assumed to be static pressure.

To make a one-to-one comparison, the K factors and friction factor used in Crane were used directly in AFT Arrow. The K factors were modeled as fitting and loss values, which evenly spreads the effect of resistance across the entire pipe. In practice, the velocity changes in the pipe can yield different answers for fitting pressure losses depending on where they are actually located.

The AFT Arrow model uses the Redlich-Kwong real gas equation of state for steam. Note that the Crane formula underpredicts the pressure drop (by about 4%), which in most applications is not conservative.

List of All Verification Models

Verification Case 3 Problem Statement

Verification Case 3

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-6, example 4-10

Crane Title Page

Pressure Drop and Velocity in Piping Systems

Example 4-10 . . . Piping Systems—Steam

Given: 600 psig steam at 850 F flows through 400 feet of horizontal 6-inch Schedule 80 pipe at a rate of 90,000 pounds per hour.

The system contains three 90 degree weld elbows having a relative radius of 1.5, one fully-open 6 x 4-inch Class 600 venturi gate valve as described in Example 4-4, and one 6-inch Class 600 y-pattern globe valve. Latter has a seat diameter equal to 0.9 of the inside diameter of Schedule 80 pipe, disc fully lifted.

Find: The pressure drop through the system.

Solution:

1. $\Delta P = \frac{28 \times 10^{-8} K W^2 \bar{V}}{d^4}$ . . . . page 3-4

2. For globe valve (see page A-27),

$K_2 = \frac{K_1 + \beta\,[0.5\,(1-\beta^2) + (1-\beta^2)^2]}{\beta^4}$

$K_1 = 55\, f_T$

$\beta = 0.9$

3. $K = 14\, f_T$ . . . . 90° weld elbows; page A-29

$K = f \frac{L}{D}$ . . . . pipe; page 3-4

$R_e = 6.31 \frac{W}{d\mu}$ . . . . page 3-2

4. $d = 5.761$ . . . . . . 6" Sched. 80 pipe; page B-17

$\bar{V} = 1.216$ . . . . 600 psi steam, 850 F; page A-17

$\mu = 0.027$ . . . . page A-2

$f_T = 0.015$ . . . . page A-26

5. For globe valve,

$K_2 = \frac{55 \times .015 + .9\,[.5\,(1-.9^2) + (1-.9^2)^2]}{.9^4}$

$K_2 = 1.44$

6. $R_e = \frac{6.31 \times 90\,000}{5.761 \times 0.027} = 3.65 \times 10^6$

$f = 0.015$ . . . . pipe; page A-25

$K = \frac{0.015 \times 400 \times 12}{5.761} = 12.5$ . . . . pipe

$K = 3 \times 14 \times 0.015 = 0.63$ 3 elbows; page A-29

$K_2 = 1.44$ . . . . . . 6 x 4" gate valve; Example 4-4

7. Summarizing K for the entire system (globe valve, pipe, venturi gate valve, and elbows),

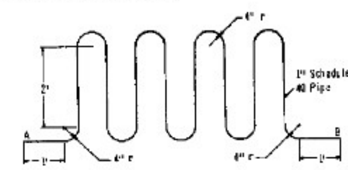
$K = 1.44 + 12.5 + 0.63 + 1.44 = 16$

8. $\Delta P = \frac{28 \times 10^{-8} \times 16 \times 9^2 \times 10^8 \times 1.216}{5.761^4}$

$\Delta P = 40.1$

Example 4-11 . . . Flat Heating Coils—Water

Given: Water at 180 F is flowing through a flat heating coil, shown in the sketch below, at a rate of 15 gallons per minute.



Find: The pressure drop from Point A to B.

Solution:

1. $\Delta P = \frac{18 \times 10^{-5} K \rho Q^2}{d^4}$ . . . . page 3-4

$R_e = \frac{50.6\, Q\rho}{d\mu}$ . . . . page 3-2

$K = f \frac{L}{D}$ . . . . straight pipe; page 3-4

$r/d = 4$ . . . . pipe bends

$K_{90} = 14\, f_T$ . . . . . . 90° bends; page A-29

$K_B = (n-1)\,(.25\,\pi f_T \frac{r}{d} + .5\,K_{90}) + K_{90}$

. . . . . . 180° bends; page A-29

2. $\rho = 60.57$ . . . . water, 180 F; page A-6

$\mu = 0.34$ . . . . water, 180 F; page A-3

$d = 1.049$ . . . . . . 1" Sched. 40 pipe; page B-16

$f_T = 0.023$ . . . . . . 1" Sched. 40 pipe; page A-26

3. $R_e = \frac{50.6 \times 15 \times 60.57}{1.049 \times 0.34} = 1.3 \times 10^5$

$f = 0.024$ . . . . pipe

$K = \frac{0.024 \times 18 \times 12}{1.049} = 4.94$ 18' straight pipe

$K = 2 \times 14 \times 0.023 = 0.64$ . . . two 90° bends

4. For seven 180° bends,

$K_B = 7[(2-1)\,(0.25\pi \times 0.023 \times 4) + (0.5 \times 0.32) + 0.32] = 3.87$

5. $K_{TOTAL} = 4.94 + 0.64 + 3.87 = 9.45$

6. $\Delta P = \frac{18 \times 10^{-5} \times 9.45 \times 60.57 \times 15^2}{1.049^4} = 1.91$

Verification Case 4

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify4.ARO

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-9, example 4-16

GAS: Air

ASSUMPTIONS: Example does not specify the heat transfer conditions, so it was assumed isothermal in this model.

RESULTS:

Parameter	Crane	AFT Arrow
Static pressure drop (psi)	2.61	2.68
Volumetric flow rate at inlet (ft3/min)	20.2	20.2
Volumetric flow rate at outlet (ft3/min)	20.9	20.9
Velocity inlet (ft/min)	3367	3368
Velocity outlet (ft/min)	3483	3485

DISCUSSION:

Crane does not make a distinction between static and stagnation pressure, and it appears that static pressure is usually assumed. Therefore, the inlet pressure of 65 psig was assumed to be static pressure.

Note that the Crane formula underpredicts the pressure drop (by about 3%), which in most applications is not conservative.

List of All Verification Models

Verification Case 4 Problem Statement

Verification Case 4

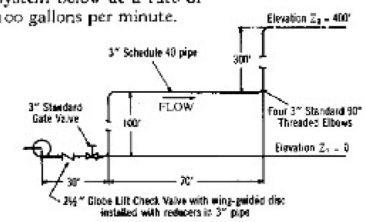
Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-9, example 4-16

Crane Title Page

Pressure Drop and Velocity in Piping Systems — continued

Example 4-15. . . Power Required for Pumping

Given: Water at 70 F is pumped through the piping system below at a rate of 100 gallons per minute.



Find: The total discharge head (H) at flowing conditions and the brake horsepower (bhp) required for a pump having an efficiency (e_p) of 70 per cent.

Solution: 1. Use Bernoulli's theorem (see page 3-2):

$$Z_1 + \frac{144\,P_1}{\rho_1} + \frac{v_1^2}{2g} = Z_2 + \frac{144\,P_2}{\rho_2} + \frac{v_2^2}{2g} + h_L$$

2. Since $\rho_1 = \rho_2$ and $v_1 = v_2$, the equation can be rewritten to establish the pump head, H:

$$\frac{144}{\rho}(P_1 - P_2) - (Z_2 - Z_1) + h_L$$

3. $h_L = \frac{0.00259\,KQ^2}{d^4}$ . . . . page 3-4

$R_e = 123.9\,\frac{d\,v\,\rho}{\mu}$ . . . . page 3-2

$v = \frac{0.408\,Q}{d^2}$ . . . . page 3-2

$bhp = \frac{Q\,H\,\rho}{247\,000\,e_p}$ . . . . page B-9

4. $K = 30 f_T$ . . . . 90° elbow; page A-29

$K_1 = 8 f_T$ . . . . gate valve; page A-27

$K = f\frac{L}{D}$ . . . . straight pipe; page 3-4

$K = 1.0$ . . . . exit; page A-29

5. $d = 3.068$ . . . . . . 3" Sched. 40 pipe; page B-16

$\rho = 62.305$ . . . . page A-6

$\mu = 0.95$ . . . . page A-3

$f_T = 0.018$ . . . . page A-26

6. $v = \frac{0.408 \times 100}{3.068^2} = 4.33$

$R_e = \frac{123.9 \times 3.068 \times 4.33 \times 62.305}{0.95} = 1.1 \times 10^5$

$f = 0.021$ . . . . page A-25

7. $K = 4 \times 30 \times 0.018 = 2.16$ . . four 90° elbows

$K_1 = 8 \times 0.018 = 0.14$ . . . . gate valve

$K = 27.0$ . . . . lift check valve with reducers; Example 4-5

For 500 feet of 3-inch Schedule 40 pipe,

$$K = \frac{0.021 \times 500 \times 12}{3.068} = 41.06$$

And,

$$K_{TOTAL} = 2.16 + 0.14 + 27.0 + 41.06 + 1 = 71.4$$

8. $h_L = \frac{0.00259 \times 71.4 \times 100^2}{3.068^4} = 21$

9. $H = 400 + 21 = \mathbf{421}$

$bhp = \frac{100 \times 421 \times 62.305}{247000 \times 0.70} = \mathbf{15.2}$

Example 4-16. . . Air Lines

Given: Air at 65 psig and 110 F is flowing through 75 feet of 1-inch Schedule 40 pipe at a rate of 100 standard cubic feet per minute (scfm).

Find: The pressure drop in pounds per square inch and the velocity in feet per minute at both upstream and downstream gauges.

Solution: 1. Referring to the table on page B-15, read pressure drop of 2.21 psi for 100 psi, 60 F air at a flow rate of 100 scfm through 100 feet of 1-inch Schedule 40 pipe.

2. Correction for length, pressure, and temperature (page B-15):

$$\Delta P = 2.21\left(\frac{75}{100}\right)\left(\frac{100 + 14.7}{65 + 14.7}\right)\left(\frac{460 + 110}{520}\right)$$

$\Delta P = \mathbf{2.61}$

3. To find the velocity, the rate of flow in cubic feet per minute at flowing conditions must be determined from page B-15.

$$q_m = q'_m\left(\frac{14.7}{14.7 + P}\right)\left(\frac{460 + t}{520}\right)$$

At upstream gauge:

$$q_m = 100\left(\frac{14.7}{14.7 + 65}\right)\left(\frac{460 + 110}{520}\right) = 20.2$$

At downstream gauge:

$$q_m = 100\left[\frac{14.7}{14.7 + (65 - 2.61)}\right]\left(\frac{460 + 110}{520}\right) = 20.9$$

4. $V = \frac{q_m}{A}$ . . . . page 3-2

5. $A = 0.006$ . . . . page B-16

6. $V = \frac{20.2}{0.006} = \mathbf{3367}$ . . . . at upstream gauge

$V = \frac{20.9}{0.006} = \mathbf{3483}$ . . . at downstream gauge

Note: Example 4-16 may also be solved by use of the pressure drop formula and nomograph shown on pages 3-2 and 3-21 respectively or the velocity formula and nomograph shown on pages 3-2 and 3-17 respectively.

Verification Case 5

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify5.ARO

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-11, example 4-18

GAS: Natural Gas (mole fractions: 75% methane, 21% ethane, and 4% propane)

ASSUMPTIONS: Isothermal flow at 40 degrees F

RESULTS:

Parameter	Crane	AFT Arrow
Mass flow rate using standard friction (MMscfd) †	107.8	125.0
Mass flow rate using Weymouth (MMscfd)	105	121.8*
Mass flow rate using Panhandle (MMscfd)	134	120.4*

† *Crane's calculation uses the "Simplified Compressible Flow Formula"*

* *Crane uses the actual Weymouth and Panhandle equations. AFT Arrow does have these equations, but instead solves the governing mass and momentum equations over pipe sections. The AFT Arrow Weymouth and Panhandle solutions above were obtained using the Weymouth and Panhandle friction factor correlation options in AFT Arrow rather than the standard Darcy-Weisbach friction factor (as used in the first case above).*

DISCUSSION:

The mixture properties for this example offer an opportunity to use the Chempak mixture capabilities. The problem statement does not say whether the fractions are on a mass or mole basis, but it does say in Crane that the mixture molecular weight is 20.1. This is consistent with mole fraction. AFT Arrow's output indicates the molecular weight of the mixture is 20.11.

As noted above (*), AFT Arrow has optional friction factor models for Weymouth and Panhandle (see Crane, page 1-8 or AFT Arrow documentation). These were used for the second and third cases above. However, regardless of what friction factor model is used, AFT Arrow differs from any of the three methods above in that it directly solves the governing equations and it does so over pipe segments. In this model, the pipe was broken into 100 segments.

Since it can be easily demonstrated that AFT Arrow's solution satisfies the mass and momentum equations for this pipe, and the solution differs from the Crane solutions, the Crane solutions do not offer a mass and/or momentum balance.

List of All Verification Models

Verification Case 5 Problem Statement

Verification Case 5

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-11, example 4-18

Crane Title Page

CRANE — CHAPTER 4 — EXAMPLES OF FLOW PROBLEMS — 4-11

Pipe Line Flow Problems — continued

Example 4-18 . . . Gas

Given: A natural gas pipe line, made of 14-inch Schedule 20 pipe, is 100 miles long. The inlet pressure is 1300 psia, the outlet pressure is 300 psia, and the average temperature is 40 F.

The gas consists of 75% methane (CH_4), 21% ethane (C_2H_6), and 4% propane (C_3H_8).

Find: The flow rate in millions of standard cubic feet per day (MMscfd).

Solutions: Three solutions to this example are presented for the purpose of illustrating the variations in results obtained by use of the Simplified Compressible Flow formula, the Weymouth formula, and the Panhandle formula.

