AFT Fathom™

Verification Cases

AFT Fathom Version 13

Incompressible Pipe Flow Modeling

Published: September 13, 2023

Applied Flow Technology

Dynamic solutions for a fluid world ™

Contents

AFT Fathom Verification Overview

There are a number of aspects to the verification process employed by Applied Flow Technology to ensure that AFT Fathom provides accurate solutions to incompressible pipe flow systems. These are discussed in Verification Methodology. A listing of all of the verified models is given in Summary of Verification Models. The verification models are taken from numerous References.

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Verification Methodology

The *AFT Fathom* software is an incompressible pipe flow analysis product intended to be used by trained engineers. As a technical software package, issues of quality and reliability of the technical data generated by the software are important. The following description summarizes the steps taken by Applied Flow Technology to ensure high quality in the technical data.

1. Comparisons with open literature examples

Numerous examples of pipe flow systems are available in the open literature which include published results. AFT Fathom results have been compared against many open literature systems, which include network systems up to 70 pipes in size. AFT Fathom predictions compare favorably in all cases.

2. Software checks results to ensure mass and energy balance

AFT Fathom uses a popular iterative method to obtain solutions to pipe network systems. The method is known as the Newton-Raphson method. As applied to pipe systems, the Newton-Raphson method employs the conservation of mass equation and the momentum balance equation (i.e., Bernoulli). Solutions are sought which satisfy these equations at all points in the system. After a solution is obtained, a final check is made by the software whereby the mass flow into each node is checked for balance. If a balance is not found, the user is warned in the output. This ensures that the results generated by the software agree with the applicable fundamental equations.

In addition, if heat transfer is modeled, AFT Fathom performs a final energy balance check for each junction.

See the AFT Fathom Help System for more information.

3. Software has been used in industry since April, 1994 with no significant technical errors

AFT Fathom became available in April, 1994 and is currently being used by companies in the following industries: chemical, petrochemical, power generation, architectural, ship construction, mining, automotive, aerospace, pulp and paper, pharmaceutical, municipal water, and environmental. Since its release, no significant technical errors have been found. AFT Fathom has been used to model a wide variety of systems and customers have reported good agreement with operating and/or test data where available.

In addition, Applied Flow Technology issues maintenance releases of the software periodically to improve performance and correct any problems that may have been discovered.

ANS Module Verification Methodology

Due to the nature of AFT Fathom's ANS (automated network sizing) module, there are several unique issues with verifying that the ANS module's solution represents the best configuration to minimize system cost.

The verification issues of the AFT Fathom ANS module can be summarized by two questions:

a. ***Will the system perform as specified?*** Because the results from each possible solution are obtained from a complete run of the hydraulic engine, the final sized system will perform as specified. The accuracy of the hydraulic solution is discussed in the Verification Methodology section and is supported by 70 verification examples and years of use in industry.
b. ***Is the sized solution the absolute optimum system?*** Because of the unique technology of ANS, this question is more difficult. The ANS module is the only automated network sizing tool available and there are no standards to compare against. We can have confidence in ANS for two reasons; first, we have verified the sizing of ANS against several literature examples of pipe and duct sizing, and, secondly, the underlying optimization technology is mature and has been used in industry for many years. If you are able to find a system that is better sized and still meets all the operational constraints of a system, please contact AFT Support.

1. Comparisons with open literature examples

To date, the ANS module has been compared with four published examples of problems that utilize the automated sizing features of the ANS module. The ANS module agrees favorably with all examples and matches exactly for discrete sizing problems where ANS is choosing from a set of nominal sizes.

2. Optimization Technology is Mature

The ANS module uses the optimization engine, VisualDOC, developed by Vanderplatts Research and Development. This optimization technology has been used by industry for many years and has been applied to a wide variety of problems in diverse engineering systems. Contact Vanderplatts Research and Development (www.vrand.com) for more information on this technology.

Verification Models

Summary of All Verification Models

Comparison of AFT Fathom predictions to the published calculation results is included herein for eighty cases from fifteen sources. Below is a summary of the cases.

Summary of Verification Models with Pumps

Summary of Verification Models with NPSH

Summary of Verification Models with PRVs

Summary of Verification Models with Slurries

Summary of Verification Models with ANS

Case	Fluid	Pipes	Pumps	NPSH	PRV's	Reference
Case 1	Water	3	0	No	0	Perry's (Tilton)
Case 2	Water	11	0	No	0	Nayyar (Swierzawski)
Case 3	Water	2	1	No	0	Miller
Case 4	Air	1	0	No	0	Miller
Case 5	Water	1	0	No	0	Miller
Case 6	Water	1	0	No	0	Brater, Williams, Lindell and Wei
Case 7	Unspecified	1	0	No	0	Brater, Williams, Lindell and Wei
Case 8	Water	4	0	No	0	Brater, Williams, Lindell and Wei
Case 9	Water	3	0	No	0	Brater, Williams, Lindell and Wei
Case 10	Water	6	0	No	0	Brater, Williams, Lindell and Wei
Case 11	Water	8	1	No	0	Brater, Williams, Lindell and Wei
Case 12	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 13	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case	Water	3	1	Yes	0	Ingersoll-Dresser Pumps

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
14						
Case 15	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 16	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 17	Water	2	1	No	0	Ingersoll-Dresser Pumps
Case 18	Specific gravity = 0.8	2	1	No	0	Karassik, Krutzsch, Fraser and Messina
Case 19	Water	1	0	No	0	Karassik, Krutzsch, Fraser and Messina
Case 20	Water	1	0	No	0	Karassik, Krutzsch, Fraser and Messina
Case 21	Water	1	0	No	0	Karassik, Krutzsch, Fraser and Messina
Case 22	Oil	2	0	No	0	Karassik, Krutzsch, Fraser and Messina
Case 23	Viscous Fluid	2	1	No	0	Hydraulic Institute
Case 24	Water	1	0	No	0	Lindeburg
Case 25	Water	1	0	No	0	Lindeburg
Case 26	Water	8	1	Yes	0	Lindeburg
Case 27	Air	1	0	No	0	Lindeburg
Case 28	Water	1	0	No	0	Crane
Case 29	Water	1	0	No	0	Crane
Case 30	SAE 10 Lube Oil	1	0	No	0	Crane
Case 31	SAE 70 Lube Oil	1	0	No	0	Crane

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 32	SAE 70 Lube Oil	4	0	No	0	Crane
Case 33	Water	10	0	No	0	Crane
Case 34	Fuel Oil	1	0	No	0	Crane
Case 35	Water	3	0	No	0	Crane
Case 36	Water	6	0	No	0	Crane
Case 37	Crude Oil 30 degree API	2	0	No	0	Crane
Case 38	Water	4	0	No	0	Crane
Case 39	Water	1	0	No	0	Fox and McDonald
Case 40	Water	2	0	No	0	Fox and McDonald
Case 41	Water	5	0	No	0	Fox and McDonald
Case 42	Water	2	0	No	0	Baumeister, Avallone and Baumeister
Case 43	Benzene	2	0	No	0	Baumeister, Avallone and Baumeister
Case 44	Ethyl Alcohol	3	0	No	0	Baumeister, Avallone and Baumeister
Case 45	Water	2	0	No	0	John and Haberman
Case 46	Water	7	0	No	0	John and Haberman
Case 47	Water	6	0	No	0	John and Haberman
Case 48	Water	2	0	No	0	John and Haberman
Case 49	Water	1	0	No	0	Janna

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 50	Water	1	0	No	0	Janna
Case 51	Benzene	1	0	No	0	Janna
Case 52	Turpentine	9	0	No	0	Janna
Case 53	Water	4	0	No	0	Janna
Case 54	Water	2	0	No	0	Janna
Case 55	Water	2	1	No	0	Janna
Case 56	Fuel Oil	1	0	No	0	Chopey
Case 57	Kerosene	1	0	No	0	Chopey
Case 58	Water	6	3	No	0	Chopey
Case 59	Water	10	3	No	0	Jeppson
Case 60	Water	9	1	No	1	Jeppson
Case 61	Water	17	1	No	0	Jeppson
Case 62	Water	48	0	No	0	Jeppson
Case 63	Water	8	1	No	0	Jeppson
Case 64	Water	10	1	No	1	Jeppson
Case 65	Water	12	0	No	2	Jeppson
Case 66	Water	16	1	No	1	Jeppson
Case 67	Water	31	3	No	0	Jeppson

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 68	Water	68	5	No	0	Jeppson
Case 69	Water	70	5	No	0	Jeppson
Case 70	Settling Slurry (Water/Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 71	Settling Slurry (Water/Coal)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 72	Settling Slurry (Water/Sand	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 73	Settling Slurry (Water/Ore)	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 74	Settling Slurry (Water/ Sand)	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 75	Settling Slurry (Water/ Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 76	Settling Slurry (Water/ Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 77	Methanol	1	0	No	0	Janna
Case 78	Water	2	1	No	0	John & Haberman
Case 79	Water	1	0	No	0	Fox & McDonald
Case 80	Air	4	0	No	0	Lindeburg

Summary of Verification Models with Pumps

Summary of All Verification Models

Comparison of AFT Fathom predictions for pumped systems to the published calculation results.

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 3	Water	2	1	No	0	Miller
Case 11	Water	8	1	No	0	Brater, Williams, Lindell and Wei
Case 12	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 13	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 14	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 15	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 16	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 17	Water	2	1	No	0	Ingersoll-Dresser Pumps
Case 18	Specific gravity = 0.8	2	1	No	0	Karassik, Krutzsch, Fraser and Messina
Case 23	Viscous Fluid	2	1	No	0	Hydraulic Institute
Case 26	Water	8	1	Yes	0	Lindeburg
Case 55	Water	2	1	No	0	Janna
Case 58	Water	6	3	No	0	Chopey
Case 59	Water	10	3	No	0	Jeppson
Case 60	Water	9	1	No	1	Jeppson
Case 61	Water	17	1	No	0	Jeppson

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 63	Water	8	1	No	0	Jeppson
Case 64	Water	10	1	No	1	Jeppson
Case 66	Water	16	1	No	1	Jeppson
Case 67	Water	31	3	No	0	Jeppson
Case 68	Water	68	5	No	0	Jeppson
Case 69	Water	70	5	No	0	Jeppson

Summary of Verification Models with NPSH

Summary of All Verification Models

Comparison of AFT Fathom predictions for pumped systems with NPSH requirements to the published calculation results.

Case	Fluid	Pipes	Pumps	NPSH	PRV's	Reference
Case 12	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 13	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 14	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 15	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 16	Water	3	1	Yes	0	Ingersoll-Dresser Pumps
Case 26	Water	8	1	Yes	0	Lindeburg

Summary of Verification Models with PRVs

Summary of All Verification Models

Comparison of AFT Fathom predictions for systems with PRVs to the published calculation results.

Case	Fluid	Pipes	Pumps	NPSH	PRV's	Reference
Case 60	Water	9	1	No	1	Jeppson
Case 64	Water	10	1	No	1	Jeppson
Case 65	Water	12	0	No	2	Jeppson
Case 66	Water	16	1	No	1	Jeppson

Summary of Verification Models with Slurries

Summary of All Verification Models

Comparison of AFT Fathom predictions for systems with slurries to the published calculation results.

Case	Fluid	Pipes	Pumps	NPSH	PRV 's	Reference
Case 70	Settling Slurry (Water-/Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 71	Settling Slurry (Water-/Coal)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 72	Settling Slurry (Water-/Sand)	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 73	Settling Slurry (Water-/Ore)	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 74	Settling Slurry (Water/ Sand)	2	1	No	0	Wilson, Addie, Sellgren & Clift
Case 75	Settling Slurry (Water/ Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift
Case 76	Settling Slurry (Water/ Sand)	1	0	No	0	Wilson, Addie, Sellgren & Clift

Summary of Verification Models with ANS

Summary of All Verification Models

Comparison of AFT Fathom predictions for systems sized using the automated network sizing (ANS) module to published sizing results.