Simplified Compressible Flow Formula
(see page 3-3)

1. $q'_h = 114.2\sqrt{\left[\frac{(P'_1)^2 - (P'_2)^2}{fL_mTS_g}\right]d^5}$
2. $d = 13.376$ . . . . page B-18
   $d^5 = 428\ 185$
3. $f = 0.0128$ turbulent flow assumed; page A-25
4. $T = 460 + t = 460 + 40 = 500$
5. Approximate atomic weights:
   Carbon . . . . . . C = 12.0
   Hydrogen . . . . H = 1.0
6. Approximate molecular weights:
   Methane (CH_4)
   $M = (1 \times 12.0) + (4 \times 1.0) = 16$
   Ethane (C_2H_6)
   $M = (2 \times 12.0) + (6 \times 1.0) = 30$
   Propane (C_3H_8)
   $M = (3 \times 12.0) + (8 \times 1.0) = 44$
   Natural Gas
   $M = (16 \times 0.75) + (30 \times 0.21) + (44 \times 0.04)$
   $M = 20.06$, or say 20.1
7. $S_g = \frac{M\ (\text{gas})}{M\ (\text{air})} = \frac{20.1}{29} = 0.693$ . . . page 3-5
8. $q'_h = 114.2\sqrt{\frac{(1300^2 - 300^2)\ 428\ 185}{0.0128 \times 100 \times 500 \times 0.693}}$
   $q'_h = 4\ 490\ 000$
9. $q'_d = \left(\frac{4\ 490\ 000\ \text{ft}^3}{1\ 000\ 000\ \text{hr}}\right)\left(\frac{24\ \text{hr}}{\text{day}}\right) = 107.8$
10. $R_e = \frac{0.482\, q'_h S_g}{d\mu}$ . . . . page 3-2
11. $\mu = 0.011$ . . . . . estimated; page A-5
12. $R_e = \frac{0.482 \times 4\ 490\ 000 \times 0.693}{13.376 \times 0.011}$
    $R_e = 10\ 190\ 000$ or 1.019×10^7
13. $f = 0.0128$ . . . . page A-25
14. Since the assumed friction factor ($f = 0.0128$) is correct, the flow rate is **107.8** MMscfd.

If the assumed friction factor were incorrect, it would have to be adjusted and Steps 8, 9, 12, and 13 repeated until the assumed friction factor was in reasonable agreement with that based upon the calculated Reynolds number.

Weymouth Formula
(see page 3-3)

15. $q'_h = 28.0\, d^{2.667}\sqrt{\left[\frac{(P'_1)^2 - (P'_2)^2}{S_g\, L_m}\right]\left(\frac{520}{T}\right)}$
16. $d^{2.667} = 1009$
17. $q'_h = 28.0 \times 1009\sqrt{\frac{(1300^2 - 300^2)}{0.693 \times 100}\left(\frac{520}{500}\right)}$
    $q'_h = 4\ 380\ 000$
18. $q'_d = \left(\frac{4\ 380\ 000\ \text{ft}^3}{1\ 000\ 000\ \text{hr}}\right)\left(\frac{24\ \text{hr}}{\text{day}}\right) = \mathbf{105.1}$

Panhandle Formula
(see page 3-3)

19. $q'_h = 36.8\, E\, d^{2.6182}\left[\frac{(P'_1)^2 - (P'_2)^2}{L_m}\right]^{0.5394}$
20. Assume average operation conditions; then efficiency is 92 per cent:
    $E = 0.92$
21. $d^{2.6182} = 889$
22. $q'_h = 36.8 \times 0.92 \times 889\left(\frac{1300^2 - 300^2}{100}\right)^{0.5394}$
    $q'_h = 5\ 570\ 000$
23. $q'_d = \left(\frac{5\ 570\ 000\ \text{ft}^3}{1\ 000\ 000\ \text{hr}}\right)\left(\frac{24\ \text{hr}}{\text{day}}\right) = \mathbf{133.7}$

Verification Case 6

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify6.ARO

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-13, example 4-20

GAS: Steam

ASSUMPTIONS:

Example does not specify the heat transfer conditions, but it appears from the calculation procedure that adiabatic flow is assumed. The AFT Arrow model assumes adiabatic.

RESULTS:

Parameter	Crane Modified Darcy Formula	Crane Sonic Velocity Formula	AFT Arrow
Mass flow rate (lbm/hr) †	11,780	11,180	11,158
Exit Static Enthalpy (Btu/lbm)	N/A	1,196	1,149
Exit Temperature (deg F)	N/A	317	243

DISCUSSION:

Crane does not make a distinction between static and stagnation pressure, and it appears that static pressure is usually assumed. However, the problem statement is that the source steam comes from a header, where conditions are more likely stagnation. Therefore, stagnation pressure was assumed in the AFT Arrow model.

The Crane Sonic Velocity Formula yields a flow rate prediction that agrees well with the AFT Arrow prediction. The Crane Modified Darcy Formula appears to yield flow rates that are too high. If the inlet conditions in the AFT Arrow model are changed to static, the AFT Arrow flow rate prediction increases slightly to 11,498.4 (lbm/hr). AFT Arrow uses real gas properties for the steam as specified in the Fluid Panel.

The Crane sonic calculation assumes an isenthalpic process, which is why the exit static enthalpy is assumed to be constant at 1,196 Btu/lbm. With this assumption, the exit temperature is 317 deg. F.

However, an isenthalpic assumption turns out to be poor when the Energy Equation is applied. From the First Law,

$$\dot{q} = \dot{m}\left(h_1 + \frac{V_1^2}{2} - h_2 - \frac{V_2^2}{2}\right)$$

If the process is adiabatic, the heat transfer is zero and therefore,

$$h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2}$$

Thus, the static enthalpy will drop because of the velocity increase. When this is accounted for, the exit static enthalpy decreases to 1149. This yields an exit static temperature of 243.0 deg. F, which is 74 degrees cooler.

List of All Verification Models

Verification Case 6 Problem Statement

Verification Case 6

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-13, example 4-20

Crane Title Page

CRANE   CHAPTER 4 — EXAMPLES OF FLOW PROBLEMS   4-13

Discharge of Fluids from Piping Systems — continued

Example 4-20...Steam at Sonic Velocity

Given: A header with 170 psia saturated steam is feeding a pulp stock digester through 30 feet of 2-inch Schedule 40 pipe which includes one standard 90 degree elbow and a fully-open conventional plug type disc globe valve. The initial pressure in the digester is atmospheric.

Find: The initial flow rate in pounds per hour, using both the modified Darcy formula and the sonic velocity and continuity equations.

Solutions—for theory, see page 1-9:

Modified Darcy Formula

1. $W = 1891\, Y d^2 \sqrt{\frac{\Delta P}{K \overline{V}_1}}$ .... page 3-4

   $K = f \frac{L}{D}$ .... pipe; page 3-4

2. $K_1 = 340 f_T$ .... globe valve; page A-27

   $K = 30 f_T$ .... 90° elbow; page A-29

   $K = 0.5$ ..... entrance from header; page A-29

   $K = 1.0$ .... exit to digester; page A-29

3. $k = 1.297$ or say 1.3 .... page A-9

   $d = 2.067$ $d^2 = 4.272$ ... 2" pipe; page B-16

   $f_T = 0.019$ .... page A-26

   $\overline{V}_1 = 2.6738$ .... page A-14

4. $K = \frac{0.019 \times 30 \times 12}{2.067} = 3.31$ .... 30 feet, 2" pipe

   $K_1 = 340 \times 0.019 = 6.46$ ...... 2" globe valve

   $K = 30 \times 0.019 = 0.57$ .... 2" 90° elbow

   and, for the entire system,

   $K = 3.31 + 6.46 + 0.57 + 0.5 + 1.0 = 11.84$

5. $\frac{\Delta P}{P'_1} = \frac{170 - 14.7}{170} = \frac{155.3}{170} = 0.914$

6. Using the chart on page A-22 for $k = 1.3$, it is found that for $K = 11.84$, the maximum $\Delta P / P'_1$ is 0.785 (interpolated from table on page A-22). Since $\Delta P / P'_1$ is less than indicated in Step 5, sonic velocity occurs at the end of the pipe, and ΔP in the equation of Step 1 is:

   $\Delta P = 0.785 \times 170 = 133.5$

7. $Y = 0.710$ .... {interpolated from table, page A-22}

8. $W = 1891 \times 0.71 \times 4.272 \sqrt{\frac{133.5}{11.84 \times 2.6738}}$

   $W = 11\,780$

Sonic Velocity and Continuity Equations

9. $v_s = \sqrt{k g\, 144\, P' \overline{V}}$ .... page 3-3

   $W = \frac{v d^2}{0.0509\, \overline{V}}$ ..... Equation 3-2; page 3-2

10. $P' = P'_1 - \Delta P$

    $P' = 170 - 133.5 = 36.5$

    ΔP determined in Step 6.

11. $h_g = 1196$ .. 170 psia saturated steam; page A-14

12. At 36.5 psia, the temperature of steam with total heat of 1196 Btu/lb equals 317 F, and $\overline{V} = 12.4$ .... pages A-13 and A-16

13. $v_s = \sqrt{1.3 \times 32.2 \times 144 \times 36.5 \times 12.4}$

    $v_s = 1652$

    $W = \frac{1652 \times 4.272}{0.0509 \times 12.4} = 11\,180$

NOTE

In Steps 11 and 12 constant total heat h_g is assumed. But the increase in specific volume from inlet to outlet requires that the velocity must increase. Source of the kinetic energy increase is the internal heat energy of the fluid. Consequently, the heat energy actually decreases toward the outlet. Calculation of the correct h_g at the outlet yields a flow rate commensurate with the answer in Step 8.

Verification Case 7

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify7.ARO

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-14, example 4-22

GAS: Air

ASSUMPTIONS: Example does not specify the heat transfer conditions. The AFT Arrow model assumes adiabatic.

RESULTS:

Parameter	Crane	AFT Arrow
Mass flow rate(scfm)	62.7	63.5

DISCUSSION:

Crane does not make a distinction between static and stagnation pressure, and it appears that static pressure is usually assumed. From the problem description, the static pressure is clearly appropriate.

The predicted flow rates agree very closely.

The Crane prediction indicates that this pipe will have subsonic velocity at the exit and hence will not choke. However, a more proper formulation of this problem shows a different mass flow will occur. This is a good example of the limitations of simplified methods such as Crane. The discrepancy comes from how to handle the exit loss of the air as it discharges to atmosphere. The Crane solution takes the appropriate K factor, equal to 1, and lumps it together with the pipe friction to obtain an overall K factor of 7.04. However, if the K factor is applied at the discharge tank and not averaged along the pipe, the mass flow differs from that calculated by Crane. The predicted flow rate using this method is 64.4 scfm. This is not drastically different from Crane's prediction, and well within typical engineering uncertainty. But the difference, which is small here, could be larger in other applications.

List of All Verification Models

Verification Case 7 Problem Statement

Verification Case 7

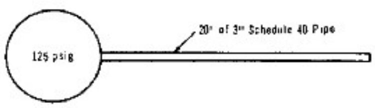
Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-14, example 4-22

Crane Title Page

Discharge of Fluids from Piping Systems — continued

Example 4-21 . . . Gases at Sonic Velocity

Given: Coke oven gas having a specific gravity of 0.42, a header pressure of 125 psig, and a temperature of 140 F is flowing through 20 feet of 3-inch Schedule 40 pipe before discharging to atmosphere. Assume ratio of specific heats, $k = 1.4$.



Find: The flow rate in standard cubic feet per hour (scfh).

Solution—for theory, see page 1-9:

1. $q'_h = 40\,700\, Y d^2 \sqrt{\dfrac{\Delta P\, P'_1}{K\, T_1\, S_g}}$ . . . . page 3-4

   $K = f\dfrac{L}{D}$ . . . . page 3-4

2. $P'_1 = 125 + 14.7 = 139.7$

3. $f = 0.0175$ . . . . page A-25

   Note: The Reynolds number need not be calculated since gas discharged to atmosphere through a short pipe will have a high R_e, and flow will always be in a fully turbulent range, in which the friction factor is constant.

4. $d = 3.068 \quad d^2 = 9.413$ . . . . . . page B-16

   $D = 0.2557$

5. $K = f\dfrac{L}{D} = \dfrac{0.0175 \times 20}{0.2557} = 1.369$ . . for pipe

   $K = 0.5$ . . . . for entrance; page A-29

   $K = 1.0$ . . . . for exit; page A-29

   $K = 1.369 + 0.5 + 1.0 = 2.87$ . . . . . total

6. $\dfrac{\Delta P}{P'_1} = \dfrac{139.7 - 14.7}{139.7} = \dfrac{125.0}{139.7} = 0.895$

7. Using the chart on page A-22 for $k = 1.4$, it is found that for $K = 2.87$, the maximum $\Delta P / P'_1$ is 0.657 (interpolated from table on page A-22). Since $\Delta P / P'_1$ is less than indicated in Step 6, sonic velocity occurs at the end of the pipe and ΔP in Step 1 is:

   $\Delta P = 0.657\, P'_1 = 0.657 \times 139.7 = 91.8$

8. $T_1 = 140 + 460 = 600$

9. $Y = 0.637$ . . . . {interpolated from table; page A-22

10. q'_h is equal to:

    $40\,700 \times 0.637 \times 9.413 \sqrt{\dfrac{91.8 \times 139.7}{2.87 \times 600 \times 0.42}}$

    $q'_h = \mathbf{1\,028\,000}$

Example 4-22 . . . Compressible Fluids at Subsonic Velocity

Given: Air at a pressure of 19.3 psig and a temperature of 100 F is measured at a point 10 feet from the outlet of a ½-inch Schedule 80 pipe discharging to atmosphere.

Find: The flow rate in standard cubic feet per minute (scfm).

Solution:

1. $q'_m = 678\, Y d^2 \sqrt{\dfrac{\Delta P\, P'_1}{K\, T_1\, S_g}}$ . . . . . . page 3-4

   $K = f\dfrac{L}{D}$ . . . . page 3-4

2. $P'_1 = 19.3 + 14.7 = 34.0$

3. $\Delta P = 19.3$

4. $d = 0.546 \quad d^2 = 0.2981$ . . . . page B-16

   $D = 0.0455$

5. $f = 0.0275$ . . fully turbulent flow; page A-25

6. $K = f\dfrac{L}{D} = \dfrac{0.0275 \times 10}{0.0455} = 6.04$ . . for pipe

   $K = 1.0$ . . . . for exit; page A-29

   $K = 6.04 + 1 = 7.04$ . . . . total

7. $\dfrac{\Delta P}{P'_1} = \dfrac{19.3}{34.0} = 0.568$

8. $Y = 0.76$ . . . . page A-22

9. $T_1 = 460 + t_1 = 460 + 100 = 560$

10. $q'_m = 678 \times 0.76 \times 0.2981 \sqrt{\dfrac{19.3 \times 34.0}{7.04 \times 560 \times 1.0}}$

    $q'_m = \mathbf{62.7}$

Verification Case 8

[Problem Statement](#)

PRODUCT: AFT Arrow

MODEL FILE: AroVerify8.ARO

REFERENCE: Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, Third Edition, John Wiley & Sons, 1985, Pages 632-633, example 12.8

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas.

RESULTS:

Parameter	Fox & McDonald	AFT Arrow
L_{1-3} – length to choking (meters)	4.99	4.99
V_1– Velocity at point 1 (m/s)	65.3	65.2
T_1– Temperature at point 1 (deg. K)	294	294
M_1– Mach number at point 1	0.19	0.19
M_2 – Mach number at point 2	0.4	0.4
P_2– Pressure at point 1 (mm Hg gage)	-18.9	-18.9
L_{1-2} – length to measured pressure (meters)	4.29	4.29*

** AFT Arrow does not have the ability to solve for pipe length, so the length was input. With this known length, the AFT Arrow Mach number at M_2 should agree with Fox & McDonald's, and it does. The resulting mass flow rate is then used as input for AFT Arrow pipe #2.*

DISCUSSION:

The predictions agree very closely.

The two pipes in the AFT Arrow model represent the solutions to stations 2 and 3.

Note that the friction factor in Fox & McDonald is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

It should also be noted that, from time to time, AFT finds it is necessary to modify the Solver used by Arrow to improve application performance, or for other reasons. These modifications to the Solver may cause slight changes to the appropriate pipe lengths determined by Arrow.

[List of All Verification Models](#)

Verification Case 8 Problem Statement

Verification Case 8

Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, Third Edition, John Wiley & Sons, 1985, Pages 632-633, example 12.8

Fox and McDonald Title Page

The ratio of local stagnation pressure to the reference stagnation pressure is given by

$$\frac{p_0}{p_0^*} = \frac{p_0}{p}\frac{p}{p^*}\frac{p^*}{p_0^*}$$

$$= \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)} \frac{1}{M}\left[\frac{\left(\frac{k+1}{2}\right)}{1 + \frac{k-1}{2}M^2}\right]^{1/2} \frac{1}{\left(1 + \frac{k-1}{2}\right)^{k/(k-1)}}$$

or

$$\frac{p_0}{p_0^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/2(k-1)} \qquad (12.18e)$$

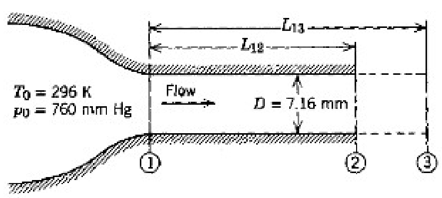
The ratios in Eqs. 12.18 are tabulated as functions of Mach number in Table D.2 of Appendix D.

Example 12.8

Air flow is induced in a smooth insulated tube of 7.16 mm diameter by a vacuum pump. The air is drawn from a room where the absolute pressure is 760 mm Hg and the temperature is 23 C through a smoothly contoured, converging nozzle. At section ①, where the nozzle joins the constant-area tube, the static gage pressure is −18.9 mm Hg. At section ②, located some distance downstream in the constant-area tube, the static pressure is −412 mm Hg (gage). The duct walls are smooth; the average friction factor, $\bar{f}$, may be taken as the value at section ①. Determine the length of duct required for choking from section ①, the Mach number at section ②, and the duct length, L_{12}, between sections ① and ②.

EXAMPLE PROBLEM 12.8

GIVEN: Air flow (with friction) in an insulated constant-area tube.



Gage pressures: $p_1 = -18.9$ mm Hg, $p_2 = -412$ mm Hg. $M_3 = 1.0$

FIND: (a) L_{13}. (b) M_2. (c) L_{12}.

SOLUTION:

Flow in the constant-area tube is frictional and adiabatic, a Fanno line flow. To find the friction factor, we need to know the flow conditions at section ①. If it is assumed that flow in the nozzle is isentropic, local properties at the nozzle exit may be computed using isentropic relations. Thus

$$\frac{p_{0_1}}{p_1} = \left(1 + \frac{k-1}{2} M_1^2\right)^{k/(k-1)}$$

Solving for M_1 we obtain

$$M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_{0_1}}{p_1}\right)^{(k-1)/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{760}{760-18.9}\right)^{0.286} - 1\right]\right\}^{1/2} = 0.190$$

$$T_1 = \frac{T_{0_1}}{1 + \frac{k-1}{2} M_1^2} = \frac{296\text{ K}}{1 + 0.2(0.190)^2} = 294\text{ K}$$

$$V_1 = M_1 c_1 = M_1\sqrt{kRT_1} = 0.190\left[1.4 \times 287\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 294\text{ K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{sec}^2}\right]^{1/2}$$

$$V_1 = 65.3\text{ m/sec}$$

$$p_1 = g\rho_{\text{Hg}} h_1 = g\text{SG}\rho_{\text{H}_2\text{O}} h_1$$

$$= 9.81\,\frac{\text{m}}{\text{sec}^2} \times 13.6 \times 999\,\frac{\text{kg}}{\text{m}^3} \times (760 - 18.9)10^{-3}\text{ m} \times \frac{\text{N}\cdot\text{sec}^2}{\text{kg}\cdot\text{m}}$$

$$p_1 = 98.8\text{ kPa (abs)}$$

$$\rho_1 = \frac{p_1}{RT_1} = 9.88 \times 10^4\,\frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\text{ N}\cdot\text{m}} \times \frac{1}{294\text{ K}} = 1.17\text{ kg/m}^3$$

At $T = 294$ K (21 C), $\mu = 1.9 \times 10^{-5}$ kg/m · sec from Fig. A.2, Appendix A. Thus

$$Re_1 = \frac{\rho_1 V_1 D_1}{\mu_1} = 1.17\,\frac{\text{kg}}{\text{m}^3} \times 65.3\,\frac{\text{m}}{\text{sec}} \times 0.00716\text{ m} \times \frac{\text{m}\cdot\text{sec}}{1.9 \times 10^{-5}\text{ kg}} = 2.88 \times 10^4$$

From Fig. 8.14, for smooth pipe, $f = 0.0235$. From Table D.2 at $M_1 = 0.19$, $p/p^* = 5.745$, and $\bar{f}L_{max}/D_h = 16.38$. Thus, assuming $\bar{f} = f_1$,

$$L_{13} = (L_{max})_1 = \left(\frac{\bar{f}L_{max}}{D_h}\right)_1 \frac{D_h}{f_1} = 16.38 \times 0.00716\text{ m} \times \frac{1}{0.0235} = 4.99\text{ m}$$ L_{13}

Since p^* is constant for Fanno line flow, conditions at section ② can be determined from the pressure ratio, $(p/p^*)_2$. Thus

$$\left(\frac{p}{p^*}\right)_2 = \frac{p_2}{p^*} = \frac{p_2}{p_1}\frac{p_1}{p^*} = \frac{p_2}{p_1}\left(\frac{p}{p^*}\right)_1 = \left(\frac{760-412}{760-18.9}\right) 5.745 = 2.698$$

From Table D.2, at $(p/p^*)_2 = 2.698$, $M_2 \simeq 0.40$ M_2

At $M_2 = 0.40$, $\bar{f}L_{max}/D_h = 2.309$ (Table D.2). Thus

$$L_{23} = (L_{max})_2 = \left(\frac{\bar{f}L_{max}}{D_h}\right)_2 \frac{D_h}{f_1} = 2.309 \times 0.00716\text{ m} \times \frac{1}{0.0235} = 0.704\text{ m}$$

Finally,

$$L_{12} = L_{13} - L_{23} = (4.99 - 0.704)\text{ m} = 4.29\text{ m}$$ L_{12}

{This is the same physical system as that analyzed in Example Problem 12.7. Use of the tables simplifies the calculations and makes it possible to determine the duct length.}

Verification Case 9

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify9.ARO

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, Seventh Edition, Professional Publications, Belmont, CA, 1984, Pages 8-11, 8-12, example 8.12

GAS: Unspecified except that the *k* value (i.e., γ) is 1.4

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas, 3) The gas is air, but for purposes of the example it only matters that the gas has k = 1.4, 4) No temperature was specified, so assume 70 deg. F

RESULTS:

Parameter	Lindeburg	AFT Arrow
M_2 – Mach number at exit	0.35	0.36
P_2– Pressure at exit (psia)	10.26	10.09

DISCUSSION:

As specified, inlet conditions are known and outlet conditions need to be determined. With the known inlet conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the exit.

The problem states that the inlet Mach number is 0.3, P_1 = 12 psia, T_1 = 70 F (assumed). From the ideal gas law, density, sonic speed and mass flow rate are:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{12\ psia}{(53.34\ \frac{ft*lbf}{lbm*R})(529.67\ R)} = 0.6116\ lbm/ft^3$$

$$a_1 = \sqrt{\gamma RT} = \sqrt{1.4(53.34\ \frac{ft*lbf}{lbm*R})529.67\ R} = 1128.1\frac{ft}{s}\ (sonic\ velocity)$$

$$\dot{m} = \rho_1 V_1 A = \rho_1 (M_1 a_1) A = (0.6116\ \frac{lbm}{ft^3})(0.3)(1128.1\ \frac{ft}{s})(\frac{\pi}{4}0.3^2\ ft^2) = 1.463\ \frac{lbm}{s}$$

With this flow rate at the exit, the predictions agree very closely.

Note that the friction factor in Lindeburg is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

List of All Verification Models

Verification Case 9 Problem Statement

Verification Case 9

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, Seventh Edition, Professional Publications, Belmont, CA, 1984, Pages 8-11, 8-12, example 8.12

Lindeburg Title Page

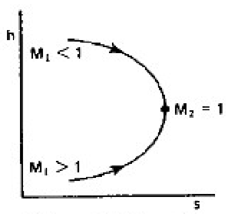


Figure 8.9 Fanno Line

The *Fanno line* is a plot of enthalpy versus entropy, derived from the energy conservation law, continuity equation, and the change in entropy relationships. Figure 8.9 illustrates the Fanno line, showing how velocity tends towards $M_2 = 1$, regardless of M_1.

With Fanno flow and an initially subsonic flow at point 1, the pressure drops as x increases. Since the density also drops, the velocity increases.[3] However, since the pressure drop is proportional to the square of the velocity, the rate of change of pressure (dp/dx) increases. Eventually, as p_{T_2} is lowered sufficiently, the velocity in the duct becomes sonic, after which no increase in velocity or mass flow can occur. Lowering p_{T_2} further will have no effect on the choked flow.[4]

Because the flow is adiabatic, the total temperature remains constant. Thus, equation 8.18 can be used to find the temperature at any point in the duct. The ratio of static pressure is given by equation 8.37.

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1}\left(1 + \left[\tfrac{1}{2}(k-1)M_1^2\right]\left[1 - \left(\frac{\rho_1}{\rho_2}\right)^2\right]\right) \qquad 8.37$$

For round ducts, the *hydraulic diameter*, D_H, is equal to the inside diameter. x_{max} is the maximum distance required for M to become unity.[5]

$$\frac{4fx_{max}}{D_H} = \left(\frac{1-M^2}{kM^2}\right) + \left(\frac{k+1}{2k}\right) \times ln\left(\frac{(k+1)M^2}{2\left[1+\frac{1}{2}(k-1)M^2\right]}\right) \qquad 8.38$$

The length of duct required to change M_1 to any other value, M_2, is

$$x = \frac{D_H}{4f}\left[\left(\frac{4fx_{max}}{D_H}\right)_1 - \left(\frac{4fx_{max}}{D_H}\right)_2\right] \qquad 8.39$$

[3] If the velocity in the duct is initially supersonic, the velocity will decrease, approaching $M = 1$.

[4] Notice that the total pressure is not constant in Fanno flow. However, since the flow is adiabatic, the total temperature does not change.

[5] The *Fanning friction factor*, f, also is known as the *coefficient of friction*, C_f. Multiplying the Fanning friction factor by 4, as has been done in figure 8.10, converts it to the *Darcy friction factor*.

The static gas properties at any point are related to the sonic properties according to equations 8.40, 8.41, and 8.42.

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{k+1}{2\left[1+\frac{1}{2}(k-1)M^2\right]}} \qquad 8.40$$

$$\frac{T}{T^*} = \frac{k+1}{2\left[1+\frac{1}{2}(k-1)M^2\right]} \qquad 8.41$$

$$\frac{p_T}{p_T^*} = \frac{1}{M}\left(\frac{2}{k+1}\left[1+\tfrac{1}{2}(k-1)M^2\right]\right)^{\frac{k+1}{2(k-1)}} \qquad 8.42$$

These equations are useful only in the simplest of problems. Tabulated or graphed values (as in figure 8.10) are required in most situations.