Case	Fluid	Pipes	Pumps	NPSH	PRV's	Reference
Case 77	Methanol	1	0	No	0	Janna
Case 78	Water	2	1	No	0	John & Haberman
Case 79	Water	1	0	No	0	Fox & McDonald
Case 80	Air	4	0	No	0	Lindeburg

Verification Cases

Verification Case 1

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify1.fth

REFERENCE: Perry's Chemical Engineers' Handbook, 7th Ed., 1997, McGraw-Hill, Robert H. Perry, Don W. Green, Eds., James N. Tilton, Ph.D., P.E., Author, Page 6-17, 18, Example 6

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Tilton	AFT Fathom
EGLinlet - Energy Gradeline at inlet (meters)	0.738	0.7397

DISCUSSION:

The flow conditions are specified in terms of velocity, at 2.0 m/s. Converting this to flow rate (for diameter of .0525 m) obtains 0.00433 cubic meters per second. The problem is stated in terms of flow requirement with an unknown inlet tank height. This is most easily modeled using an Assigned Flow junction at the inlet. The EGL (energy gradeline) at the inlet will yield the tank height.

The inlet loss factor of 0.5 is modeled as part of the inlet Assigned Flow.

The friction factor in the reference is the Fanning friction factor, whereas AFT Fathom uses the Moody friction factor. The two differ by a factor of 4. If this ratio is applied, results using either friction factor are the same.

The problem does not state the discharge boundary condition very clearly, but reading through the solution procedure it is evident that discharge condition is static pressure at atmospheric pressure. This is modeled using an Assigned Pressure junction at 1 atm.

The loss factor at the elbow, as developed in the reference, is 0.37.

The problem solution as given in the reference is 0.73 feet. However, if the numbers as given in the reference are multiplied, a value of 0.738 m results. The author apparently rounded the value down to 0.73.

List of All Verification Models

Verification Case 1 Problem Statement

Verification Case 1

Perry' Chemical Engineers' Handbook, 7th Ed., 1997, McGraw-Hill, Robert H. Perry, Don W. Green, Eds., James N. Tilton, Ph.D., P.E., Author, Page 6-17, 18, Example 6

Perry's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Equation (6-93) agrees with experimental data (Kemblowski and Kiljanski, *Chem. Eng. J.* (*Lausanne*), **9,** 141–151 [1975]) for $\alpha < 11°$. For Newtonian liquids, Eq. (6-93) simplifies to

$$\Delta p = \mu \left(\frac{8V_2}{D_2} \right) \left\{ \frac{1}{6 \tan (\alpha/2)} \left[1 - \left(\frac{D_2}{D_1} \right)^3 \right] \right\} \qquad (6\text{-}94)$$

For creeping flow through rectangular or two-dimensional converging channels, the differential form of the Hagen-Poiseulle equation with equivalent diameter given by Eq. (6-49) may be used, provided the convergence is gradual.

Expansion and Exit Losses For ducts of any cross section, the frictional loss for a **sudden enlargement** (Fig. 6-13*c*) with turbulent flow is given by the Borda-Carnot equation:

$$l_v = \frac{V_1^2 - V_2^2}{2} = \frac{V_1^2}{2} \left(1 - \frac{A_1}{A_2} \right)^2 \qquad (6\text{-}95)$$

where V_1 = velocity in the smaller duct
V_2 = velocity in the larger duct
A_1 = cross-sectional area of the smaller duct
A_2 = cross-sectional area of the larger duct

Equation (6-95) is valid for incompressible flow. For compressible flows, see Benedict, Wyler, Dudek, and Gleed (*J. Eng. Power,* **98,** 327–334 [1976]). For an infinite expansion, $A_1/A_2 = 0$, Eq. (6-95) shows that the **exit loss** from a pipe is 1 velocity head. This result is easily deduced from the mechanical energy balance Eq. (6-90), noting that $p_1 = p_2$. This exit loss is due to the dissipation of the discharged jet; there is no pressure drop at the exit.

For creeping Newtonian flow (Re < 1), the frictional loss due to a sudden enlargement should be obtained from the same equation for a sudden contraction (Eq. [6-92]). Note, however, that Boger, Gupta, and Tanner (ibid.) give an exit friction equivalent length of 0.12 diameter, increasing for power law fluids as the exponent decreases. For laminar flows at higher Reynolds numbers, the pressure drop is twice that given by Eq. (6-95). This results from the velocity profile factor α in the mechanical energy balance being 2.0 for the parabolic laminar velocity profile.

If the transition from a small to a large duct of any cross-sectional shape is accomplished by a **uniformly diverging duct** (see Fig. 6-13*d*) with a straight axis, the total frictional pressure drop can be computed by integrating the differential form of Eq. (6-89), $dl_v/dx = 2fV^2/D$ over the length of the expansion, provided the total angle α between the diverging walls is less than 7°. For angles between 7 and 45°, the loss coefficient may be estimated as 2.6 sin (α/2) times the loss coefficient for a sudden expansion; see Hooper (*Chem. Eng.*, Nov. 7, 1988). Gibson (*Hydraulics and Its Applications*, 5th ed., Constable, London 1952, p. 93) recommends multiplying the sudden enlargement loss by 0.13 for $5° < \alpha < 7.5°$ and by $0.0110\alpha^{1.22}$ for $7.5° < \alpha < 35°$. For angles greater than 35 to 45°, the losses are normally considered equal to those for a sudden expansion, although in some cases the losses may be greater. Expanding flow through standard pipe reducers should be treated as sudden expansions.

Trumpet-shaped enlargements for **turbulent flow** designed for constant decrease in velocity head per unit length were found by Gibson (ibid., p. 95) to give 20 to 60 percent less frictional loss than straight taper pipes of the same length.

A special feature of expansion flows occurs when **viscoelastic** liquids are extruded through a die at a low Reynolds number. The extrudate may expand to a diameter several times greater than the die diameter, whereas for a Newtonian fluid the diameter expands only 10 percent. This phenomenon, called **die swell,** is most pronounced with short dies (Graessley, Glasscock, and Crawley, *Trans. Soc. Rheol.*, **14,** 519–544 [1970]). For velocity distribution measurements near the die exit, see Goulden and MacSporran (*J. Non-Newtonian Fluid Mech.*, **1,** 183–198 [1976]) and Whipple and Hill (*AIChE J.*, **24,** 664–671 [1978]). At high flow rates, the extrudate becomes distorted, suffering **melt fracture** at wall shear stresses greater than 10^5 N/m^2. This phenomenon is reviewed by Denn (*Ann. Review Fluid Mech.*, **22,** 13–34 [1990]). Ramamurthy (*J. Rheol.*, **30,** 337–357 [1986]) has found a dependence of apparent stick-slip behavior in melt fracture to be dependent on the material of construction of the die.

Fittings and Valves For **turbulent flow,** the frictional loss for fittings and valves can be expressed by the equivalent length or velocity head methods. As fitting size is varied, K values are relatively more constant than L_e/D values, but since fittings generally do not achieve geometric similarity between sizes, K values tend to decrease with increasing fitting size. Table 6-4 gives K values for many types of fittings and valves.

Manufacturers of valves, especially control valves, express valve capacity in terms of a flow coefficient C_v, which gives the flow rate through the valve in gal/min of water at 60°F under a pressure drop of 1 lbf/in^2. It is related to K by

$$C_v = \frac{C_1 d^2}{\sqrt{K}} \qquad (6\text{-}96)$$

where C_1 is a dimensional constant equal to 29.9 and d is the diameter of the valve connections in inches.

For **laminar flow,** data for the frictional loss of valves and fittings are meager. (Beck and Miller, *J. Am. Soc. Nav. Eng.*, **56,** 62–83 [1944]; Beck, ibid., **56,** 235–271, 366–388, 389–395 [1944]; De Craene, *Heat. Piping Air Cond.*, **27**[10], 90–95 [1955]; Karr and Schutz, *J. Am. Soc. Nav. Eng.*, **52,** 239–256 [1940]; and Kittredge and Rowley, *Trans. ASME,* **79,** 1759–1766 [1957]). The data of Kittredge and Rowley indicate that K is constant for Reynolds numbers above 500 to 2,000, but increases rapidly as Re decreases below 500. Typical values for K for laminar flow Reynolds numbers are shown in Table 6-5.

Methods to calculate losses for **tee and wye junctions** for dividing and combining flow are given by Miller (*Internal Flow Systems,* 2d ed., Chap. 13, BHRA, Cranfield, 1990), including effects of Reynolds number, angle between legs, area ratio, and radius. Junctions with more than three legs are also discussed. The sources of data for the loss coefficient charts are Blaisdell and Manson (*U.S. Dept. Agric. Res. Serv. Tech. Bull.* 1283 [August 1963]) for combining flow and Gardel (*Bull. Tech. Suisses Romande,* **85**[9], 123–130 [1957]; **85**[10], 143–148 [1957]) together with additional unpublished data for dividing flow.

Miller (*Internal Flow Systems*, 2d ed., Chap. 13, BHRA, Cranfield, 1990) gives the most complete information on losses in **bends and curved pipes.** For turbulent flow in circular cross-section bends of constant area, as shown in Fig. 6-14*a*, a more accurate estimate of the loss coefficient K than that given in Table 6-4 is

$$K = K^* C_{Re} C_o C_f \qquad (6\text{-}97)$$

where K^*, given in Fig. 6-14*b*, is the loss coefficient for a smooth-walled bend at a Reynolds number of 10^6. The Reynolds number correction factor C_{Re} is given in Fig. 6-14*c*. For $0.7 < r/D < 1$ or for $K^* < 0.4$, use the C_{Re} value for $r/D = 1$. Otherwise, if $r/D < 1$, obtain C_{Re} from

$$C_{Re} = \frac{K^*}{K^* + 0.2(1 - C_{Re,\, r/D=1})} \qquad (6\text{-}98)$$

The correction C_o (Fig. 6-14*d*) accounts for the extra losses due to developing flow in the outlet tangent of the pipe, of length L_o. The total loss for the bend plus outlet pipe includes the bend loss K plus the straight pipe frictional loss in the outlet pipe $4fL_o/D$. Note that $C_o = 1$ for L_o/D greater than the termination of the curves on Fig. 6-14*d*, which indicate the distance at which fully developed flow in the outlet pipe is reached. Finally, the roughness correction is

$$C_f = \frac{f_{rough}}{f_{smooth}} \qquad (6\text{-}99)$$

where f_{rough} is the friction factor for a pipe of diameter D with the roughness of the bend, at the bend inlet Reynolds number. Similarly, f_{smooth} is the friction factor for smooth pipe. For Re > 10^6 and $r/D \geq 1$, use the value of C_f for Re = 10^6.

Example 6: Losses with Fittings and Valves It is desired to calculate the liquid level in the vessel shown in Fig. 6-15 required to produce a discharge velocity of 2 m/s. The fluid is water at 20°C with ρ = 1,000 kg/m^3 and μ = 0.001 Pa · s, and the butterfly valve is at $\theta = 10°$. The pipe is 2-in Schedule 40, with an inner diameter of 0.0525 m. The pipe roughness is 0.046 mm. Assuming the flow is turbulent and taking the velocity profile factor $\alpha = 1$, the engineering Bernoulli equation Eq. (6-16), written between surfaces 1 and 2, where the

TABLE 6-4 Additional Frictional Loss for Turbulent Flow through Fittings and Valves[a]

Type of fitting or valve	Additional friction loss, equivalent no. of velocity heads, K
45° ell, standard[b,c,d,e,f]	0.35
45° ell, long radius[c]	0.2
90° ell, standard[b,c,e,f,g,h]	0.75
Long radius[b,c,d,e]	0.45
Square or miter[h]	1.3
180° bend, close return[b,c,e]	1.5
Tee, standard, along run, branch blanked off[e]	0.4
Used as ell, entering run[g,i]	1.0
Used as ell, entering branch[c,g,i]	1.0
Branching flow[i,j,k]	1[l]
Coupling[c,e]	0.04
Union[e]	0.04
Gate valve,[b,e,m] open	0.17
¾ open[n]	0.9
½ open[n]	4.5
¼ open[n]	24.0
Diaphragm valve,[o] open	2.3
¾ open[n]	2.6
½ open[n]	4.3
¼ open[n]	21.0
Globe valve,[e,m]	
Bevel seat, open	6.0
½ open[n]	9.5
Composition seat, open	6.0
½ open[n]	8.5
Plug disk, open	9.0
¾ open[n]	13.0
½ open[n]	36.0
¼ open[n]	112.0
Angle valve,[b,e] open	2.0
Y or blowoff valve,[b,m] open	3.0
Plug cock[p]	
$\theta = 5°$	0.05
$\theta = 10°$	0.29
$\theta = 20°$	1.56
$\theta = 40°$	17.3
$\theta = 60°$	206.0
Butterfly valve[p]	
$\theta = 5°$	0.24
$\theta = 10°$	0.52
$\theta = 20°$	1.54
$\theta = 40°$	10.8
$\theta = 60°$	118.0
Check valve,[b,e,m] swing	2.0[q]
Disk	10.0[q]
Ball	70.0[q]
Foot valve[e]	15.0
Water meter,[h] disk	7.0[r]
Piston	15.0[r]
Rotary (star-shaped disk)	10.0[r]
Turbine-wheel	6.0[r]

[a] Lapple, *Chem. Eng.*, **56**(5), 96–104 (1949), general survey reference.
[b] "Flow of Fluids through Valves, Fittings, and Pipe," Tech. Pap. 410, Crane Co., 1969.
[c] Freeman, *Experiments upon the Flow of Water in Pipes and Pipe Fittings*, American Society of Mechanical Engineers, New York, 1941.
[d] Giesecke, *J. Am. Soc. Heat. Vent. Eng.*, **32,** 461 (1926).
[e] *Pipe Friction Manual*, 3d ed., Hydraulic Institute, New York, 1961.
[f] Ito, *J. Basic Eng.*, **82,** 131–143 (1960).
[g] Giesecke and Badgett, *Heat. Piping Air Cond.*, **4**(6), 443–447 (1932).
[h] Schoder and Dawson, *Hydraulics*, 2d ed., McGraw-Hill, New York, 1934, p. 213.
[i] Hoopes, Isakoff, Clarke, and Drew, *Chem. Eng. Prog.*, **44,** 691–696 (1948).
[j] Gilman, *Heat. Piping Air Cond.*, **27**(4), 141–147 (1955).
[k] McNown, *Proc. Am. Soc. Civ. Eng.*, **79,** Separate 258, 1–22 (1953); discussion, ibid., **80,** Separate 396, 19–45 (1954). For the effect of branch spacing on junction losses in dividing flow, see Hecker, Nystrom, and Qureshi, *Proc. Am. Soc. Civ. Eng., J. Hydraul. Div.*, **103**(HY3), 265–279 (1977).
[l] This is pressure drop (including friction loss) between run and branch, based on velocity in the mainstream before branching. Actual value depends on the flow split, ranging from 0.5 to 1.3 if mainstream enters run and from 0.7 to 1.5 if mainstream enters branch.
[m] Lansford, *Loss of Head in Flow of Fluids through Various Types of 1½-in. Valves*, Univ. Eng. Exp. Sta. Bull. Ser. 340, 1943.

pressures are both atmospheric and the fluid velocities are 0 and V = 2 m/s, respectively, and there is no shaft work, simplifies to

$$gZ = \frac{V^2}{2} + l_v$$

Contributing to l_v are losses for the entrance to the pipe, the three sections of straight pipe, the butterfly valve, and the 90° bend. Note that no exit loss is used because the discharged jet is outside the control volume. Instead, the $V^2/2$ term accounts for the kinetic energy of the discharging stream. The Reynolds number in the pipe is

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{0.0525 \times 2 \times 1000}{0.001} = 1.05 \times 10^5$$

From Fig. 6-9 or Eq. (6-38), at $\epsilon/D = 0.046 \times 10^{-3}/0.0525 = 0.00088$, the friction factor is about 0.0054. The straight pipe losses are then

$$l_{v(sp)} = \left(\frac{4fL}{D}\right)\frac{V^2}{2}$$
$$= \left(\frac{4 \times 0.0054 \times (1+1+1)}{0.0525}\right)\frac{V^2}{2}$$
$$= 1.23\,\frac{V^2}{2}$$

The losses from Table 6-4 in terms of velocity heads K are $K = 0.5$ for the sudden contraction and $K = 0.52$ for the butterfly valve. For the 90° standard radius (r/D = 1), the table gives $K = 0.75$. The method of Eq. (6-94), using Fig. 6-14, gives

$$K = K^* C_{\text{Re}} C_o C_f$$
$$= 0.24 \times 1.24 \times 1.0 \times \left(\frac{0.0054}{0.0044}\right)$$
$$= 0.37$$

This value is more accurate than the value in Table 6-4. The value $f_{smooth} = 0.0044$ is obtainable either from Eq. (6-37) or Fig. 6-9.

The total losses are then

$$l_v = (1.23 + 0.5 + 0.52 + 0.37)\,\frac{V^2}{2} = 2.62\,\frac{V^2}{2}$$

and the liquid level Z is

$$Z = \frac{1}{g}\left(\frac{V^2}{2} + 2.62\,\frac{V^2}{2}\right) = 3.62\,\frac{V^2}{2g}$$
$$= \frac{3.62 \times 2^2}{2 \times 9.81} = 0.73 \text{ m}$$

Curved Pipes and Coils For flow through curved pipe or coil, a secondary circulation perpendicular to the main flow called the **Dean effect** occurs. This circulation increases the friction relative to straight pipe flow and stabilizes laminar flow, delaying the transition Reynolds number to about

$$\text{Re}_{crit} = 2{,}100\left(1 + 12\sqrt{\frac{D}{D_c}}\right) \tag{6-100}$$

where D_c is the coil diameter. Equation (6-100) is valid for $10 < D_c/D < 250$. The **Dean number** is defined as

$$\text{De} = \frac{\text{Re}}{(D_c/D)^{1/2}} \tag{6-101}$$

In laminar flow, the friction factor for curved pipe f_c may be expressed in terms of the straight pipe friction factor $f = 16/\text{Re}$ as (Hart, *Chem. Eng. Sci.*, **43,** 775–783 [1988])

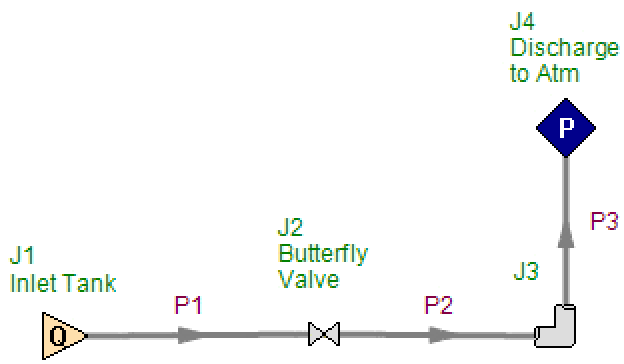
TABLE 6-5 Additional Frictional Loss for Laminar Flow through Fittings and Valves

Type of fitting or valve	Additional frictional loss expressed as K			
	Re = 1,000	500	100	50
90° ell, short radius	0.9	1.0	7.5	16
Gate valve	1.2	1.7	9.9	24
Globe valve, composition disk	11	12	20	30
Plug	12	14	19	27
Angle valve	8	8.5	11	19
Check valve, swing	4	4.5	17	55

SOURCE: From curves by Kittredge and Rowley, *Trans. Am. Soc. Mech. Eng.*, **79,** 1759–1766 (1957).

View Verification Case 1 Model

Verification Case 1


J4
Discharge
to Atm
P
P3
J2
Butterfly
Valve
J1
Inlet Tank
J3
P1
P2
Q

Verification Case 2

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify2.fthFthVerify2.fth

REFERENCE: Piping Handbook, 7th Ed., 2000, McGraw-Hill, Mohinder L. Nayyar, P.E., Ed., Tadeusz J. Swierzawski, Author, Page B.375-381, Example B8.1

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Swierzawski	AFT Fathom
Pressure drop (psid)	61.48	61.33

DISCUSSION:

The exit loss factor of 1.0 is modeled as part of the exit Assigned Flow.

The overall pressure drop in AFT Fathom is obtained by subtracting the discharge pressure (438.67 psia) from the supply (500 psia). It is also displayed in the General Results list at the top of the Output window. Finally, the pressure difference is given in the Junction Deltas table at the top of the Output window. The junction delta was setup in the Output Control window.

On page B.380 the Piping Handbook calculates the Flow Resistance Coefficients as 9.77 as shown in the table on that page. However, if one adds up the individual K factors, 9.84 is obtained, not 9.77. To better match the overall resistance assumed for the handbook calculation, the tee K factor was therefore reduced from 0.33 to 0.26 so that a K factor of 9.77 is obtained.

The difference in the final answer (61.48 vs. 61.33) is mostly due to round-off errors in the handbook calculations. If more digits are saved in the handbook calculations, the answer will be much closer to 61.33.

List of All Verification Models

Verification Case 2 Problem Statement

Verification Case 2

Piping Handbook, 7th Ed., 2000, McGraw-Hill, Mohinder L. Nayyar, P.E., Ed., Tadeusz J. Swierzawski, Author, Page B.375-381, Example B8.1

Nayyar Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

$d2/d1$	$(d1/d2)^5$
0.99	1.050
0.98	1.106
0.97	1.165
0.96	1.230
0.95	1.292
0.90	1.694
0.80	3.052
0.70	5.950

in Fig. B8.7. In addition, there is the cost of the pump and the pump drive, which increases as greater pumping power is required, as represented by cost *c*.

The total cost, which is the sum of these three costs ($T_{COST} = a + b + c$) in $/year, is shown on Fig. B8.7 as reaching a minimum value at the optimum flow velocity. The analysis for each piping system should consider the optimization of flow velocity (optimum internal diameter) of the pipe under consideration. It is important to note that cost *b* depends strongly on the plant operating mode or the load factor, and on other economic indicators for a particular project. Industry data containing updated prices of equipment, piping, and labor are needed to implement this optimization procedure.

Sample Problem B8.1. What is the pressure drop $p_A - p_B$ (see Fig. B8.8), when water at 200°F (93.3°C) flows in a piping system at the rate of $\dot{m}$ = 450,200 lb_m/h (204,207 kg/h), p_A = 500 psia (34.47 bar)?

Pipe Data. For 6-in nominal size, schedule 40 pipe, the internal diameter d = 6.065 in, D = 0.5054 ft, A = 0.200 ft².

Properties of Fluid. From the ASME Steam Tables software (Ref. 5) for T = 200°F and p = 500 psia, the dynamic viscosity of water (see attached computer printout, Table B8.9) is

$$\mu = 63.43 * 10^{-7}\ (\mathrm{lb_f \cdot s})/\mathrm{ft}^2$$

From the same computer printout, the specific volume of water at T = 200°F and p = 500 psia is

$$v = 0.01661\ \mathrm{ft^3/lb_m}$$

Then, the kinematic viscosity of water is [see Eq. (B8.3)]:

$$\nu = \frac{\mu}{\rho} g_c = (63.43 * 10^{-7})\,(0.01661)\,(32.174)$$

$$= 3.39 * 10^{-6}\ \mathrm{ft^2/s}$$

The flow velocity is calculated from the continuity equation

$$w = \frac{\dot{m}}{\rho A}$$

$$\text{where } \dot{m} = \frac{450{,}200}{3600} = 125.06\ \mathrm{lb_m/s}$$

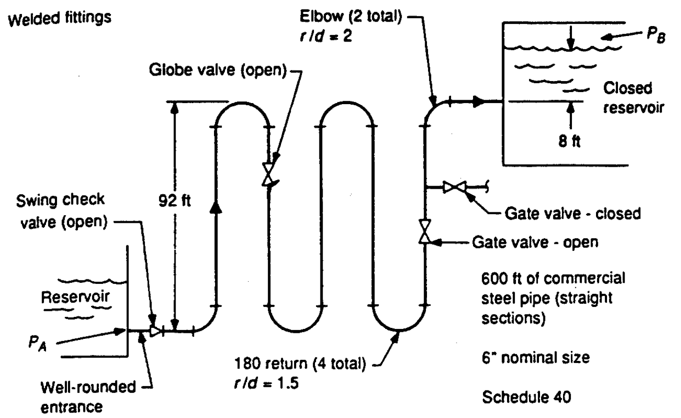


FIGURE B8.8 Reference for sample problem B8.1.