With Fanno flow and an initially sonic flow at point 1, the friction causes the pressure, density, and temperature to increase and the velocity to decrease. The frictional effects cause the fluid to reach $M = 1$ at the exit.

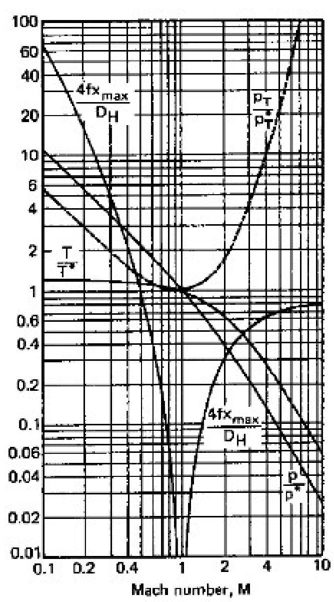


Figure 8.10 Fanno Flow Relationships, $k = 1.4$

Example 8.12

Gas $(k = 1.4)$ enters an adiabatic, constant-area 0.3 foot diameter duct at 12 psia and $M = .3$. If the

Fanning friction factor is .003, what is the Mach number and pressure 50 feet downstream?

From figure 8.10 or equation 8.38, the value of $4fx_{max}/D_H$ is found to be 5.3.

50 feet downstream,

$$\frac{4fx}{D_H} = \frac{(4)(.003)(50)}{.3} = 2.0$$

From equation 8.39,

$$\left(\frac{4fx_{max}}{D_H}\right)_2 = 5.3 - 2.0 = 3.3$$

Solving for M_2 from equation 8.38 is not easy, but M_2 can be found from figure 8.10. It is approximately .35. At this speed, p/p^* (from the chart or equation 8.40) is approximately 3.09. At the entrance, p/p^* is 3.62, or

$$p^* = \frac{12}{3.62} = 3.32$$

Therefore, $p_2 = (3.32)(3.09) = 10.26$ psia.

Example 8.13

A duct (D_i = 2.0 inches, f = .005) receives air at M = 2. The total pressure and temperature at the entrance are 78.25 psia and 1080°R.

(a) Find the temperature and pressure at the entrance.

(b) Find the total temperature, total pressure, static temperature, and static pressure at a point where M = 1.75.

(c) Find the distance between the points where $M = 2$ and $M = 1.75$.

(a) From the isentropic flow table for $M = 2$,

$$T_1 = (.5556)(1080) = 600°R$$
$$p_1 = (.1278)(78.25) = 10 \text{ psia}$$

(b) Although it isn't known if the duct is long enough for choked flow ($M = 1$) to exist, that point still can be used as a reference point. The following values are read from a Fanno flow table. (Figure 8.10 can be used also.)

	M = 2.0	M = 1.75
$\frac{p^*}{p}$	.40825	.49295
$\frac{p_T}{p_T^*}$	1.6875	1.3865
$\frac{T}{T^*}$	.66667	.74419
$\frac{4fL}{D}$	.30499	.22504

$T_{T1} = T_{T2} = 1080°$R since the flow is adiabatic.

$$T_2 = T_1\left(\frac{T_2}{T^*}\right)\left(\frac{T^*}{T_1}\right) = (600)\left(\frac{.74419}{.66667}\right) = 670°R$$

$$p_2 = p_1\left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_1}\right) = (10)\left(\frac{.49295}{.40825}\right) = 12.1 \text{ psia}$$

$$p_{T2} = p_{T1}\left(\frac{p_{T2}}{p_T^*}\right)\left(\frac{p_T^*}{p_{T1}}\right) = (78.25)\left(\frac{1.3865}{1.6875}\right) = 64.3 \text{ psia}$$

(c) At $M = 2$, the distance to reach $M = 1$ is

$$L_1 = \frac{(.30499)(2)}{(4)(.005)} = 30.5 \text{ inches}$$

At $M = 1.75$, the distance to reach $M = 1$ is

$$L_2 = \frac{(.22504)(2)}{(4)(.005)} = 22.5 \text{ inches}$$

The distance between points 1 and 2 is

$$L = L_1 - L_2 = 30.5 - 22.5 = 8 \text{ inches}$$

10 ISOTHERMAL FLOW WITH FRICTION

As the Mach number approaches unity, an infinite heat-transfer rate is required to keep the flow isothermal. That is why the gas flow is closer to adiabatic than to isothermal. A notable exception is encountered with long-distance pipelines where the earth supplies the needed heat to keep the flow isothermal.[8]

For any given pressure drop, the mass flow rate is given by the *Weymouth equation*.

$$\dot{m} = \sqrt{\frac{(p_1^2 - p_2^2)\,D^5 g_c(\pi/4)^2}{4fxRT}} \qquad 8.43$$

The Fanning friction factor, f, in the Weymouth equation is assumed to be given by equation 8.44.

$$f = \frac{.00349}{(D)^{.333}} \qquad 8.44$$

Example 8.14

A 40 inch (inside diameter) pipe is placed into service carrying natural gas between pumping stations 75 miles apart. The gas leaves the pumping stations at 650 psia, but the pressure drops to 450 psia by the time it reaches the next station. The gas temperature remains constant at 40°F. Assume the gas is methane (MW=16).

[8]The low velocities (less than 20 ft/sec) typical of natural gas pipelines hardly seem appropriate in this chapter. However, the gas experiences a density change along the length of pipe.

Verification Case 10

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify10.ARO

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, Seventh Edition, Professional Publications, Belmont, CA, 1984, Pages 8-12, 8-13, example 8.13b

GAS: Methane

ASSUMPTIONS: 1) Isothermal flow, 2) Perfect gas

RESULTS:

Parameter	Lindeburg	AFT Arrow
Mass flow rate (lbm/s)	456	457

DISCUSSION:

The predictions agree closely. Part "a" makes a comparison to the Bernoulli equation, which Arrow does not solve. So part “a” was skipped.

Note that the friction factor in Lindeburg is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

List of All Verification Models

Verification Case 10 Problem Statement

Verification Case 10

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, Seventh Edition, Professional Publications, Belmont, CA, 1984, Pages 8-12, 8-13, example 8.13b

Lindeburg Title Page

Fanning friction factor is .003, what is the Mach number and pressure 50 feet downstream?

From figure 8.10 or equation 8.38, the value of $4fx_{max}/D_H$ is found to be 5.3.

50 feet downstream,

$$\frac{4fx}{D_H} = \frac{(4)(.003)(50)}{.3} = 2.0$$

From equation 8.39,

$$\left(\frac{4fx_{max}}{D_H}\right)_2 = 5.3 - 2.0 = 3.3$$

Solving for M_2 from equation 8.38 is not easy, but M_2 can be found from figure 8.10. It is approximately .35. At this speed, p/p^* (from the chart or equation 8.40) is approximately 3.09. At the entrance, p/p^* is 3.62, or

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A duct (D_i = 2.0 inches, f = .005) receives air at M = 2. The total pressure and temperature at the entrance are 78.25 psia and 1080°R.

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$$T_1 = (.5556)(1080) = 600°R$$
$$p_1 = (.1278)(78.25) = 10 \text{ psia}$$

(b) Although it isn't known if the duct is long enough for choked flow ($M = 1$) to exist, that point still can be used as a reference point. The following values are read from a Fanno flow table. (Figure 8.10 can be used also.)

	M = 2.0	M = 1.75
$\frac{p^*}{p}$	.40825	.49295
$\frac{p_T}{p_T^*}$	1.6875	1.3865
$\frac{T}{T^*}$	.66667	.74419
$\frac{4fL}{D}$	.30499	.22504

$T_{T1} = T_{T2} = 1080°R$ since the flow is adiabatic.

$$T_2 = T_1\left(\frac{T_2}{T^*}\right)\left(\frac{T^*}{T_1}\right) = (600)\left(\frac{.74419}{.66667}\right) = 670°R$$

$$p_2 = p_1\left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_1}\right) = (10)\left(\frac{.49295}{.40825}\right) = 12.1 \text{ psia}$$

$$p_{T2} = p_{T1}\left(\frac{p_{T2}}{p_T^*}\right)\left(\frac{p_T^*}{p_{T1}}\right) = (78.25)\left(\frac{1.3865}{1.6875}\right) = 64.3 \text{ psia}$$

(c) At $M = 2$, the distance to reach $M = 1$ is

$$L_1 = \frac{(.30499)(2)}{(4)(.005)} = 30.5 \text{ inches}$$

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The distance between points 1 and 2 is

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10 ISOTHERMAL FLOW WITH FRICTION

As the Mach number approaches unity, an infinite heat-transfer rate is required to keep the flow isothermal. That is why the gas flow is closer to adiabatic than to isothermal. A notable exception is encountered with long-distance pipelines where the earth supplies the needed heat to keep the flow isothermal.[6]

For any given pressure drop, the mass flow rate is given by the *Weymouth equation*.

$$\dot{m} = \sqrt{\frac{(p_1^2 - p_2^2)D^5 g_c(\pi/4)^2}{4fxRT}} \qquad 8.43$$

The Fanning friction factor, f, in the Weymouth equation is assumed to be given by equation 8.44.

$$f = \frac{.00349}{(D)^{.333}} \qquad 8.44$$

Example 8.14

A 40 inch (inside diameter) pipe is placed into service carrying natural gas between pumping stations 75 miles apart. The gas leaves the pumping stations at 650 psia, but the pressure drops to 450 psia by the time it reaches the next station. The gas temperature remains constant at 40°F. Assume the gas is methane (MW=16).

[6] The low velocities (less than 20 ft/sec) typical of natural gas pipelines hardly seem appropriate in this chapter. However, the gas experiences a density change along the length of pipe.

Use the Bernoulli equation and the discharge conditions to calculate the flow rate. Compare the Bernoulli flow rate with the flow rate predicted by the Weymouth equation.

The pipe diameter and area are

$$D = \frac{40}{12} = 3.333$$
$$A = \frac{\pi}{4}(3.333)^2 = 8.73 \text{ ft}^2$$

The pressures in psf are

$$p_1 = (650)(144) = 9.36 \text{ EE4 psf}$$
$$p_2 = (450)(144) = 6.48 \text{ EE4 psf}$$
$$\rho_1 = \frac{p}{RT} = \frac{9.36 \text{ EE4}}{(1545/16)(40+460)}$$
$$= 1.938 \text{ lbm/ft}^3$$

(a) The entire pressure drop is due to friction. Neglecting the low kinetic energy and the changes to potential energy, the Bernoulli equation based on the pump discharge conditions is

$$\frac{9.36 \text{ EE4}}{1.938} = \frac{6.48 \text{ EE4}}{1.938} + h_f$$
$$h_f = 14,861 \text{ ft}$$

Assume $f_{\text{Darcy}} = 0.015$. Then,

$$h_f = \frac{fLv^2}{2Dg}$$
$$14,861 = \frac{(0.015)(75)(5280)v^2}{(2)(3.333)(32.2)}$$
$$v = 23.17 \text{ ft/sec}$$

The mass flow rate is

$$\dot{m} = \rho v A = (1.938)(23.17)(8.73) = 392 \text{ lbm/sec}$$

(b) The Fanning friction factor from equation 8.44 is

$$f = \frac{.00349}{(3.333)^{.333}} = .00234$$

From equation 8.43,

$$\dot{m} = \sqrt{\frac{[(9.36 \text{ EE4})^2 - (6.48 \text{ EE4})^2](3.333)^5(32.2)\left(\frac{\pi}{4}\right)^2}{(4)(0.00234)(75)(5280)(1545/16)(460+40)}}$$
$$= 456 \text{ lbm/sec}$$

11 FRICTIONLESS FLOW WITH HEATING

If heat is added to a fluid flowing without friction through a constant-area duct, the flow is said to be a *Rayleigh flow*.[7] The heat comes from chemical reactions, phase changes, electrical heating, or external sources. (Frictional heating is not Rayleigh flow.) The usual conservation of energy, mass, and momentum equations hold. The following ratios of static to critical values can be calculated and plotted. (See figure 8.11.)

$$\dot{q} = c_p(T_{T2} - T_{T1}) \quad 8.45$$

$$\frac{T}{T^*} = \left(\frac{1+k}{1+kM^2}\right)^2 M^2 \quad 8.46$$

$$\frac{T_T}{T_T^*} = \frac{2(k+1)M^2[1+\frac{1}{2}(k-1)M^2]}{(1+kM^2)^2} \quad 8.47$$

$$\frac{p}{p^*} = \frac{1+k}{1+kM^2} \quad 8.48$$

$$\frac{p_T}{p_{T^*}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \left(\frac{1+k}{1+kM^2}\right) \times \left[1+\tfrac{1}{2}(k-1)M^2\right]^{\frac{k}{k-1}} \quad 8.49$$

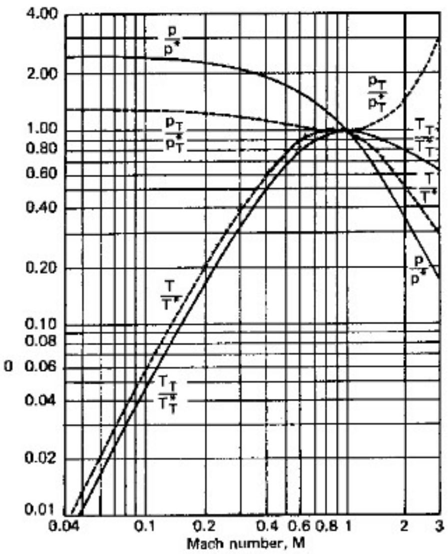

Figure 8.11 Rayleigh Flow Parameters

For Rayleigh flow, the Mach number approaches unity for both subsonic and supersonic flow. From the T/T^* curve, it is apparent that the maximum temperature occurs at $M = \sqrt{k}$. Also, the total pressure always decreases, even while the heat is being added. The *Rayleigh line* is plotted in figure 8.12.

[7]This is known as *diabatic flow*—flow *with* heating.