TABLE B8.9 Computer Printout of Steam/Water Properties

STEAM PROPERTIES		INPUT	*	RESULTS
ABS. PRESSURE,	bar	.0000		34.4738
	psia	500.0000		500.0000
TEMPERATURE,	C	.0000		93.3333
	F	200.0000		200.0000
ENTHALPY,	kJ/kg	.0000		393.5460
	Btu/lbm	.0000		169.1943
ENTROPY,	kJ/(kg.K)	.000000		1.228453
	Btu/(lbm.R)	.000000		.293411
SPEC.VOLUME	m3/kg	.000000		.001037
	ft3/lbm	.000000		.016609
QUALITY,	decimal	.000000		-1.000000
DYNAM. VISC	N.s/m2	.000000000		.000303681
	lbf.s/ft2	.000000000		.000006343

UNITS FOR INPUT: UNT = 1-SI, 2-BRITISH * 1995/TJS
UNT PRESSURE TEMPERAT ENTHALPY ENTROPY QUALITY

Then:

$$w = \frac{125.06}{0.200} 0.01661 = 10.39 \text{ ft/s}$$

Using Eq. (B8.16), the Reynolds number is

$$\text{Re} = \frac{Dw}{\nu} = \frac{(0.5054)(10.39)}{3.39 * 10^{-6}} = 1.549 * 10^6$$

D'Arcy-Weisbach Friction Factor. For commercial steel pipe from Ref. 7, the relative roughness is

$$\frac{\varepsilon}{D} = 0.00015/0.5054 = 0.0003$$

and the friction factor in zone of complete turbulence is

$$f_T = 0.015$$

From Eq. (B8.17), using, for example, the Newton-Raphson method for solving equations [or using the Moody diagram, Fig. (B8.5)], for calculated values of Re and ε/D, the D'Arcy-Weisbach friction factor, f, is found:

$$f = 0.0154$$

Flow Resistance Coefficients[7]:

Local disturbance	$K(f_T = 0.015)$
Entrance	$K = 0.04$
4 180° turns	$(4)(22.18\, f_T) = 1.33$
1 globe valve (open)	$340\, f_T = 5.10$
1 tee flow through run	$20\, f_T = 0.33$
1 swing check valve (open)	$100\, f_T = 1.50$
2 long radius elbows	$(2)(14 f_T) = 0.42$
1 gate valve (open)	$8\, f_T = 0.12$
Exit	$K = 1.00$
	$\sum K = 9.77$

Total Resistance

$$K_{tot} = \sum K + f\frac{L}{D}$$

$$= 9.77 + 0.0154 \frac{600}{0.5054}$$

$$= 28.05$$

"Friction" Pressure Drop in Pipe. From Eqs. (B8.14) and (B8.18):

$$\Delta p = K_{\text{tot}}\,\rho\,\frac{w^2}{2g_c}$$

$$= 28.05\,\frac{(10.39)^2}{(0.01661)(2)(32.174)}$$

$$= 2833.08\,\frac{\text{lb}_f}{\text{ft}^2}$$

$$= \frac{2833.08}{144} = 19.67 \text{ psi}$$

Pressure Difference $p_A - p_B$. From Bernoulli's equation (Eq. B8.12 with $H_f = 0$ and $H_p = 0$):

$$\frac{\rho_A w_A^2}{2g_c} + p_A + \frac{g}{g_c} z_A \rho_A$$

$$= \frac{\rho_B w_B^2}{2g_c} + p_B + \frac{g}{g_c} z_B \rho_B + \Delta \rho$$

Assuming $\rho_A = \rho_B = \rho$ and $g = 32.174\,\frac{\text{ft}}{\text{s}^2}$

$$p_A - p_B = \frac{g}{g_c}(z_B - z_A)\frac{\rho}{144} + \Delta p$$

$$= \frac{100}{(144)(0.01661)} + 19.67 = 61.48 \text{ psi (4.24 bar)}$$

Applications: Oil and Other Liquid Systems

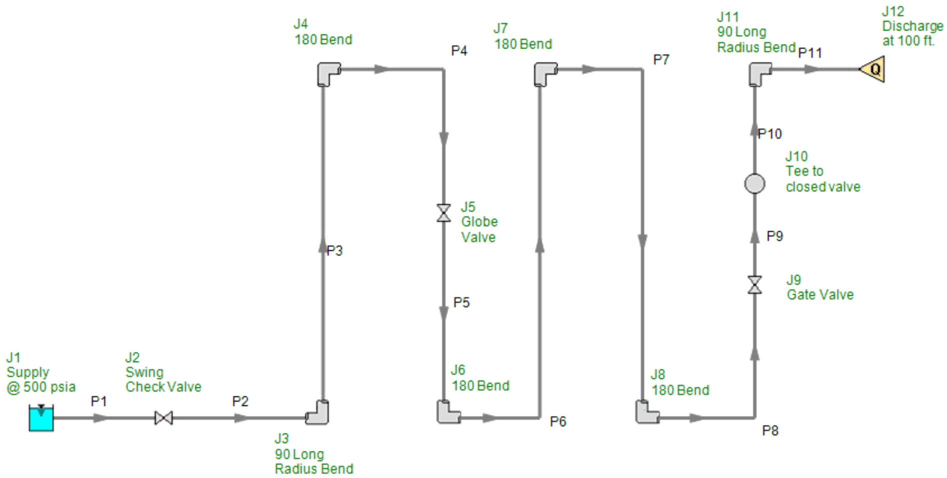
The calculation of oil flow through pipes is a much more complicated process than a similar calculation for water flow. Water properties are well-defined and many, including viscosity, are nearly constant within liquid temperature ranges, whereas oil is quite different. No two oils have the same physical properties, and any given oil is subject to important physical changes at expected variable temperatures. When considering oil flow in pipes, the most important variable physical property is viscosity.

Ordinary crude oil is not a homogeneous liquid; it is a very complex fluid composed of compounds of carbon and hydrogen which exist in petroleum in a wide variety of combinations. Physical properties change accordingly, and they also change as a result of temperature variations. Therefore, this handbook assumes that the user will research the fluid of interest in other sourcebooks (such as the *Petroleum Processing Handbook*[9]) to determine important properties including density and viscosity.

The "Viscosity" subsection in this handbook refers to Tables B8.5 and B8.6 for conversion among several common units of viscosity. The special names and symbols for derived CGS units (such as dyne, erg, poise, and stokes) should not be used

View Verification Case 2 Model

Verification Case 2



Verification Case 3

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify3.fth

REFERENCE: D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 28-29

FLUID: Water

ASSUMPTIONS: Size pump for given flow

RESULTS:

Parameter	Miller	AFT Fathom
System head loss (meters)	21.6	21.51
Required pump head rise (meters)	28.6	28.51
Requred pump pressure rise (bar)	2.8	2.796

DISCUSSION:

The reflux valve K value of 0.5 was lumped into pipe 2 as a fitting and loss value.

The overall system head loss in AFT Fathom is obtained by subtracting the discharge EGL (1.5 m) from the supply EGL (8.5 m), then subtracting this difference from the pump head rise (28.51 m).

The pump head rise and pressure rise in AFT Fathom is given in the pump summary at the top of the Output window, and also in the junction output table in the lower part of the window.

There is some slight disagreement with Miller due to round-off errors.

List of All Verification Models

Verification Case 3 Problem Statement

Verification Case 3

D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 28-29

Miller Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

and D is the hydraulic diameter or pipe diameter. For a passage D is given by

$$D = \frac{4 \times \text{Passage cross-sectional area}}{\text{Passage perimeter}}$$

The loss coefficients, K, in equation (3.1) are provided as a function of the Reynolds number. The coefficients can be found from Part 2 or from additional sources.

The overall system head loss, H, or pressure loss, P, is found by summing the head and pressure losses for individual components. For a system with n components

$$H_{\text{overall}} = \sum_{i=1}^{i=n} \Delta H_i \tag{3.2a}$$

$$P_{\text{overall}} = \sum_{i=1}^{i=n} \Delta P_i = \rho g H_{\text{overall}} \tag{3.2b}$$

To obtain the duty of a pump or fan it is necessary to add the static lift or pressure difference across the ends of the system to the values obtained from equations (3.2a) or (3.2b).

3.3. SYSTEM CALCULATION AND PUMP OR FAN SELECTION

The method adopted for carrying out a calculation of overall pressure loss depends on individual preference and the suitability of the method for particular types of system. For instance, if the mean velocity is constant throughout the system, the individual loss coefficients could be summed and multiplied by the velocity head to obtain the overall head loss.

Complexities arise mainly from interaction between closely spaced components involving a departure from simple summing of individual component losses. For systems consisting mainly of straight pipes or passages, interaction effects are seldom important as regards pressure losses. If the distance between components is more than four diameters, neglecting interaction effects will usually result in the loss being slightly overestimated.

As an example of one calculation procedure, consider the simplified system in Fig. 3.1. For calculation purposes the components are best represented connected together at nodes as in Fig. 3.2, pipes being treated in the same manner as components. This type of system representation has advantages when carrying out computer solutions of complex systems and pipe networks.

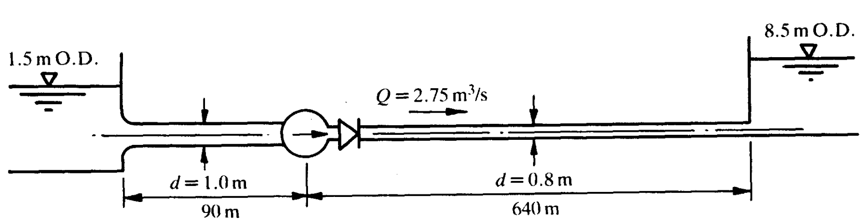


Fig. 3.1. Simple system

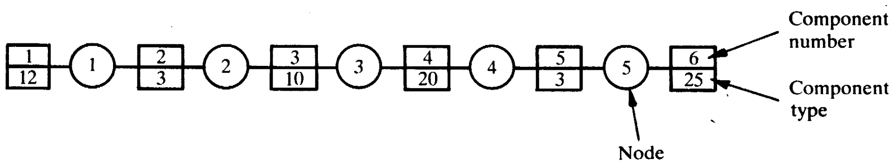


Fig. 3.2. Node and component representation of the system in Fig. 3.1

Table 3.1. Calculation for the system shown in Fig. 3.1

($\rho = 1000$ kg/m^3, $\nu = 1.1 \times 10^{-6}$ m^2/s, friction coefficient taken as 0.015, including an allowance for fouling)

Node	Component No.	Component Type	Diameter (m)	Velocity (m/s)	Reynolds number	$U^2/2g$ (m)	K*	Head loss (m)	Pressure loss (kN/m^2)
1	1	12–reservoir inlet	1.0	3.50	3.2×10^6	0.625	0.1	0.06	0.589
1–2	2	3–pipe	1.0	3.50	3.2×10^6	0.625	1.35	0.84	8.240
2–3	3	10–pump	—	—	—	—	—	—	—
3–4	4	20–reflux valve	0.8	5.47	4.0×10^6	1.53	0.5	0.76	7.456
4–5	5	3–pipe	0.8	5.47	4.0×10^6	1.53	12.0	18.36	180.112
5	6	25–reservoir outlet	0.8	5.47	4.0×10^6	1.53	1.0	1.53	15.000
							Total loss =	21.55	211

*For pipes $K = fL/D$.