Verification Case 11

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify11.ARO

REFERENCE: Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Pages 213-215, example 5.2

GAS: Air

ASSUMPTIONS: 1) Adiabatic, 2) Perfect gas

RESULTS:

Parameter	Saad	AFT Arrow
M_2 – Mach number at exit	0.685	0.686
M_1 – Mach number at inlet	0.347	0.346
P_1– Static pressure at inlet (kPa)	306	308
T_1– Static temperature at inlet (deg. K)	312.76	312.9

DISCUSSION:

As specified, exit conditions are known and inlet conditions need to be determined for specified volume flow at exit. With the known exit conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the inlet.

The problem states that the exit volume flow rate, Q_2, is 1000 m^3/min, P_2 = 150 kPa, T_2 = 293 K. From the ideal gas law, density, and mass flow rate are:

$$\rho_2 = \frac{P_2}{RT_2} = \frac{150\ kPa}{\left(0.2868\ \frac{kJ}{kg \cdot K}\right)(293\ K)} = 1.784\ kg/m^3$$

$$\dot{m} = \rho_2 V_2 A = \rho_2 Q_2 = \left(1.784\ \frac{kg}{m^3}\right)\left(1000\ \frac{m^3}{min}\right) = 29.73\ \frac{kg}{s}$$

In AFT Arrow, the discharge pressure can be specified. The temperature can be specified at the exit junction, but the actual discharge is what is displayed for the pipe exit. The pipe exit temperature depends on the inlet temperature and the thermodynamic process in the pipe, which is adiabatic. Therefore, to solve this problem the inlet static temperature at J1 must be guessed until the pipe delivers 293 K at the exit. This results in the 312.9 K displayed in the above table.

All results agree closely.

Note that the friction factor in Saad is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

List of All Verification Models

Verification Case 11 Problem Statement

Verification Case 11

Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Pages 213-215, example 5.2

Saad Title Page

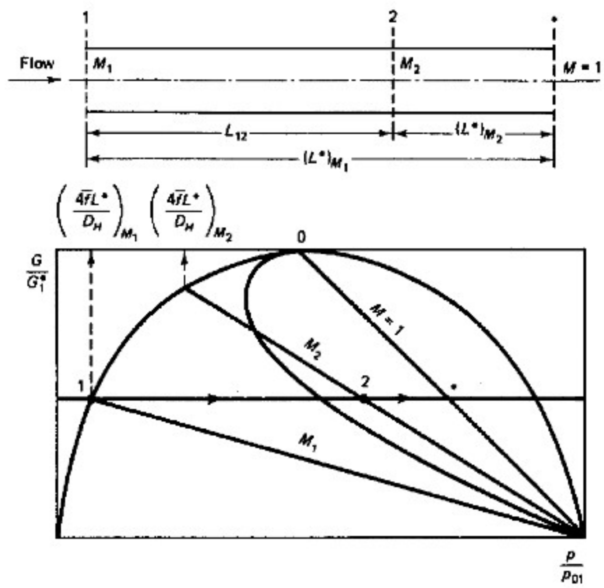

Figure 5.5 Duct length necessary for choking.

Therefore the distance between sections 1 and 2 is:

$$L_{1\text{-}2} = \frac{D}{4\bar{f}}(0.43197 - 0.13605)$$

$$= \frac{5 \times 10^{-2}}{4 \times 0.002} \times 0.29592 = 1.85 \text{ m}$$

The maximum length is therefore:

$$L_1^* = \frac{5 \times 10^{-2}}{4 \times 0.002} \times 0.43197 = 2.70 \text{ m}$$

Example 5.2

It is required to deliver 1000 m^3/min of air at 293 K and 150 kPa at the exit of a constant-area duct. The inside diameter of the duct is 0.3 m and its length is 50 m. If the flow is adiabatic and the average friction factor is $\bar{f} = 0.005$, determine:

(a) The Mach number at the exit of the duct.
(b) The inlet pressure of the air.
(c) The inlet temperature of the air.
(d) The total change of entropy.

Solution

(a) Referring to Fig. 5.6:

$$V_2 = \frac{Q}{A} = \frac{\frac{1000}{60}}{\frac{\pi}{4}[(0.3)^2]} = 235.79 \text{ m/s}$$

$$M_2 = \frac{V_2}{20.1\sqrt{T_2}} = \frac{235.79}{20.1\sqrt{293}} = 0.685$$

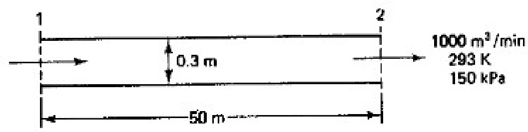

Figure 5.6

(b) and **(c)** At $M_2 = 0.685$:

$$\frac{4\bar{f}L_2^*}{D} = 0.239, \qquad \frac{p_2}{p^*} = 1.529, \qquad \frac{T_2}{T^*} = 1.097$$

Therefore:

$$\frac{4\bar{f}L_1^*}{D} = \frac{4\bar{f}L_{1\text{-}2}}{D} + \frac{4\bar{f}L_2^*}{D}$$

$$= \frac{4 \times 0.005 \times 50}{0.3} + 0.239 = 3.572$$

At $4\bar{f}L_1^*/D = 3.572$:

$$M_1 = 0.347, \qquad \frac{p_1}{p^*} = 3.12, \qquad \frac{T_1}{T^*} = 1.171$$

The inlet pressure and temperature are:

$$p_1 = \frac{\frac{p_1}{p^*}}{\frac{p_2}{p^*}} p_2 = \frac{3.12}{1.529} \times 150 = 306 \text{ kPa}$$

$$T_1 = \frac{\frac{T_1}{T^*}}{\frac{T_2}{T^*}} T_2 = \frac{1.171}{1.097} \times 293 = 312.76 \text{ K}$$

(d) The change in entropy is:

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$= 1000 \ln \frac{293}{312.76} - 287 \ln \frac{150}{306}$$

$$= -65.26 + 204.65 = 139.39 \text{ J/kg K}$$

but

$$\dot{m} = \rho_2 A V_2 = \left(\frac{p_2}{RT_2}\right) A V_2$$

$$= \left(\frac{1.5 \times 10^5}{287 \times 293}\right) \left[\frac{\pi}{4}(0.3)^2\right] (235.79) = 29.74 \text{ kg/s}$$

Therefore:

$$\Delta \dot{S} = (139.39)(29.74) = 4145.77 \text{ J/K·s}$$

In the above examples the friction factor was assumed constant. But for a duct of a certain relative roughness the value of f is a function of Reynolds number Re as depicted by the Moody diagram. In a constant-area duct, Re in turn depends on the velocity, density, and viscosity, which change as the fluid flows in the duct. But from continuity the product ρV is constant, so that the only variable in Re is the viscosity and, unless viscosity changes drastically, the variations in f are small. An additional factor to be considered is that most engineering applications involve turbulent flow, where f depends on the relative roughness of the duct but is essentially insensitive to the magnitude of Re. The following example illustrates these effects.

Example 5.3

Methane ($\gamma = 1.3$, $R = 0.5184$ kJ/kg K) flows adiabatically in a 0.3 m commercial steel pipe. At the inlet the pressure $p_1 = 0.8$ MPa, the temperature $T_1 = 320$ K (viscosity $= 0.011 \times 10^{-3}$ kg/m-s), and the velocity $V_1 = 30$ m/s. Find:

(a) The maximum possible length of the pipe.

(b) The pressure and velocity at the exit of the pipe.

Solution

(a) The conditions at the exit are sonic ($M = 1$):

$$\rho_1 = \frac{p_1}{RT_1} = \frac{800}{0.5184(320)} = 4.823 \text{ kg/m}^3$$

$$M_1 = \frac{V_1}{c_1} = \frac{30}{\sqrt{(1.3)(518.4)(320)}} = \frac{30}{464.386} = 0.0646$$

Verification Case 12

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify12.ARO

REFERENCE: Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Pages 226-227, example 5.5

GAS: Air

ASSUMPTIONS: 1) Adiabatic, 2) Perfect gas

RESULTS:

Parameter	Saad	AFT Arrow
Mass flow rate when choked (kg/s)	2.11	2.10
M_1 – Mach number at inlet	0.603	0.603
P_1– Static pressure at inlet (MPa)	2.106	2.114
T_1– Static temperature at inlet (deg. K)	427.8	429.5
$P_{2,choke}$– Static back pressure for choking (MPa)	1.203	1.220

DISCUSSION:

All results agree closely. The AFT Arrow static pressure below which choking occurs is the pipe exit static pressure.

Note that the friction factor in Saad is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

List of All Verification Models

Verification Case 12 Problem Statement

Verification Case 12

Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Pages 226-227, example 5.5

Saad Title Page

Therefore:

$$p_2 = p_1 \frac{\frac{p_2}{p^*}}{\frac{p_1}{p^*}} = 101.3 \left(\frac{1.48}{0.40825}\right) = 367.2 \text{ kPa}$$

Example 5.5

Air at a stagnation temperature of 460 K and a stagnation pressure of 2.7 MPa flows isentropically through a convergent nozzle which feeds an insulated constant-area duct. The duct is 0.025 m in diameter and 0.6 m long. If the average friction factor in the duct, $\bar{f}$, is 0.005, determine the maximum air flow rate and compare it with the flow through the nozzle in the absence of the duct. What is the maximum back pressure for choking to occur in both cases?

Solution For maximum flow rate, the Mach number at the duct exit is unity. Referring to Fig. 5.14:

$$\frac{4\bar{f}L_1^*}{D} = \frac{4 \times 0.005 \times \overset{0.6}{\cancel{0.06}}}{0.025} = 0.48$$

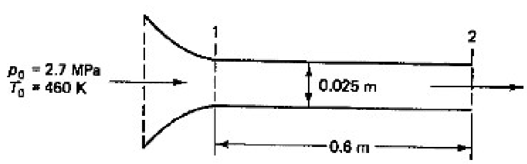

Figure 5.14

At this value $M_1 = 0.603$:

$$\frac{p_1}{p_{01}} = 0.78 \quad \text{from which} \quad p_1 = 2.106 \text{ MPa}$$

$$\frac{T_1}{T_0} = 0.93 \quad \text{from which} \quad T_1 = 427.8 \text{ K}$$

The mass rate of flow is:

$$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_1}{RT_1}\right) A_1 M_1 c_1$$

$$= \left(\frac{2.106 \times 10^6}{287 \times 427.8}\right)\left[\frac{\pi}{4} \times (0.025)^2\right](0.603)(20.1\sqrt{427.8}) = 2.11 \text{ kg/s}$$

At $M_1 = 0.603$:

$$\frac{p_1}{p^*} = 1.75$$

so that:

$$p^* = \frac{2.106}{1.75} = 1.203 \text{ MPa}$$

The system is therefore choked if the back pressure is equal or lower than 1.203 MPa.

For the convergent nozzle alone, the Mach number is unity at the nozzle exit, so that:

$$p^* = 0.528 p_0 = 1.4256 \text{ MPa}$$

and

$$T^* = 0.833 T_0 = 383.18 \text{ K}$$

The maximum mass rate of flow, according to Eq. (3.25), is

$$\dot{m} = 0.0404 A \frac{p_0}{\sqrt{T_0}} = 0.0404 \left[\frac{\pi}{4} \times (0.025)^2\right] \frac{2.7 \times 10^6}{\sqrt{460}} = 2.5 \text{ kg/s}$$

The nozzle is choked if the back pressure is equal or lower than 1.4256 MPa.

5.6 ADIABATIC FLOW WITH FRICTION IN A VARIABLE-AREA DUCT

In many applications, frictional effects are accompanied by changes in area, and this section is concerned with the resultant changes in flow properties when both of these effects occur simultaneously. Consider flow through the control volume shown in Fig. 5.15. The continuity equation is:

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad (5.39)$$

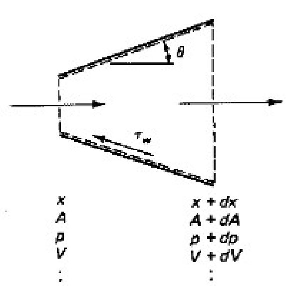

Figure 5.15 Frictional flow in a variable-area duct.

Verification Case 13

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify13.ARO

REFERENCE: Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Page 270, example 6.5

GAS: Natural Gas

ASSUMPTIONS: 1) Isothermal, 2) Perfect gas, 3) Natural gas can be represented with methane

RESULTS:

Parameter	Saad	AFT Arrow
P_2– Static back pressure at exit (kPa)	263.16	262.0
M_2 – Mach number at exit	0.38	0.38
L_{1-2} – length to reach 500 kPa (m)	544.79	558.6
L_T – length to reach sonic choke point (m)	710.0	709.8
P_T– Static pressure choke point (kPa)	114.46	115.7
M_T – Mach number at isothermal choke point	0.874	0.865

DISCUSSION:

As specified, inlet conditions are known and outlet conditions need to be determined. With the known inlet conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the exit.

The problem states that the inlet Mach number is 0.1, P_1 = 1 MPa, T_1 = 293 K. From the ideal gas law, density, sonic speed and mass flow rate are:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{1\ MPa}{\left(0.5179\ \frac{kJ}{kg \cdot K}\right)(293\ K)} = 6.585\ kg/m^3$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4\left(0.5179\ \frac{kJ}{kg \cdot K}\right) 293\ K} = 445.9 \frac{m}{s} \quad (sonic\ velocity)$$

$$\dot{m} = \rho_1 V_1 A = \rho_1 (M_1 a_1) A = \left(6.585\ \frac{kg}{m^3}\right)(0.1)\left(445.9\ \frac{m}{s}\right)\left(\frac{\pi}{4}\ 0.08^2\ m^2\right) = 1.4764\ \frac{kg}{s}$$

The first pipe represents the pipe from point 1 to 2. The second pipe represents the pipe from point 1 to the choking point. AFT Arrow does not solve for pipe length. To obtain the maximum pipe length, different lengths must be guessed with lengths that exceed sonic flow discarded.

The results for part b) were obtained by interpolating the Internal Pipe Results for the first pipe.

All results agree closely. The AFT Arrow static pressure below which choking occurs is the pipe exit static pressure.

Note that the friction factor in Saad is the Fanning friction factor. To obtain the Darcy-Weisbach friction factor used in AFT Arrow, multiply the Fanning friction factor by 4.

It should also be noted that, from time to time, AFT finds it is necessary to modify the Solver used by Arrow to improve application performance, or for other reasons. These modifications to the Solver may cause slight changes to the appropriate pipe lengths determined by Arrow.

List of All Verification Models

Verification Case 13 Problem Statement

Verification Case 13

Michel A. Saad, Compressible Fluid Flow, 2nd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1993, Page 270, example 6.5

Saad Title Page

270 Flow with Heat Interaction and Generalized Flow Chap. 6

Example 6.5

Natural gas ($\gamma = 1.31$) flows isothermally at 293 K through a pipeline of diameter 0.08 m. At the inlet the pressure is 1.0 MPa and the Mach number is 0.1. The length of the pipeline is 684 m and the average friction factor is 0.002. Calculate:

(a) The pressure and Mach number at the exit.
(b) The distance from the inlet where the pressure has dropped to 500 kPa.
(c) The maximum length of the duct for which isothermal flow is possible.
(d) The Mach number and pressure for part (c).

Solution

(a) Referring to Fig. 6.18, Eq. (6.72) gives:

$$\frac{4\bar{f}L_{1\text{-}2}}{D_H} = \frac{4 \times 0.002 \times 684}{0.08} = \frac{1 - \left(\frac{0.1}{M_2}\right)^2}{1.31 \times 0.1^2} - \ln\left(\frac{M_2}{0.1}\right)^2$$

from which $M_2 = 0.38$.

$$p_2 = p_1 \frac{M_1}{M_2} = 263.16 \text{ kPa}$$

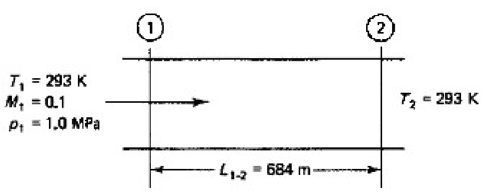

Figure 6.18

(b) Using Eq. (6.73):

$$L_{1\text{-}2} = \frac{0.08}{4 \times 0.002}\left[\frac{1 - (0.5)^2}{1.31 \times (0.1)^2} - \ln 4\right] = 544.79 \text{ m}$$

(c) $$L_T^* = \frac{0.08}{4 \times 0.002}\left[\frac{1 - 1.31 \times (0.1)^2}{1.31 \times (0.1)^2} + \ln(1.31 \times 0.1^2)\right] = 710 \text{ m}$$

(d) The Mach number and pressure are:

$$M_T^* = \frac{1}{\sqrt{\gamma}} = \frac{1}{\sqrt{1.31}} = 0.874$$

$$p_T^* = p_1\sqrt{\gamma}M_1 = 10^6 \times \sqrt{1.31} \times 0.1 = 114.46 \text{ kPa}$$

Verification Case 14

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify14.ARO

REFERENCE: Robert H. Perry and Don W. Green Editors, Author James. N. Tilton, Perry's Chemical Engineer's Handbook, Seventh Edition, Page 6-25, example 8

GAS: Air

ASSUMPTIONS: 1) Adiabatic, 2) Perfect gas

RESULTS:

Parameter	Tilton	AFT Arrow
Mass flow rate (kg/s)	2.7	2.7

DISCUSSION:

The objective is to solve for the choked flow rate. The results agree closely.

List of All Verification Models

Verification Case 14 Problem Statement

Verification Case 14

Robert H. Perry and Don W. Green Editors, Author James. N. Tilton, Perry's Chemical Engineer's Handbook, Seventh Edition, Page 6-25, example 8

Perry's Title Page

difficult to interpolate precisely. While they are quite useful for rough estimates, precise calculations are best done using the equations for one-dimensional adiabatic flow with friction, which are suitable for computer programming. Let subscripts 1 and 2 denote two points along a pipe of diameter D, point 2 being downstream of point 1. From a given point in the pipe, where the Mach number is M, the additional length of pipe required to accelerate the flow to sonic velocity ($M = 1$) is denoted L_{max} and may be computed from

$$\frac{4fL_{max}}{D} = \frac{1-M^2}{kM^2} + \frac{k+1}{2k} \ln \left(\frac{\frac{k+1}{2} M^2}{1 + \frac{k-1}{2} M^2} \right) \qquad (6\text{-}123)$$

With L = length of pipe between points 1 and 2, the change in Mach number may be computed from

$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_1 - \left(\frac{4fL_{max}}{D}\right)_2 \qquad (6\text{-}124)$$

Eqs. (6-116) and (6-113), which are valid for adiabatic flow with friction, may be used to determine the temperature and speed of sound at points 1 and 2. Since the mass flux $G = \rho v = \rho c M$ is constant, and $\rho = PM_w/RT$, the pressure at point 2 (or 1) can be found from G and the pressure at point 1 (or 2).

The additional frictional losses due to pipeline fittings such as elbows may be added to the velocity head loss $N = 4fL/D_H$ using the same velocity head loss values as for incompressible flow. This works well for fittings which do not significantly reduce the channel cross-sectional area, but may cause large errors when the flow area is greatly reduced, as, for example, by restricting orifices. Compressible flow across restricting orifices is discussed in Sec. 10 of this *Handbook*. Similarly, elbows near the exit of a pipeline may choke the flow even though the Mach number is less than unity due to the nonuniform velocity profile in the elbow. For an abrupt contraction rather than rounded nozzle inlet, an additional 0.5 velocity head should be added to N. This is a reasonable approximation for G, but note that it allocates the additional losses to the pipeline, even though they are actually incurred in the entrance. It is an error to include one velocity head exit loss in N. The kinetic energy at the exit is already accounted for in the integration of the balance equations.

Example 8: Compressible Flow with Friction Losses Calculate the discharge rate of air to the atmosphere from a reservoir at 10^6 Pa gauge and 20°C through 10 m of straight 2-in Schedule 40 steel pipe (inside diameter = 0.0525 m), and 3 standard radius, flanged 90° elbows. Assume 0.5 velocity heads lost for the elbows.

For commercial steel pipe, with a roughness of 0.046 mm, the friction factor for fully rough flow is about 0.0047, from Eq. (6-38) or Fig. 6-9. It remains to be verified that the Reynolds number is sufficiently large to assume fully rough flow. Assuming an abrupt entrance with 0.5 velocity heads lost,

$$N = 4 \times 0.0047 \times \frac{10}{0.0525} + 0.5 + 3 \times 0.5 = 5.6$$

The pressure ratio p_3/p_0 is

$$\frac{1.01 \times 10^5}{(1 \times 10^6 + 1.01 \times 10^5)} = 0.092$$

From Fig. 6-21*b* at $N = 5.6$, $p_3/p_0 = 0.092$ and $k = 1.4$ for air, the flow is seen to be choked. At the choke point with $N = 5.6$ the critical pressure ratio p_2/p_0 is about 0.25 and G/G^* is about 0.48. Equation (6-122) gives

$$G^* = 1.101 \times 10^6 \times \sqrt{\left(\frac{2}{1.4+1}\right)^{(1.4+1)/(1.4-1)} \left(\frac{1.4 \times 29}{8{,}314 \times 293.15}\right)} = 2{,}600 \text{ kg/m}^2 \cdot \text{s}$$

Multiplying by $G/G^* = 0.48$ yields $G = 1{,}250$ kg/m² · s. The discharge rate is $w = GA = 1{,}250 \times \pi \times 0.0525^2/4 = 2.7$ kg/s.

Before accepting this solution, the Reynolds number should be checked. At the pipe exit, the temperature is given by Eq. (6-120) since the flow is choked. Thus, $T_2 = T^* = 244.6$ K. The viscosity of air at this temperature is about 1.6×10^{-5} Pa · s. Then

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{DG}{\mu} = \frac{0.0525 \times 1{,}250}{1.6 \times 10^{-5}} = 4.1 \times 10^6$$

At the beginning of the pipe, the temperature is greater, giving greater viscosity and a Reynolds number of 3.6×10^6. Over the entire pipe length the Reynolds number is very large and the fully rough flow friction factor choice was indeed valid.

Once the mass flux G has been determined, Fig. 6-21*a* or 6-21*b* can be used to determine the pressure at any point along the pipe, simply by reducing $4fL/D_H$ and computing p_2 from the figures, given G, instead of the reverse. Charts for calculation between two points in a pipe with known flow and known pressure at either upstream or downstream locations have been presented by Loeb (*Chem. Eng.*, **76**[5], 179–184 [1969]) and for known downstream conditions by Powley (*Can. J. Chem. Eng.*, **36**, 241–245 [1958]).

Convergent/Divergent Nozzles (De Laval Nozzles) During frictionless adiabatic one-dimensional flow with changing cross-sectional area A the following relations are obeyed:

$$\frac{dA}{A} = \frac{dp}{\rho V^2}(1 - M^2) = \frac{1-M^2}{M^2}\frac{d\rho}{\rho} = -(1-M^2)\frac{dV}{V} \qquad (6\text{-}125)$$

Equation (6-125) implies that in converging channels, subsonic flows are accelerated and the pressure and density decrease. In diverging channels, subsonic flows are decelerated as the pressure and density increase. In subsonic flow, the converging channels act as nozzles and diverging channels as diffusers. In supersonic flows, the opposite is true. Diverging channels act as nozzles accelerating the flow, while converging channels act as diffusers decelerating the flow.

Figure 6-23 shows a converging/diverging nozzle. When p_2/p_0 is less than the critical pressure ratio (p^*/p_0), the flow will be subsonic in the converging portion of the nozzle, sonic at the throat, and supersonic in the diverging portion. At the throat, where the flow is critical and the velocity is sonic, the area is denoted A^*. The cross-sectional area and pressure vary with Mach number along the converging/diverging flow path according to the following equations for isentropic flow of a perfect gas:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/2(k-1)} \qquad (6\text{-}126)$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)} \qquad (6\text{-}127)$$

The temperature obeys the adiabatic flow equation for a perfect gas,

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad (6\text{-}128)$$

Equation (6-128) does not require frictionless (isentropic) flow. The sonic mass flux through the throat is given by Eq. (6-122). With A set equal to the nozzle exit area, the exit Mach number, pressure, and temperature may be calculated. Only if the exit pressure equals the ambient discharge pressure is the ultimate expansion velocity reached in the nozzle. Expansion will be incomplete if the exit pressure exceeds the ambient discharge pressure; shocks will occur outside the nozzle. If the calculated exit pressure is less than the ambient discharge pressure, the nozzle is overexpanded and compression shocks within the expanding portion will result.

The shape of the converging section is a smooth trumpet shape similar to the simple converging nozzle. However, special shapes of the diverging section are required to produce the maximum supersonic exit velocity. Shocks result if the divergence is too rapid and excessive boundary layer friction occurs if the divergence is too shallow. See Liepmann and Roshko (*Elements of Gas Dynamics*, Wiley, New York, 1957, p. 284). If the nozzle is to be used as a thrust device, the diverg-

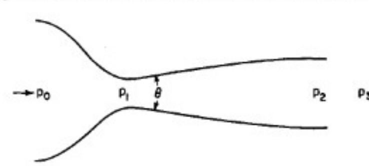

FIG. 6-23 Converging/diverging nozzle.

Verification Case 15

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify15.ARO

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, PWS Publishers, Belmont, CA 1983, Pages 317-319, example 8.8

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Janna	AFT Arrow
M_2– Mach number at exit	0.14	0.14
P_2– Pressure at exit (psia)	9.63	9.84
T_2– Temperature at exit (deg. R)	528.8	528.9

DISCUSSION:

As specified, inlet conditions are known and outlet conditions need to be determined. With the known inlet conditions, an implied mass flow rate exists. To pose the problem in AFT Arrow terms, a few simple calculations are needed to obtain the mass flow rate. Once obtained, it is applied as a flow demand at the exit.

The problem states that the inlet velocity is 100 ft/s, P_1 = 15 psia, T_1 = 530 R. From the ideal gas law, density, and mass flow rate are:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{15\ psia}{\left(53.34\ \frac{ft \cdot lbf}{lbm \cdot R}\right)(530\ R)} = 0.0764\ lbm/ft^3$$

$$\dot{m} = \rho_1 V_1 A = \rho_1 (M_1 a_1) A = \left(0.0764\ \frac{lbm}{ft^3}\right)\left(100\ \frac{ft}{s}\right)\left(\frac{\pi}{4}\ 2.067^2\ in^2\right) = 0.17814\ \frac{lbm}{s}$$

With this flow rate at the exit, the predictions agree very closely.

List of All Verification Models

Verification Case 15 Problem Statement

Verification Case 15

William S. Janna, Introduction to Fluid Mechanics, PWS Publishers, Belmont, CA 1983, Pages 317-319, example 8.8

Janna Title Page

lent flow, which is usually the case in compressible flow, the change in f is considerably smaller. It is therefore reasonable to assume that friction factor f is a constant when integrating Equation 8.35 and that f is equal to some average or the initial value in the pipe.

We are now ready to integrate Equation 8.35; it is convenient to choose as our limits the following:

$$0 \leq dx \leq L_{max}$$

$$M \leq M \leq 1$$

Thus the flow at zero has a Mach number of M and is accelerated to M = 1, where $x = L_{max}$, at the end of the pipe. The point where Mach number is unity is selected as a reference. Integrating Equation 8.35 gives

$$\frac{fL_{max}}{D} = \int_{M}^{1} \frac{2(1 - M^2)(dM/M)}{\left(1 + \frac{\gamma - 1}{2} M^2\right)\gamma M^2} \qquad (8.36)$$

Using this equation, values of fL_{max}/D versus M for $\gamma = 1.4$ have been tabulated in Appendix Table D-3.