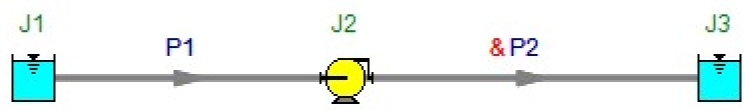
From Part 2 the loss coefficients, K, in Table 3.1 are appropriate for the various components of Fig. 3.1. Other relevant data are given in Table 3.1 to which, in some cases, it will be necessary to add details of pipe roughness and of interaction correction factors if components are close together. An example using interaction correction factors is included in Chapter 10.

$$\text{Pump head} = \text{Total head loss} + \text{Static lift}$$
$$= 21.6 \text{ m} + 7 \text{ m}$$
$$= 28.6 \text{ m}$$
$$= 28.6 \times 1000 \times 9.81 = 280 \text{ kN/m}^2 = 2.8 \text{ bar}$$

A pump or fan that most closely meets a design duty will usually be selected from a standard range. The head-flow characteristic of a suitable pump for the system in the example is shown in Fig. 3.3. Also shown in Fig. 3.3 is the system characteristic. For most purposes

View Verification Case 3 Model

Verification Case 3



Verification Case 4

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify4.fth

REFERENCE: D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 197-199

FLUID: Air

ASSUMPTIONS: Air is incompressible

RESULTS:

Parameter	Miller	AFT Fathom
System head loss (m of air)	40.3	40.36
System head loss (mm of water)	49.6	49.64

DISCUSSION:

The velocity and diameter are specified. For use in AFT Fathom, this is converted to a flow rate, obtaining 0.8 m3/sec.

The head loss in the pipe in AFT Fathom can be easily obtained from the pipe dP Stag and dH output values.

List of All Verification Models

Verification Case 4 Problem Statement

Verification Case 4

D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 197-199

Miller Title Page

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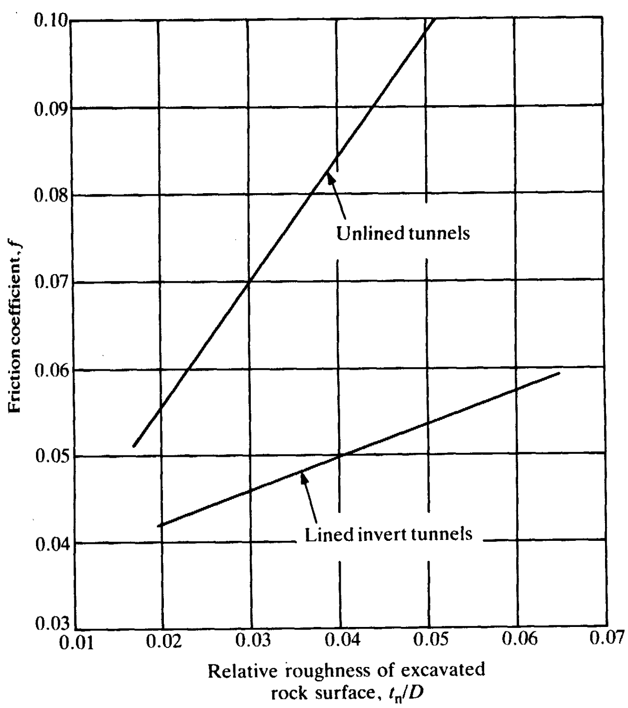


Fig. 8.5(1). Friction coefficients for rock tunnels

8.6. EXAMPLES

EXAMPLE 1: GIVEN *Q, L, D, v* AND *k*, FIND THE HEAD LOSS

Calculate the head loss for air flowing in a 0.2 m square cross-section passage 25 m long, given:

$Q = 0.8\ \text{m}^3/\text{s}$ Air density $= 1.23\ \text{kg/m}^3$

Kinematic viscosity $= 1.45 \times 10^{-5}\ \text{m}^2/\text{s}$ Surface roughness, $k = 0.025$ mm

From equation (8.2)

$$D = 4A/P_r = 4 \times 0.2^2/4 \times 0.2 = 0.2\ \text{m}$$

Relative roughness

$$k/D = 0.025 \times 10^{-3}/0.2 = 1.2 \times 10^{-4}$$

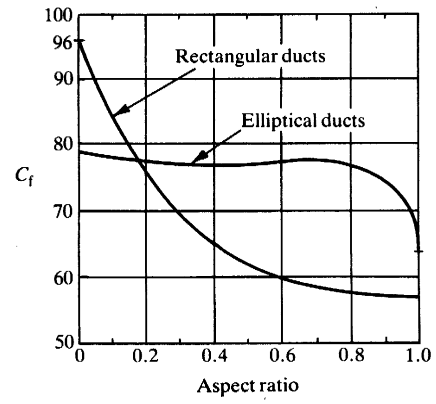


Fig. 8.6. Laminar flow coefficients — rectangular and elliptical cross-sections

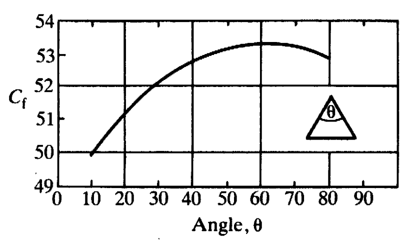


Fig. 8.7. Laminar flow coefficients — isosceles triangle

Mean velocity

$$U = Q/A = 0.8/0.2 \times 0.2 = 20 \text{ m/s}$$

Reynolds number

$$\text{Re} = DU/\nu = 0.2 \times 20/1.45 \times 10^{-5} = 2.8 \times 10^5$$

From equation (8.4)

$$f = 0.25\bigg/\left[\log\left(\frac{k}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2 = 0.25\bigg/\left[\log\left(\frac{1.2 \times 10^{-4}}{3.7} + \frac{5.74}{(2.8 \times 10^5)^{0.9}}\right)\right]^2 = 0.0158$$

For a Reynolds number of 2.8×10^5 and a relative roughness of 1.2×10^{-4} the friction coefficient from Fig. 8.1 is also 0.0158.

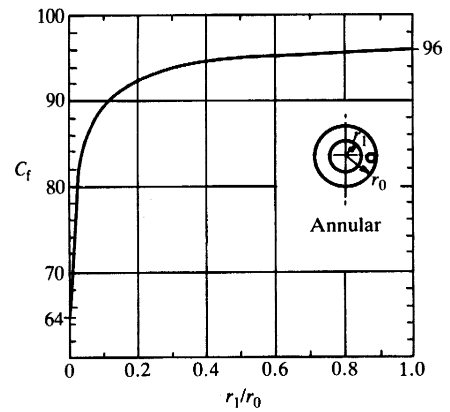


Fig. 8.8. Laminar flow coefficients — annular cross-sections

From equation (8.3)

$$K_f = fL/D = 0.0158 \times 25/0.2 = 1.975$$

Head loss

$$\Delta H = K_f U^2/2g = 1.975 \times 20^2/(2 \times 9.81)$$
$$= 40.3 \text{ m of air} = 49.6 \text{ mm of water}$$

EXAMPLE 2: GIVEN ΔH, L, D, ν AND k, FIND THE FLOW RATE Q

Water at 15°C flows by gravity between two tanks in a system consisting of 50 mm diameter pipes and components. The combined loss coefficient for the components is $K_t = 6.5$ for Reynolds numbers greater than 10^5. If the total length of the straight pipes is 40 m, determine the flow for a head differential between the tanks of 6 m. Assume that the pipe is smooth. Kinematic viscosity = 1.14×10^{-6} m^2/s.

First trial

Assume $f = 0.015$, which lies in the middle of the smooth curve in Fig. 8.1.
Pipe loss coefficient

$$K_f = fL/D = 0.015 \times 40/50 \times 10^{-3} = 12$$

View Verification Case 4 Model

Verification Case 4

Verification Case 5

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify5.fth

REFERENCE: D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 199-200

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Miller	AFT Fathom
Flow rate (m3/sec) x 1000	4.68	4.674

DISCUSSION:

The K factor of 6.5 is entered in pipe 1 as a fitting and loss value.

The answer differs slightly because Miller reads the friction factor (0.0179) off of a chart, whereas AFT Fathom uses a roughness of zero to obtain the friction factor (0.01784).

List of All Verification Models

Verification Case 5 Problem Statement

Verification Case 5

D.S. Miller, Internal Flow Systems, 2nd Ed., 1990, Gulf Publishing Co., Page 199-200

Miller Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Friction in pipes and passages 199

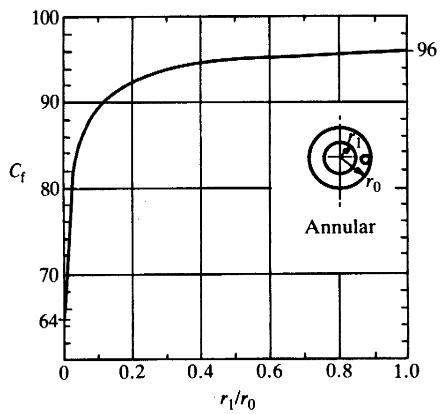


Fig. 8.8. Laminar flow coefficients — annular cross-sections

From equation (8.3)

$$K_f = fL/D = 0.0158 \times 25/0.2 = 1.975$$

Head loss

$$\Delta H = K_f U^2/2g = 1.975 \times 20^2/(2 \times 9.81)$$
$$= 40.3 \text{ m of air} = 49.6 \text{ mm of water}$$

EXAMPLE 2: GIVEN ΔH, L, D, ν AND k, FIND THE FLOW RATE Q

Water at 15°C flows by gravity between two tanks in a system consisting of 50 mm diameter pipes and components. The combined loss coefficient for the components is $K_t = 6.5$ for Reynolds numbers greater than 10^5. If the total length of the straight pipes is 40 m, determine the flow for a head differential between the tanks of 6 m. Assume that the pipe is smooth. Kinematic viscosity = 1.14×10^{-6} m^2/s.

First trial

Assume $f = 0.015$, which lies in the middle of the smooth curve in Fig. 8.1.
Pipe loss coefficient

$$K_f = fL/D = 0.015 \times 40/50 \times 10^{-3} = 12$$

Head loss

$$\Delta H = 6\ \text{m} = (K_f + K_t)U^2/2g$$

Mean velocity

$$U = \sqrt{\frac{6 \times 2 \times 9.81}{(12+6.5)}} = 2.52\ \text{m/s}$$

Reynolds number

$$\text{Re} = \frac{UD}{\nu} = \frac{2.52 \times 50 \times 10^{-3}}{1.14 \times 10^{-6}} = 0.111 \times 10^6$$

From Fig. 8.1

$$f = 0.0178$$

Second trial

Assume $f = 0.0178$

$$K_f = 12 \times 0.0178/0.015 = 14.2$$

$$U = 2.52\sqrt{\frac{12+6.5}{14.2+6.5}} = 2.38\ \text{m/s}$$

$$\text{Re} = 0.111 \times 10^6 \times 2.38/2.52 = 0.105 \times 10^6$$

from Fig. 8.1, $f = 0.0179$, which agrees with the assumed value to within the accuracy of Fig. 8.1.