A similar analysis can be developed to obtain an expression between pressure and Mach number. Using Equation 8.31 and substituting from continuity for dV/V in terms of M yields the following after simplification:

$$\frac{dp}{p} = -\frac{dM}{M}\left[\frac{1 + (\gamma - 1)M^2}{1 + \frac{\gamma - 1}{2} M^2}\right] \qquad (8.37)$$

Integrating between p and p* corresonding to limits M and 1 gives a relationship for p/p^* versus M, which is tabulated also in Appendix Table D-3 for $\gamma = 1.4$. Again M = 1 is a reference point and the corresponding pressure is denoted with an asterisk. Similarly from Equation 8.34 we can derive an expression for T/T^* versus M. Finally, combining Equations 8.33 and 8.34 yields an integrable expression for V/V^* versus M. These too are tabulated in Appendix Table D-3 for $\gamma = 1.4$. With the equations thus described and tabulated, compressible flow with friction (referred to as **Fanno flow**) can be adequately modeled.

EXAMPLE 8.8

An airflow enters a constant-area pipe at 100 ft/s, 15 psia, and 530°R. The pipe is 500 ft long and made of 2-nominal schedule 40 PVC. Determine conditions at the pipe exit.

SOLUTION

From the appendix tables:

$\mu = 0.3801 \times 10^{-6}$ lbf-s/ft^2 (Table A-3 for air)

$D = 0.1723$ ft $A = 0.02330$ ft^2 (Table C-1 for pipe)

Also for air,

$$\rho = \frac{p}{RT} = \frac{15(144)}{53.3(530)} = 0.0765 \text{ lbm/ft}^3$$

The sonic velocity at inlet is

$$a_1 = \sqrt{\gamma R T_1 g_c} = \sqrt{1.4(53.3)(530)(32.2)} = 1128 \text{ ft/s}$$

Therefore

$$M_1 = \frac{V_1}{a_1} = \frac{100}{1128} = 0.09$$

From Appendix Table D-3 at $M_1 = 0.09$, we get

$$\frac{T_1}{T^*} = 1.1981 \qquad \frac{p_1}{p^*} = 12.162 \qquad \left.\frac{fL_{max}}{D}\right|_1 = 83.496$$

where L_{max} is the duct length required for the flow to achieve $M = 1$, at which the pressure is p^* and the temperature is T^*.

The Reynolds number at inlet is

$$Re_1 = \frac{\rho V_1 D}{\mu g_c} = \frac{0.0765(100)(0.1723)}{0.3801 \times 10^{-6}(32.2)} = 1.08 \times 10^5$$

For PVC pipe, assume the surface is very smooth. At $Re_1 = 1.08 \times 10^5$, the Moody diagram of Chapter 5 gives

$$f = 0.0175$$

and for the actual pipe in this example we have

$$\left.\frac{fL}{D}\right|_{ac} = \frac{0.0175\,(500)}{0.1723} = 50.78$$

To find the properties at the pipe exit, we combine fL/D terms using the $M = 1$ point as a reference. As illustrated in Figure 8.18,

$$\left.\frac{fL_{max}}{D}\right|_1 = \left.\frac{fL}{D}\right|_{ac} + \left.\frac{fL_{max}}{D}\right|_2$$

Thus

$$\left.\frac{fL_{max}}{D}\right|_2 = 83.496 - 50.78 = 32.71$$

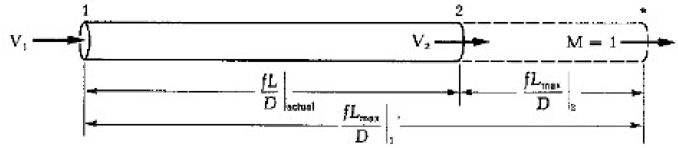

Figure 8.18. Sketch for Example 8.8.

From Appendix Table D-3,

$$\underline{M_2 = 0.14} \qquad \frac{T_2}{T^*} = 1.1953 \qquad \frac{p_2}{p^*} = 7.8093$$

With these ratios, we find

$$p_2 = \frac{p_2}{p^*}\frac{p^*}{p_1}p_1 = \frac{7.8093}{12.162}(15 \text{ psia}) = \underline{9.63 \text{ psia}}$$

$$T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = \frac{1.1953}{1.1981}(530°\text{R}) = \underline{528.8°\text{R}}$$

As seen from these calculations, temperature did not change significantly. We were justified in assuming a constant value of friction factor, evaluated in this case at the pipe inlet.

8.6 / SUMMARY

In this chapter we have examined some concepts associated with compressible flow. We determined an expression for sonic velocity in a compressible medium and also developed equations for isentropic flow. We examined in detail the behavior of a compressible fluid as it goes through a nozzle. We derived equations for normal shock waves and, moreover, we have seen the effect of friction on compressible flow through a constant-area duct.

8.7 / PROBLEMS: CHAPTER 8

1. Determine the sonic velocity in air at a temperature of 0°F.
2. Calculate the sonic velocity in argon at a temperature of 25°C.
3. Calculate the sonic velocity in carbon dioxide at a temperature of 100°C.
4. Determine the sonic velocity in helium at a temperature of 50°C.
5. Calculate the sonic velocity in hydrogen at a temperature of 75°F.
6. Determine the sonic velocity in oxygen at a temperature of 32°F.

Verification Case 16

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify16.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.394, B.397, B.398, Example B8.2

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
M_1– Mach number at valve	0.317	0.317
P_1– Pressure at valve (psia)	128.46	128.58

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate is known and the stagnation temperature. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 120 F stagnation temperature, the inlet static temperature was iterated a few times.

The conditions result in sonic choking at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 16 Problem Statement

Verification Case 16

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.394, B.397, B.398, Example B8.2

Nayyar Title Page

the boiler reheater should be 7 to 9 percent of high pressure turbine exhaust pressure. It is desirable to use a smaller diameter hot reheat line and a larger diameter cold reheat line, taking a greater pressure drop in the more expensive, alloy, hot reheat line.

Extraction steam piping also affects heat rate and output, and norally this piping should be sized so that the pressure drop does not exceed about 5 percent of turbine stage pressure for the low pressure and 3 to 4 percent for the higher pressure lines.

Extraction steam lines should be designed for the pressure shown on full-load heat balance diagram at 5 percent overpressure and valves wide open (VWO).

Continuously operating steam lines in process projects shall be designed on the basis of reasonable total pressure drop and, except for short leads such as to turbines and pumps, shall not generally exceed the conditions noted in Table B8.13.

Applications: Air and Other Gas Systems

As indicated in the last section, "Applications: Steam Systems," in a case of compressible fluids such as air or steam when density changes are small, the fluid may be considered as incompressible. Therefore, all design rules described in the subsections "Characteristics of Incompressible Flow," "Applications: Water Systems," and "Applications: Steam Systems" are applicable to this section. For steam and gas systems, where the fluid density is small, static head is negligible and may be omitted in pressure drop calculations.

Table B8.14 (from Ref. 7) presents pressure drop for some typical values of air flow rates through piping from ⅛ to 12 in diameter.

Applications: Sample Problems B8.2–B8.7

The most frequent application of single-phase compressible flow steam line analysis normally encountered by engineers is the sizing of safety valve vent lines. This analysis can be done either by using a computer program which is based on procedures discussed, or a hand calculation similar to that presented in this section. Users may compare their results with those obtained from an approximate, semi-empirical procedure based on tables in App. 11 of Ref. 12. It is the responsibility of the designer to make sure that the method used yields conservative results.

The primary consideration in these analyses is to ensure that the vent line will pass the required flow without exceeding recommended backpressure limitations on valves with solidly connected vents or without blowing back in the case of open vent stacks.

Problem B8.2. During abnormal operation of a system, 72,000 lb_m/h (32,659 kg/h) of air must be released from a high-pressure air tank through a 4-in (Schedule 40) commercial steel bypass line (vent line) into the atmosphere. Stagnation conditions in the vessel during this operation are: p_o = 600 psig (41.4) bar gauge (kept at a constant level by compressors), t_F = 120°F (48.9°C). Equivalent length of the vent line is 90 ft (27.43 m). Calculate the pressure p_1 that exists at the valve discharge and make your comments on the vent line size. Air may be treated as a perfect gas. Assume that this is not a typical safety valve and that the valve backpressure should not exceed 50 percent of the valve set pressure. (The safety valve vent line design criteria are well documented in the ASME Boiler and Pressure Vessel Code.)

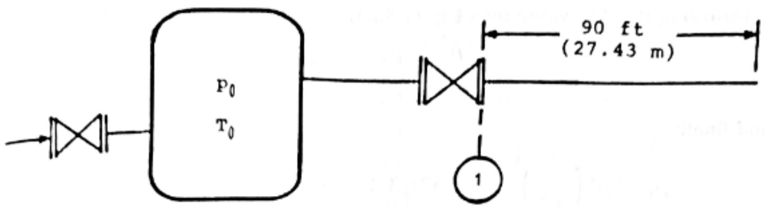

FIGURE B8.18 Reference for sample problem B8. 2.

$\dot{m} = 72{,}000$ lb/hr $= 20$ lb$_m$/sec

$T_0 = 120°F = (460 + 120) = 580°R$

$p_0 = 600$ psig $= 614.7$ psia (assumed atmospheric pressure of 14.7 psia)

Internal pipe diameter $D = 4.026$ in ($A = 0.0884$ ft^2).

Critical pressure at the pipe discharge (see Fig. B8.18) is found by using Eq. (B8.25):

$$p^* = \frac{\dot{m}}{A}\sqrt{\frac{2RT_0}{g_c k(k+1)}} \ \text{lb}_f/\text{ft}^2$$

$$= \frac{20}{0.0884}\sqrt{\frac{(2)\,(53.3)\,(580)}{(32.174)\,(1.4)\,(2.4)}}$$

$$= 5410.35 \ \text{lb}_f/\text{ft}^2 = 37.57 \ \text{psia}$$

The line is choked because $p^* > p_{amb} = 14.7$ psia. Pipe $f(L/D)$, from choked exit to the valve discharge, is calculated by assuming complete turbulence of flow. From Ref. 7, for a commercial steel pipe:

$$\frac{\varepsilon}{D} = \frac{(0.00015)\,(12.0)}{4.026} = 0.00045$$

and the friction factor $f = f_T = 0.017$. Then the value of $(f\,L)/D$, counting from the choked pipe exit to the valve outlet is:

$$f\frac{L}{D} = 0.017\frac{(90)\,(12)}{4.026} = 4.5604$$

Using a computerized method (for example the Newton-Raphson method) of solving Eq. (B8.28) (notice that for Ma = 1 at the pipe discharge, the second term on the right hand side of Eq.(B8.28) is zero), with $k = 1.4$, having on the left side of this equation the above calculated $(f\,L)/D = 4.5604$ and changing the Mach number, the value of Mach number for air entering the pipe, which satisfies the equation, is found to be:

$$\text{Ma} = 0.317$$

Substituting this Ma value into Eq. (B8.26) results in

$$\frac{p}{p^*} = \frac{p_1}{p^*} = 3.4193$$

and finally:

$$p_1 = p^*\left(\frac{p}{p^*}\right) = (37.57)(3.4193) = 128.46 \text{ psia } (8.86 \text{ bar})$$

Fanno line tables for $k = 1.4$ (Table B8.12) may be used instead, but this procedure would require cumbersome interpolations. Also, note that in Ref. 11 (the source of Table B8.12) the Fanning friction factor is used, which is four times less than that of D'Arcy-Weisbach. This is the reason why Table B8.12 uses 4 fL_{max}/D) value in the last column.

Problem B8.3. Twice as much flow rate must be released from the tank described in Problem B8.2. Check if the piping is adequate.

$$p^* = (2)(37.57) = 75.14 \text{ psia}$$

Then,

$$p_1 = (75.14)(3.4193) = 256.93 \text{ psia } (17.72 \text{ bar})$$

The piping is still adequate because the calculated $p_1 < 0.5\ p_0$.

Problem B8.4. Keep the mass flow as in Problem B8.2, but double the length of the pipe. Check the pressure p_1:

$$fL/D = (2)(4.5604) = 9.1208$$

at this $f\ L/D$,

$$(p_1/p^*) = 4.4854$$

$$\text{Ma} = 0.243$$

and

$$p_1 = (37.57)(4.4854) = 168.52 \text{ psia } (11.62 \text{ bar})$$

which means that the line size is sufficient.

Problem B8.5. For the same mass flow and the same pipe as in Problem B8.2, assume that the air temperature in the vessel $t_F = 500°\text{F}$ (260°C).

$$p^* = \frac{20}{0.0884}\sqrt{\frac{(2)(53.3)(460+500)}{(32.174)(1.4)(2.4)}}$$

$$= 6960.97 \text{ lb}_f/\text{ft}^2 = 48.34 \text{ psia}$$

$$p_1 = (3.4193)(48.34) = 165.29 \text{ psia } (11.40 \text{ bar})$$

The line size is sufficient.

Verification Case 17

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify17.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.3

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
M_1 – Mach number at valve	0.317	0.317
P_1 – Pressure at valve (psia)	256.93	257.15

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate is known and the stagnation temperature. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 120 F stagnation temperature, the inlet static temperature was iterated a few times.

The conditions result in sonic choking at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 17 Problem Statement

Verification Case 17

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.3

Nayyar Title Page

B.398 GENERIC DESIGN CONSIDERATIONS

Substituting this Ma value into Eq. (B8.26) results in

$$\frac{p}{p^*} = \frac{p_1}{p^*} = 3.4193$$

and finally:

$$p_1 = p^*\left(\frac{p}{p^*}\right) = (37.57)(3.4193) = 128.46 \text{ psia } (8.86 \text{ bar})$$

Fanno line tables for $k = 1.4$ (Table B8.12) may be used instead, but this procedure would require cumbersome interpolations. Also, note that in Ref. 11 (the source of Table B8.12) the Fanning friction factor is used, which is four times less than that of D'Arcy-Weisbach. This is the reason why Table B8.12 uses $4\,fL_{max}/D$) value in the last column.

Problem B8.3. Twice as much flow rate must be released from the tank described in Problem B8.2. Check if the piping is adequate.

$$p^* = (2)(37.57) = 75.14 \text{ psia}$$

Then,

$$p_1 = (75.14)(3.4193) = 256.93 \text{ psia } (17.72 \text{ bar})$$

The piping is still adequate because the calculated $p_1 < 0.5\,p_0$.

Problem B8.4. Keep the mass flow as in Problem B8.2, but double the length of the pipe. Check the pressure p_1:

$$f\,L/D = (2)(4.5604) = 9.1208$$

at this $f\,L/D$,

$$(p_1/p^*) = 4.4854$$

$$\text{Ma} = 0.243$$

and

$$p_1 = (37.57)(4.4854) = 168.52 \text{ psia } (11.62 \text{ bar})$$

which means that the line size is sufficient.

Problem B8.5. For the same mass flow and the same pipe as in Problem B8.2, assume that the air temperature in the vessel $t_F = 500°\text{F}$ (260°C).

$$p^* = \frac{20}{0.0884}\sqrt{\frac{(2)(53.3)(460+500)}{(32.174)(1.4)(2.4)}}$$

$$= 6960.97 \text{ lb}_f/\text{ft}^2 = 48.34 \text{ psia}$$

$$p_1 = (3.4193)(48.34) = 165.29 \text{ psia } (11.40 \text{ bar})$$

The line size is sufficient.

Verification Case 18

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify18.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.4

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
M_1 – Mach number at valve	0.243	0.243
P_1 – Pressure at valve (psia)	168.52	168.61

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate and the stagnation temperature are known. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 120 F stagnation temperature, the inlet static temperature was iterated a few times.

The conditions result in sonic choking at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 18 Problem Statement

Verification Case 18

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.4

Nayyar Title Page

B.398 GENERIC DESIGN CONSIDERATIONS

Substituting this Ma value into Eq. (B8.26) results in

$$\frac{p}{p^*} = \frac{p_1}{p^*} = 3.4193$$

and finally:

$$p_1 = p^*\left(\frac{p}{p^*}\right) = (37.57)(3.4193) = 128.46 \text{ psia (8.86 bar)}$$

Fanno line tables for $k = 1.4$ (Table B8.12) may be used instead, but this procedure would require cumbersome interpolations. Also, note that in Ref. 11 (the source of Table B8.12) the Fanning friction factor is used, which is four times less than that of D'Arcy-Weisbach. This is the reason why Table B8.12 uses $4\,fL_{max}/D$) value in the last column.

Problem B8.3. Twice as much flow rate must be released from the tank described in Problem B8.2. Check if the piping is adequate.

$$p^* = (2)(37.57) = 75.14 \text{ psia}$$

Then,

$$p_1 = (75.14)(3.4193) = 256.93 \text{ psia (17.72 bar)}$$

The piping is still adequate because the calculated $p_1 < 0.5\ p_0$.

Problem B8.4. Keep the mass flow as in Problem B8.2, but double the length of the pipe. Check the pressure p_1:

$$f\,L/D = (2)(4.5604) = 9.1208$$

at this $f\ L/D$,

$$(p_1/p^*) = 4.4854$$

$$\text{Ma} = 0.243$$

and

$$p_1 = (37.57)(4.4854) = 168.52 \text{ psia (11.62 bar)}$$

which means that the line size is sufficient.

Problem B8.5. For the same mass flow and the same pipe as in Problem B8.2, assume that the air temperature in the vessel $t_F = 500°\text{F}$ (260°C).

$$p^* = \frac{20}{0.0884}\sqrt{\frac{(2)(53.3)(460+500)}{(32.174)(1.4)(2.4)}}$$

$$= 6960.97 \text{ lb}_f/\text{ft}^2 = 48.34 \text{ psia}$$

$$p_1 = (3.4193)(48.34) = 165.29 \text{ psia (11.40 bar)}$$

The line size is sufficient.

Verification Case 19

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify19.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.5

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
P_1 – Pressure at valve (psia)	165.29	165.60

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate is known and the stagnation temperature. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 500 F stagnation temperature, the inlet static temperature was iterated a few times.

The conditions result in sonic choking at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 19 Problem Statement

Verification Case 19

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.398, Example B8.5

Nayyar Title Page

B.398 GENERIC DESIGN CONSIDERATIONS

Substituting this Ma value into Eq. (B8.26) results in

$$\frac{p}{p^*} = \frac{p_1}{p^*} = 3.4193$$

and finally:

$$p_1 = p^*\left(\frac{p}{p^*}\right) = (37.57)(3.4193) = 128.46 \text{ psia } (8.86 \text{ bar})$$

Fanno line tables for $k = 1.4$ (Table B8.12) may be used instead, but this procedure would require cumbersome interpolations. Also, note that in Ref. 11 (the source of Table B8.12) the Fanning friction factor is used, which is four times less than that of D'Arcy-Weisbach. This is the reason why Table B8.12 uses $4\,fL_{max}/D$) value in the last column.

Problem B8.3. Twice as much flow rate must be released from the tank described in Problem B8.2. Check if the piping is adequate.

$$p^* = (2)(37.57) = 75.14 \text{ psia}$$

Then,

$$p_1 = (75.14)(3.4193) = 256.93 \text{ psia } (17.72 \text{ bar})$$

The piping is still adequate because the calculated $p_1 < 0.5\,p_0$.

Problem B8.4. Keep the mass flow as in Problem B8.2, but double the length of the pipe. Check the pressure p_1:

$$f\,L/D = (2)(4.5604) = 9.1208$$

at this $f\,L/D$,

$$(p_1/p^*) = 4.4854$$

$$\text{Ma} = 0.243$$

and

$$p_1 = (37.57)(4.4854) = 168.52 \text{ psia } (11.62 \text{ bar})$$

which means that the line size is sufficient.

Problem B8.5. For the same mass flow and the same pipe as in Problem B8.2, assume that the air temperature in the vessel $t_F = 500°F$ (260°C).

$$p^* = \frac{20}{0.0884}\sqrt{\frac{(2)(53.3)(460+500)}{(32.174)(1.4)(2.4)}}$$

$$= 6960.97 \text{ lb}_f/\text{ft}^2 = 48.34 \text{ psia}$$

$$p_1 = (3.4193)(48.34) = 165.29 \text{ psia } (11.40 \text{ bar})$$

The line size is sufficient.

Verification Case 20

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify20.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.399, Example B8.6

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
M_1 – Mach number at valve	0.235	0.235
P_1 – Pressure at valve (psia)	661.96	662.27

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate is known and the stagnation temperature. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 120 F stagnation temperature, the inlet static temperature was iterated a few times.

The conditions result in sonic choking at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 20 Problem Statement

Verification Case 20

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.399, Example B8.6

Nayyar Title Page

Problem B8.6. For the same mass flow and the same pipe as in Problem B8.2, use the pipe diameter of 2 in (Schedule 40) and calculate p_1.

$$D = 2.067 \text{ in } (A = 0.0233 \text{ ft}^2)$$

$$p^* = 37.57 \frac{0.0884}{0.0233} = 142.54 \text{ psia}$$

$$f\frac{L}{D} = (0.019)\frac{(90)(12)}{2.067} = 9.9274$$

Then

$$(p/p^*) = 4.644$$

$$\text{Ma} = 0.235$$

and

$$p_1 = (142.54)(4.644) = 661.96 \text{ psia } (45.64 \text{ bar})$$

The calculated pressure p_1 is too high. It is even higher than the pressure p_0 within the vessel. This line cannot be used for releasing the required flow.

Problem B8.7. For the same flow and the same pipe length as in Problem B8.2, use the pipe diameter of 8 in (Schedule 40), and calculate p_1.

$$D = 7.981 \text{ in } (A = 0.3474 \text{ ft}^2)$$

$$f = f_T = 0.014$$

$$p^* = 37.57 \frac{0.0884}{0.3474} = 9.56 \text{ psia}$$

Because $p^* < p_{atm}$, the line is not choked. To calculate the pressure p_1 after the valve, the additional length, L_{add}, of the pipe, required for choked conditions at the exit, should be calculated first.

Assuming

$$p_{amb} = p_{atm} = 14.7 \text{ psia}$$

$$\frac{p_{atm}}{p^*} = \frac{14.7}{9.56} = 1.5376$$

From Fanno line tables, for $(p/p^*) = 1.5376$, the following is found for the existing pipe outlet

$$fL_{add}/D = 0.2467$$

$$\text{Ma} = 0.6814$$

Verification Case 21

Problem Statement

PRODUCT: AFT Arrow

MODEL FILE: AroVerify21.ARO

REFERENCE: Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.399-400, Example B8.7

GAS: Air

ASSUMPTIONS: 1) Adiabatic flow, 2) Perfect gas

RESULTS:

Parameter	Nayar	AFT Arrow
M_1– Mach number at valve	0.4096	0.413
P_1– Pressure at valve (psia)	25.15	24.94

DISCUSSION:

The problem assumes an unusual inlet boundary condition where the flow rate is known and the stagnation temperature. AFT Arrow uses the static temperature at the inlet because it is typically associated with a flow rate. To match the 120 F stagnation temperature, the inlet static temperature was iterated a few times.

The result is sub-sonic conditions at the discharge.

The predictions agree very closely.

List of All Verification Models

Verification Case 21 Problem Statement

Verification Case 21

Mohinder L. Nayyar, Piping Handbook, Seventh Edition, McGraw-Hill, New York, 2000, Page B.399-400, Example B8.7

Nayyar Title Page

FLOW OF FLUIDS **B.399**

Problem B8.6. For the same mass flow and the same pipe as in Problem B8.2, use the pipe diameter of 2 in (Schedule 40) and calculate p_1.

$$D = 2.067 \text{ in } (A = 0.0233 \text{ ft}^2)$$

$$p^* = 37.57 \frac{0.0884}{0.0233} = 142.54 \text{ psia}$$

$$f\frac{L}{D} = (0.019)\frac{(90)(12)}{2.067} = 9.9274$$

Then

$$(p/p^*) = 4.644$$

$$\text{Ma} = 0.235$$

and

$$p_1 = (142.54)(4.644) = 661.96 \text{ psia } (45.64 \text{ bar})$$

The calculated pressure p_1 is too high. It is even higher than the pressure p_0 within the vessel. This line cannot be used for releasing the required flow.

Problem B8.7. For the same flow and the same pipe length as in Problem B8.2, use the pipe diameter of 8 in (Schedule 40), and calculate p_1.

$$D = 7.981 \text{ in } (A = 0.3474 \text{ ft}^2)$$

$$f = f_T = 0.014$$

$$p^* = 37.57 \frac{0.0884}{0.3474} = 9.56 \text{ psia}$$

Because $p^* < p_{atm}$, the line is not choked. To calculate the pressure p_1 after the valve, the additional length, L_{add}, of the pipe, required for choked conditions at the exit, should be calculated first.

Assuming

$$p_{amb} = p_{atm} = 14.7 \text{ psia}$$

$$\frac{p_{atm}}{p^*} = \frac{14.7}{9.56} = 1.5376$$

From Fanno line tables, for $(p/p^*) = 1.5376$, the following is found for the existing pipe outlet

$$fL_{add}/D = 0.2467$$

$$\text{Ma} = 0.6814$$

Therefore,

$$L_{\text{add}} = \frac{\left(f\frac{L_{\text{add}}}{D}\right)}{\frac{f}{D}} = \frac{(0.2467)(7.981)}{(0.014)(12)} = 11.72 \text{ ft}$$

and the fictitious pipe length from the choked pipe exit to the valve outlet

$$L_{\text{fict}} = 90 + 11.72 = 101.72 \text{ ft}$$

The corresponding $(f\ L/D)_{\text{fict}}$ value, starting from the choked exit to the valve, is

$$\left(f\frac{L}{D}\right)_{\text{fict}} = \frac{(0.014)(101.72)(12)}{7.981} = 2.1412$$

For this $(f\ L/D)_{\text{fict}}$, the following is obtained from the Fanno line computerized calculation, similar to that explained in Problem B8.2 (or from Fanno line tables for $k = 1.4$):

$$(p/p^*) = 2.6306$$

$$\text{Ma} = 0.4096$$

Therefore,

$$p_1 = (9.56)(2.6306) = 25.15 \text{ psia } (1.73 \text{ bar})$$

SINGLE-PHASE FLOW IN NOZZLES, VENTURI TUBES, AND ORIFICES

Theoretical Background

Liquid Service. A nozzle or an orifice in a tank or reservoir may be installed in the wall (Fig. B8.19) or in the bottom.

In the case of a nozzle, the fluid emerges in the form of a cylindrical jet of the same diameter as the throat of the nozzle, but in the case of a sharp-edged orifice, the jet contracts after passing through it, attaining its smallest diameter (vena contracta) and greatest velocity some distance (about one-half of a diameter) downstream from the opening. When installed in the bottom, the distance z from the opening in the bottom of the tank to the liquid free surface must include the length of a nozzle or the distance of the vena contracta from the bottom of the tank. The ratio of jet area A_2 at vena contracta to the area of an orifice A is called the *coefficient of contraction* C_c.

$$C_c = A_2/A \tag{B8.34}$$

For a nozzle $C_c = 1$.

Bernoulli's equation [(Eq. B8.12) without H_p and H_f] applied from a point 1 on the free surface to the center of the vena contracta, point 2, yields the following