Flow

$$Q = UA = 2.38 \times 0.785 \times 50^2 \times 10^{-6}$$
$$= 4.68 \times 10^{-3}\ \text{m}^3/\text{s}$$

EXAMPLE 3: GIVEN ΔH, Q, L, ν AND k, FIND THE DIAMETER

0.58 m^3 of water at 15°C has to flow through a constant diameter system with a head loss of not more than 3 m of water-gauge. The system consists of a number of sections of straight new steel pipe with a total length of 145 m, and a number of components. The estimated total loss coefficient for the components is $K_t = 2.9$ at Reynolds numbers greater than 10^6. Determine the minimum pipe and component diameter required. Kinetic viscosity $= 1.14 \times 10^{-6}\ \text{m}^2/\text{s}$. From Table 8.1, hydraulic roughness, $k = 0.025$ mm.

First trial

Assume diameter $D = 0.6$ m.
Mean velocity

$$U = Q/A = 0.58/(0.785 \times 0.6^2) = 2.052\ \text{m/s}$$

View Verification Case 5 Model

Verification Case 5

Verification Case 6

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify6.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.30, Example 6.1a

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (meters)	1.53	1.614

DISCUSSION:

The velocity and diameter are specified. For use in AFT Fathom, this is converted to a flow rate, obtaining 0.5655 m3/s.

The answers differ slightly because the handbook authors read the friction factor (0.015) off of a chart, whereas AFT Fathom solves for the friction factor using the Colebrook-White equation (obtaining 0.01583, 5.5% higher).

If the pipe is modeled as an explicit friction factor at 0.015, the exact same answer as the handbook is obtained.

No pressure boundary conditions are specified. In determining pressure drop, the boundary pressure plays no role. Therefore any value is suitable. The model assumes 1 MPa at the inlet.

List of All Verification Models

Verification Case 6 Problem Statement

Verification Case 6

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.30, Example 6.1a

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

ferred to[10,16] for corrugated-metal pipes resulted in values of f varying from 0.051 to 0.078, a range of 53 percent based on the smaller value, whereas for the same tests the values of n varied from 0.0229 to 0.0248, a range of only 8 percent based on the smaller value. Obviously, it is simpler and perhaps more accurate to use an n of 0.024 for all corrugated-metal pipes in the range of diameters and Reynolds numbers tested than to use the somewhat more cumbersome Darcy-Weisbach equation. Furthermore, extrapolation to other pipe sizes might also be more safely accomplished in this case by using the Manning equation. However, particularly for smoother pipe and large values of the Reynolds number, more dependable results can be obtained by means of the Darcy-Weisbach equation.

Although the developments described in the preceding section have vastly improved the understanding of the mechanics of energy dissipation in conduits, the problem is not yet fully understood, and the engineer should realize that the solution of all problems outside the laminar range must depend on the interpolation or extrapolation of experimental data determined under conditions which may have been dissimilar to that in which they will be used. For materials and pipe sizes commonly used, the curves of f and tables of n will give good results. When unusual surfaces or very large pipe sizes are involved, and especially for very long conduits in which the energy loss becomes a vital design consideration, the literature should be searched for tests on similar conduits. The U.S. Bureau of Reclamation has presented much information on friction factors.[8] Other test results of interest have been reported by Burke[9] and Elder.[11]

The computation of energy loss will be illustrated by the following examples. Section 13, Examples 13.6 and 13.7, illustrate similar computations using a digital computer.

Example 6.1. Determine the energy loss in 300 m of new 60-cm concrete pipe when water at 21°C is flowing at an average velocity of 2 m/s (1) using the Darcy-Weisbach equation and (2) using the Manning equation.

1. Obtaining the kinematic viscosity from Table 1.2,

$$R = \frac{dV}{\nu} = \frac{0.60 \times 2.0}{0.984 \times 10^{-6}} = 1.2 \times 10^6$$

From Table 6.1, $\epsilon = 0.2$ mm. Then $\epsilon/d = 0.0002/0.60 = 0.00033$. From Fig. 6.4, $f = 0.015$. Then, from Eq. (6.19),

$$h = f\frac{l}{d}\frac{V^2}{2g} = 0.015 \times \frac{300}{0.60} \times \frac{4}{19.6} = 1.53 \text{ m}$$

View Verification Case 6 Model

Verification Case 6

Verification Case 7

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify7.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.31, Example 6.2

FLUID: Unspecified

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (meters)	62.8	62.65

DISCUSSION:

The velocity and diameter are specified. For use in AFT Fathom, this is converted to a flow rate, obtaining 0.8482 cm3/sec. The unit weight is specified as 9580 N/m3. At 9.81 meters/sec2 gravitational acceleration, the density is thus 976.55 kg/m3.

The answers differ slightly because the handbook rounds off the Reynolds number to 3 digits.

No pressure boundary conditions are specified. In determining pressure drop, the boundary pressure plays no role. Therefore any value is suitable. The model assumes 1 MPa at the inlet.

List of All Verification Models

Verification Case 7 Problem Statement

Verification Case 7

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.31, Example 6.2

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

2. From Table 6.4, minimum $n = 0.010$. Then from Eq. (6.26c),

$$h = \frac{6.25n^2lV^2}{d^{4/3}} = \frac{6.25 \times 0.010^2 \times 300 \times 4}{(0.60)^{4/3}} = 1.48 \text{ m}$$

Table 6.3 may also be used to obtain this solution. The discharge is obtained as follows:

$$Q = aV = 0.283 \times 2 = 0.565 \text{ m}^3/\text{s}$$

Then four-way interpolation in Table 6.3 for $n = 0.010$ gives $s = 0.0049$ and $sl = 1.47$ m.

Example 6.2. Determine the energy loss in 15 m of 6-mm tubing for a liquid having a unit weight of 9580 N/m³ and a viscosity of 1.50 N·s/m² when the velocity is 3 cm/s.

$$R = \frac{dv\,\rho}{\mu} = \frac{0.006 \times 0.03 \times 9580}{1.5 \times 9.8} = 0.117$$

and from Eqs. (6.19) and (6.20),

$$h = f\frac{l}{d}\frac{V^2}{2g} = \frac{64}{R}\frac{l}{d}\frac{V^2}{2g} = \frac{64}{0.117} \times \frac{15}{0.006} \times \frac{0.03^2}{19.6} = 62.8 \text{ m}$$

MINOR LOSSES

Energy losses resulting from rapid changes in the direction or magnitude of the velocity are called *minor* losses, or *local* losses. The term minor loss is appropriate for pipelines that include long reaches of uniform straight pipe. However, for short pipes it is a misnomer, because the minor losses may be greater than the friction losses.

Minor losses are usually expressed in terms of the kinetic energy and a coefficient, as illustrated by the equation

$$h_m = K_m \frac{V^2}{2g} \tag{6.29}$$

When a change in pipe size occurs, two velocities are involved, and both may be included in the expression for energy loss, as will be shown for the case of enlargements and contractions. Under certain conditions, particularly for the compound-pipe problem (see Compound Pipes), it may be convenient to express the energy loss in terms

View Verification Case 7 Model

Verification Case 7

Verification Case 8

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify8.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.40-42, Example 6.3a

FLUID: Water at 21 deg. C

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (meters)	41.99	42.14

DISCUSSION:

The exit loss factor of 1.0 is modeled as part of the exit Assigned Flow.

The overall head loss in AFT Fathom is obtained by subtracting the discharge EGL (17.86 meters) from the supply (60 meters). It is also displayed in the General Results list at the top of the Output window. Finally, the head loss is given in the Junction Deltas table at the top of the Output window. The junction delta was setup in the Output Control window.

The difference in the final answer (61.48 vs. 61.33) is due to round-off errors in the handbook calculations.

List of All Verification Models

Verification Case 8 Problem Statement

Verification Case 8

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.40-42, Example 6.3a

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

A curve of reduction factors for bends of less than 90° given by the U.S. Bureau of Reclamation shows that the 90° bend loss should be multiplied by 0.83 for 60° bends, by 0.70 for 45° bends, and by 0.42 for 22.5° bends.

The solution of a pipe problem involving minor losses is illustrated by the following example.

Example 6.3. In Fig. 6.1 pipes a and b are steel pipes having diameters of 15 and 30 cm, respectively. The discharge is 0.10 m³/s, and the fluid is water at 21°C. The radii of the bends are twice the respective pipe diameters. Pipe lengths are

2–3	60 m
3–4	75 m
4–5	90 m
5–6	90 m

1. Determine the difference in elevations of the water surfaces in the two reservoirs ($z_1 - z_7$).

$$V_a = 0.10/0.0177 = 5.65 \text{ m/s}$$

$$V_b = 0.10/0.071 = 1.41 \text{ m/s}$$

Then

$$\frac{V_a^2}{2g} = 1.63 \text{ m} \qquad \text{and} \qquad \frac{V_b^2}{2g} = 0.10 \text{ m}$$

The various elements in the total energy loss are shown in Eq. (6.3) and identified below Eq. (6.3). They will now be evaluated.

From Eq. (6.34),

$$h_2 = 0.5\frac{V_a^2}{2g} = 0.82 \text{ m}$$

From the section on turbulent flow,

$$h_{2-3} = f\frac{l}{d}\frac{V^2}{2g} = f\frac{60}{0.15} \times 1.63$$

The value of viscosity is obtained from Table 1.2. Then

$$R_a = \frac{dV}{\nu} = \frac{0.15 \times 5.65}{0.984 \times 10^{-6}} = 0.86 \times 10^6$$

From Table 6.1,

$$\epsilon = 0.00005$$

and therefore

$$\frac{\epsilon}{d} = \frac{0.00005}{0.15} = 0.0003$$

From Fig. 6.4, $f = 0.016$. Then, from the preceding,

$$h_{2-3} = 0.016 \times 60/0.15 \times 1.63 = 10.43 \text{ m}$$

and, by similarity,

$$h_{3-4} = 13.04 \text{ m} \quad \text{and} \quad h_{4-5} = 15.65 \text{ m}$$

From Fig. 6.5,

$$h_3 = h_4 = 0.2 \frac{V_a^2}{2g} = 0.33 \text{ m}$$

From Table 6.5,

$$h_5 = 0.5 \times 1.63 = 0.82 \text{ m}$$

The Reynolds number for pipe b is

$$R_b = \frac{dV}{\nu} = \frac{0.30 \times 1.41}{0.984 \times 10^{-6}} = 0.43 \times 10^6$$

and from Fig. 6.4, $f = 0.0156$ and

$$h_{5-6} = 0.0156 \times 90/0.30 \times 0.10 = 0.47 \text{ m}$$

The outlet loss from Table 6.5 is

$$h_6 = \frac{V_b^2}{2g} = 0.10 \text{ m}$$

Then, as shown by Eq. (6.2), the difference in the elevation of the water surfaces can be found by summation of all the losses as follows:

$$z_1 - z_7 = 0.82 + 10.43 + 0.33 + 13.04 + 0.33 + 15.65 + 0.82 + 0.47 + 0.10 = 41.99 \text{ m}$$

It should be noted that several of the so-called minor losses are greater than the friction loss in pipe *b*.

2. Find the pressure at x if the distance from point 4 to x is 50 m, $z_1 = 60$ m, and $z_x = 15$ m.

The solution of this problem is discussed under Fundamental Principles. The summation of the energy losses shown in Eq. (6.7) can be evaluated from the values in part 1 of this example, except for h_{4-x}, which is

$$h_{4-x} = 0.016 \times 50/0.15 \times 1.63 = 8.69 \text{ m}$$

Then

$$\Sigma h_l = 0.82 + 10.43 + 0.33 + 13.04 + 0.33 + 8.69 = 33.64 \text{ m}$$

Then, from Eq. (6.6),

$$\frac{p_x}{w} = z_1 - \left(\frac{V_a^2}{2g} + z_x + \Sigma h_l\right)$$
$$= 60 - (1.63 + 15 + 33.64) = 9.73 \text{ m}$$

and

$$p_x = 9.8 \times 9.73 = 95.4 \text{ kN/m}^2$$

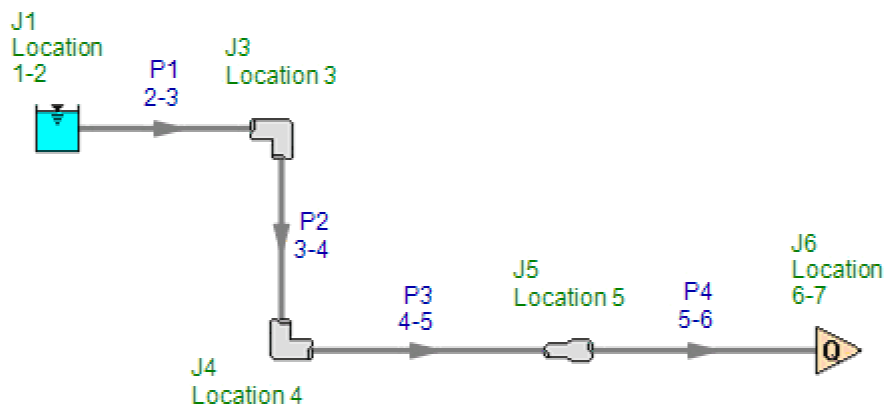
COMPOUND PIPES

Two types of compound-pipe problems may be encountered. One type occurs in pipe networks when two or more paths are available for water flowing between two points, such as points 1 and 5 in Fig. 6.6. A second type, illustrated by Fig. 6.7, occurs when water can flow to or from a junction of three or more pipes from independent sources or outlets.

The first type, often called the *parallel-pipe problem*, can be solved by making use of the fact that the total energy loss through the sys-

View Verification Case 8 Model

Verification Case 8



Verification Case 9

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify9.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.40-42, Example 6.3b

FLUID: Water at 21 deg. C

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Pressure at location x (kPa gauge)	95.4	94.13

DISCUSSION:

From the equation in the handbook, it is evident the pressure at location x is the static pressure and that it is gauge pressure. The units used, kN/m2 are the same as kPa. The proper pressure to review in AFT Fathom for comparison to the handbook is the exit from junction 6.

The difference in the final answer is due to round-off errors in the handbook calculations.

List of All Verification Models

Verification Case 9 Problem Statement

Verification Case 9

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.40-42, Example 6.3b

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

A curve of reduction factors for bends of less than 90° given by the U.S. Bureau of Reclamation shows that the 90° bend loss should be multiplied by 0.83 for 60° bends, by 0.70 for 45° bends, and by 0.42 for 22.5° bends.

The solution of a pipe problem involving minor losses is illustrated by the following example.

Example 6.3. In Fig. 6.1 pipes a and b are steel pipes having diameters of 15 and 30 cm, respectively. The discharge is 0.10 m^3/s, and the fluid is water at 21°C. The radii of the bends are twice the respective pipe diameters. Pipe lengths are

2–3	60 m
3–4	75 m
4–5	90 m
5–6	90 m

1. Determine the difference in elevations of the water surfaces in the two reservoirs ($z_1 - z_7$).

$$V_a = 0.10/0.0177 = 5.65 \text{ m/s}$$

$$V_b = 0.10/0.071 = 1.41 \text{ m/s}$$

Then

$$\frac{V_a^2}{2g} = 1.63 \text{ m} \qquad \text{and} \qquad \frac{V_b^2}{2g} = 0.10 \text{ m}$$

The various elements in the total energy loss are shown in Eq. (6.3) and identified below Eq. (6.3). They will now be evaluated.

From Eq. (6.34),

$$h_2 = 0.5\frac{V_a^2}{2g} = 0.82 \text{ m}$$

From the section on turbulent flow,

$$h_{2-3} = f\frac{l}{d}\frac{V^2}{2g} = f\frac{60}{0.15} \times 1.63$$

The value of viscosity is obtained from Table 1.2. Then

$$R_a = \frac{dV}{\nu} = \frac{0.15 \times 5.65}{0.984 \times 10^{-6}} = 0.86 \times 10^6$$

From Table 6.1,

$$\epsilon = 0.00005$$

and therefore

$$\frac{\epsilon}{d} = \frac{0.00005}{0.15} = 0.0003$$

From Fig. 6.4, $f = 0.016$. Then, from the preceding,

$$h_{2-3} = 0.016 \times 60/0.15 \times 1.63 = 10.43 \text{ m}$$

and, by similarity,

$$h_{3-4} = 13.04 \text{ m} \quad \text{and} \quad h_{4-5} = 15.65 \text{ m}$$

From Fig. 6.5,

$$h_3 = h_4 = 0.2 \frac{V_a^2}{2g} = 0.33 \text{ m}$$

From Table 6.5,

$$h_5 = 0.5 \times 1.63 = 0.82 \text{ m}$$

The Reynolds number for pipe b is

$$R_b = \frac{dV}{\nu} = \frac{0.30 \times 1.41}{0.984 \times 10^{-6}} = 0.43 \times 10^6$$

and from Fig. 6.4, $f = 0.0156$ and

$$h_{5-6} = 0.0156 \times 90/0.30 \times 0.10 = 0.47 \text{ m}$$

The outlet loss from Table 6.5 is

$$h_6 = \frac{V_b^2}{2g} = 0.10 \text{ m}$$

Then, as shown by Eq. (6.2), the difference in the elevation of the water surfaces can be found by summation of all the losses as follows:

$$z_1 - z_7 = 0.82 + 10.43 + 0.33 + 13.04 + 0.33 + 15.65 + 0.82 + 0.47 + 0.10 = 41.99 \text{ m}$$

It should be noted that several of the so-called minor losses are greater than the friction loss in pipe *b*.

2. Find the pressure at x if the distance from point 4 to x is 50 m, $z_1 = 60$ m, and $z_x = 15$ m.

The solution of this problem is discussed under Fundamental Principles. The summation of the energy losses shown in Eq. (6.7) can be evaluated from the values in part 1 of this example, except for h_{4-x}, which is

$$h_{4-x} = 0.016 \times 50/0.15 \times 1.63 = 8.69 \text{ m}$$

Then

$$\Sigma h_l = 0.82 + 10.43 + 0.33 + 13.04 + 0.33 + 8.69 = 33.64 \text{ m}$$

Then, from Eq. (6.6),

$$\frac{p_x}{w} = z_1 - \left(\frac{V_a^2}{2g} + z_x + \Sigma h_l\right) = 60 - (1.63 + 15 + 33.64) = 9.73 \text{ m}$$

and

$$p_x = 9.8 \times 9.73 = 95.4 \text{ kN/m}^2$$

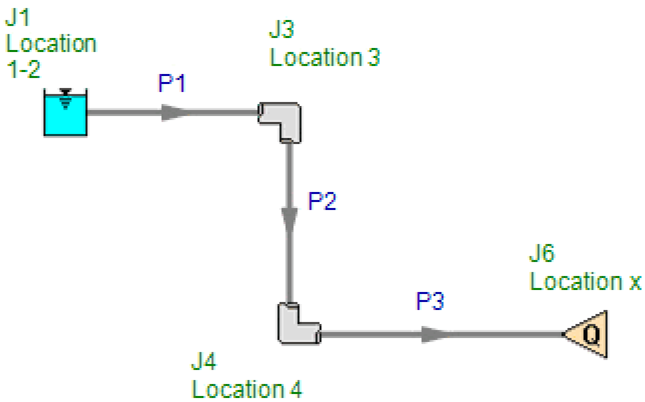
COMPOUND PIPES

Two types of compound-pipe problems may be encountered. One type occurs in pipe networks when two or more paths are available for water flowing between two points, such as points 1 and 5 in Fig. 6.6. A second type, illustrated by Fig. 6.7, occurs when water can flow to or from a junction of three or more pipes from independent sources or outlets.

The first type, often called the *parallel-pipe problem,* can be solved by making use of the fact that the total energy loss through the sys-

View Verification Case 9 Model

Verification Case 9



Verification Case 10

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify10.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.47-48, Example 6.4

FLUID: Water

ASSUMPTIONS: Temperature unspecified, so assume 20 deg. C

RESULTS:

Pipe Flow Rate (m3/s)	**1**	**2**	**3**	**4**	**5**	**6**
Handbook	2.8	2.2	0.7	3.2	3.2	1.8
AFT Fathom	2.8	2.2	0.4	3.2	3.2	1.8

Node EGL (m)	**1**	**2**	**3**	**4**	**5**
Handbook	10.98	10	9.37	10.04	8.32
AFT Fathom	10.96	10	9.37	10.02	8.32

DISCUSSION:

The Handbook of Hydraulics solves the network using a different method than AFT Fathom, and also uses slightly different friction laws. The head loss relationship in the handbook assumes that the head loss in each pipe varies as the flow rate to the 1.85 power. AFT Fathom assumes it varies by the 2nd power. This slight difference accounts for the slightly different results.

Results for AFT Fathom vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom Help File.

List of All Verification Models

Verification Case 10 Problem Statement

Verification Case 10

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 6.47-48, Example 6.4

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

b. Find the algebraic sum of the head loss around the loop, after establishing a sign convention.

c. Compute the sum of the quantities nkQ_1^{n-1} from each pipe in the loop without regard to the direction.

d. Calculate the flow adjustment for the loop from Eq. (6.55).

4. Apply step 3 to each circuit in the system, make the flow adjustments to each pipe, and repeat the process until the desired accuracy is obtained.

Example 6.4. Determine the distribution of flow in the network of Fig. 6.8 with the inflow and outflow shown. Also, determine the elevation of the hydraulic grade line at points 1 and 5 if the reservoir elevation is 10 m. There is no flow into or out of the reservoir.

This network contains six pipes and two loops. The Hazen-Williams formula is used, and the coefficient is assumed to be 100 in each pipe. The lengths, diameters, and values of k for each pipe are listed in the following table:

Pipe	Length, m	Diameter, m	k	Assumed Q	ΔQ	Corrected Q
1	2000	2.0	0.146	−2.5	−0.3	−2.8
2	3000	2.0	0.218	+2.5	−0.3	+2.2
3	1000	2.0	0.073	+1.0	−0.3	+0.7
				−1.0	+0.3	−0.7
4	1000	2.0	0.073	−3.5	+0.3	−3.2
5	1000	1.8	0.122	−3.5	+0.3	−3.2
6	2000	1.5	0.590	+1.5	+0.3	+1.8

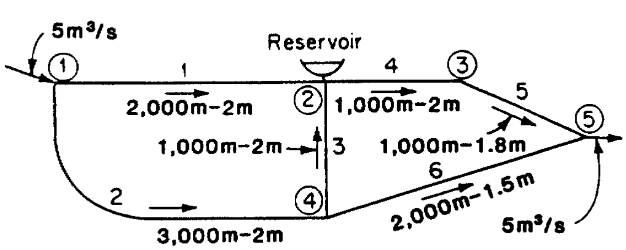


FIGURE 6.8 Distribution network.

The exponent in Eq. (6.49) is $n = 1.85$. Flows in the counterclockwise direction were taken as positive. The assumed flows with the corresponding signs for the flow directions shown in Fig. 6.8 are also listed in the table. Note that the discharge in pipe 3 has a plus sign for the left-hand loop and a minus sign for the right-hand loop. Flow directions are computed for the left and right loops in accordance with steps 3*a* to 3*d* in the procedure outlined. The computed correction factors and the corrected discharges are also listed. The correction factors for both loops are applied to pipe 3, and both decrease the discharge.

The process is illustrated in the following table. It should be repeated until the pipe flows do not change significantly.

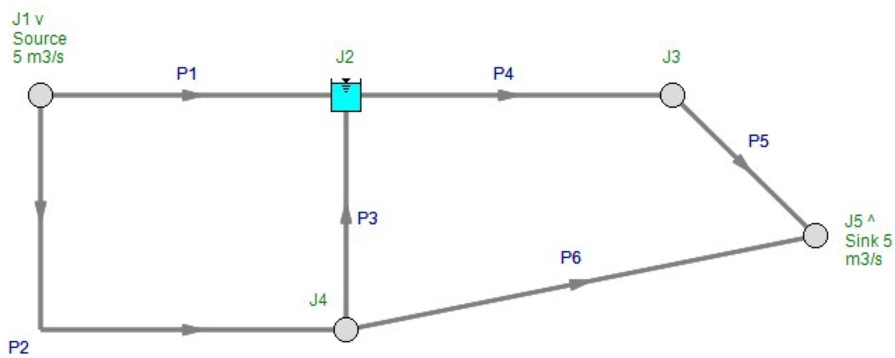
Pipe	ΣkQ^n	ΣnkQ^{n-1}
	Left loop	
1	$0.146 \times 2.5^{1.85} = -0.80$	$1.85 \times 0.146 \times 2.5^{0.85} = 0.59$
2	$0.218 \times 2.5^{1.85} = +1.19$	$1.85 \times 0.218 \times 2.5^{0.85} = 0.88$
3	$0.073 \times 1.0^{1.85} = +0.073$	$1.85 \times 0.073 \times 1.0^{0.85} = 0.14$
		1.61
	$\Delta Q = \frac{-(-0.80 + 1.19 + 0.073)}{1.61} = -0.3$	
	Right loop	
3	$0.073 \times 1.0^{1.85} = -0.073$	$1.85 \times 0.073 \times 1.0^{0.85} = 0.14$
4	$0.073 \times 3.5^{1.85} = -0.74$	$1.85 \times 0.073 \times 3.5^{0.85} = 0.39$
5	$0.122 \times 3.5^{1.85} = -1.24$	$1.85 \times 0.122 \times 3.5^{0.85} = 0.65$
6	$0.59 \times 1.5^{1.85} = +1.25$	$1.85 \times 0.59 \times 1.5^{0.85} = 1.54$
		2.72
	$\Delta Q = \frac{-(-0.073 - 0.74 - 1.24 + 1.25)}{2.72} = +0.3$	

In this example the flows are balanced after the first adjustment since the originally assumed flows were quite close.

The pressure level at junction 3 is $10.0 - 0.073 \times 3.2^{1.85} = 9.37$ m, and at the outflow junction it is $9.37 - 0.122 \times 3.2^{1.85} = 8.32$ m. At node 1 it is $10.0 + 0.146 \times 2.8^{1.85} = 10.98$ m.

View Verification Case 10 Model

Verification Case 10



Verification Case 11

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify11.fth

REFERENCE: Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 13.14, Example 13.9, results on page 13.39

FLUID: Water

ASSUMPTIONS: Temperature unspecified, so assume 20 deg. C

RESULTS:

Pipe Flow Rate (m3/s)	1	2	3	4	5	6
Handbook	3.42	2.68	0.89	3.21	3.21	1.79
AFT Fathom	3.48	2.73	0.94	3.21	3.21	1.79

Node EGL (m)	1	2	3	4	5
Handbook	11.42	10.00	9.37	10.06	8.32
AFT Fathom	11.46	10.00	9.38	10.06	8.33

DISCUSSION:

The Handbook of Hydraulics solves the network using a different method than AFT Fathom, and also uses slightly different friction laws. The head loss relationship in the handbook assumes that the head loss in each pipe varies as the flow rate to the 1.85 power. AFT Fathom assumes it varies by the 2nd power. This slight difference accounts for the slightly different results.

Results for AFT Fathom vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom Help File.

List of All Verification Models

Verification Case 11 Problem Statement

Verification Case 11

Ernest Brater, Horace Williams, James Lindell, C.Y. Wei, Handbook of Hydraulics, 7th Ed., 1996, McGraw-Hill, Page 13.14, Example 13.9, results on page 13.39

Brater, Williams, Lindell and Wei Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Nodal Method). The following example illustrates the development of a computer program to solve the pipe network problems used as examples in Sec. 6.

Example 13.9. Determine the flow distribution in the network presented in Example 6.5 by use of the nodal method. A schematic diagram of the system is shown in Fig. 6.8. Note that a pump, with characteristics shown in Fig. 6.9, is located at node 1, and the flow at the reservoir (junction 2) is determined from the analysis.

The program listing and input data are shown in the Appendix. One of the primary difficulties associated with a computer solution of a complex network is the adequate description of the physical and geometric properties of the system. In this program the network is described by assigning a nominal upstream and downstream node to each pipe. Each junction is assigned a number indicating the type of junction: 1—node with one or more pipes connected, 2—node with a pump, 3—node at a reservoir, and 4—node with outflow.

For each pipe the upstream and downstream nodes are specified as well as pipe length, diameter, and Hazen-Williams roughness coefficient. For each junction the type is given along with a first guess of the hydraulic grade line and, depending on the type, the demand Q_v or the elevation on the suction side of the pump. If a pump is present, then the rating is input as a table of discharge versus head.

Execution of the program is stopped by exceeding the number of iterations given by `maxit`, by finding the sum of the junction head corrections, or by any iteration that is less than a tolerance `toler`.

The output for this example are shown in the Appendix. The first table shows the variation in hydraulic grade line with each iteration and the convergence tolerance. The second table shows the flow in each pipe.

For large complex pipe networks more advanced computer programs are available, as indicated in Sec. 14.

UNSTEADY FLOW IN PIPES AND OPEN CHANNELS

The method of characteristics can be used to solve the unsteady flow equations.[8] The equations of motion and continuity are combined and then, using the method of characteristics, two new equations are derived,

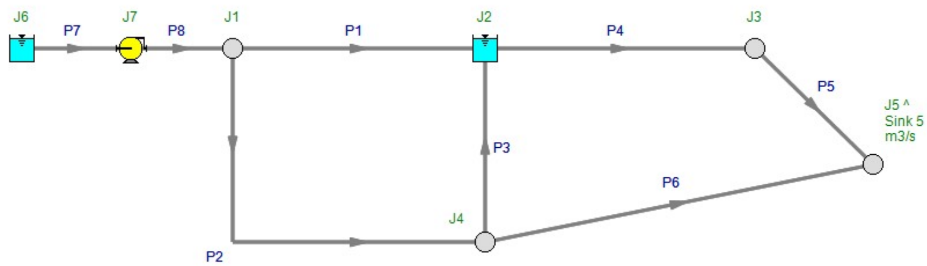
Output Data

Iteration	Tolerance	H(1..5)				
0	0.0000	11.00	10.00	9.00	10.50	8.50
1	1.8343	11.47	10.00	9.43	9.89	8.18
2	0.6517	11.39	10.00	9.32	10.20	8.33
3	0.2895	11.44	10.00	9.37	10.03	8.31
4	0.0642	11.41	10.00	9.37	10.06	8.31
5	0.0105	11.42	10.00	9.37	10.06	8.32
6	0.0026	11.42	10.00	9.37	10.06	8.32

Pipe	Flow
1	3.42
2	2.68
3	0.89
4	3.21
5	3.21
6	1.79

View Verification Case 11 Model

Verification Case 11



Verification Case 12

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify12.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-13, Example 1

FLUID: Water at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
NPSHA (feet)	20.26	20.23
Suction Head (feet)	-12.92	-12.92

DISCUSSION:

The example in the book does not actually perform a flow calculation. To demonstrate how AFT Fathom determines NPSHA, a few unusual steps were taken.

1. The head loss up the pump is given in Cameron as 2.92 feet. This was modeled in valve J2, as a fixed pressure drop of 2.92 feet.
2. All three pipes were modeled as frictionless so that all pressure loss comes from valve J2.
3. A discharge pipe and reservoir were modeled to complete the system, even though the discharge piping does not affect the answer to the problem.
4. The pump was modeled as an assigned flow at 100 gpm. Since the pressure drop is fixed at J2 as 2.92 feet, the flow rate in the pump (and piping for that matter) does not affect the answer. In typical system analysis, the flow in the piping affects the results because as the flow changes the pressure drop changes. In this model this did not occur because all pipes were frictionless.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the EGL inlet to the pump, J3. It is the negative of the suction lift.

List of All Verification Models

Verification Case 12 Problem Statement

Verification Case 12

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-13, Example 1

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Hydraulics

1

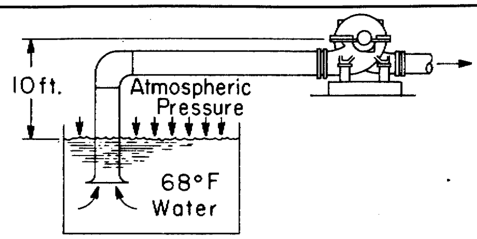


Fig. 4. (Example No 1)

Example No 1 (Fig 4)

Open system, source below pump; 68°F water at sea level. Atmospheric pressures 14.696 psia, 33.96 ft abs. Vapor pressure of liquid 0.339 psia = 0.783 ft abs.

NPSHA = 33.96 − 0.783 − 10.00 − 2.92 = 20.26 ft

Suction Lift = 10.00 + 2.92 = 12.92 ft—this is to be added to discharge head to obtain total head.

Note: No pump can actually lift water on the suction side. In this case, water is forced in by an excess of atmospheric pressure over the vapor pressure less 12.92 ft net static lift.

Example No 2 (Fig 5)

Open system, source above pump; 68°F water at sea level.

NPSHA = 33.96 − 0.783 + 10.00 − 2.92 = 40.26 ft.

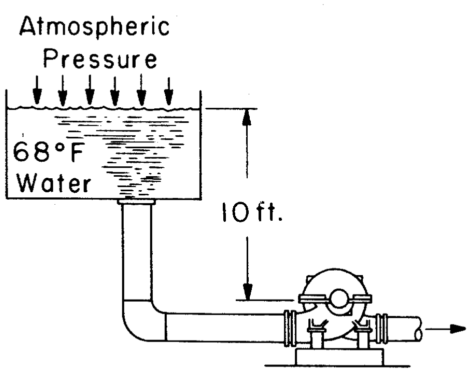
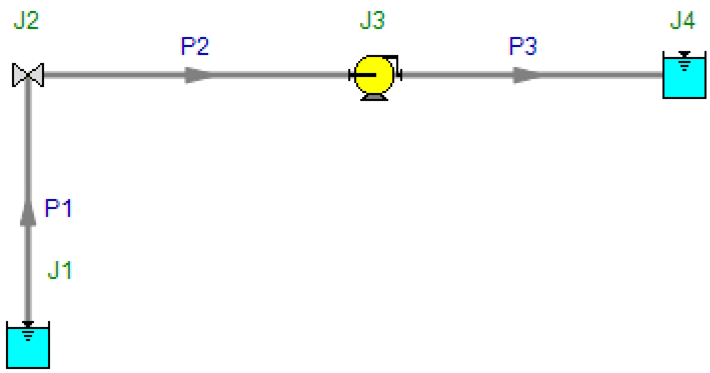


Fig. 5 (Example 2)

1-13

View Verification Case 12 Model

Verification Case 12



Verification Case 13

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify13.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-13, 14, Example 2

FLUID: Water at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
NPSHA (feet)	40.26	40.23
Suction Head (feet)	7.08	7.08

DISCUSSION:

The example in the book does not actually perform a flow calculation. To demonstrate how AFT Fathom determines NPSHA, a few unusual steps were taken.

1. The head loss up the pump is given in Cameron as 2.92 feet. This was modeled in the valve J2, as a fixed pressure drop of 2.92 feet.
2. All three pipes were modeled as frictionless so that all pressure loss comes from valve J2.
3. A discharge pipe and reservoir were modeled to complete the system, even though the discharge piping does not affect the answer to the problem.
4. The pump was modeled as an assigned flow at 100 gpm. Since the pressure drop is fixed at J2 as 2.92 feet, the flow rate in the pump (and piping for that matter) does not affect the answer. In typical system analysis, the flow in the piping affects the results because as the flow changes the pressure drop changes. In this model this did not occur because all pipes were frictionless.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the EGL inlet to the pump, J3.

List of All Verification Models

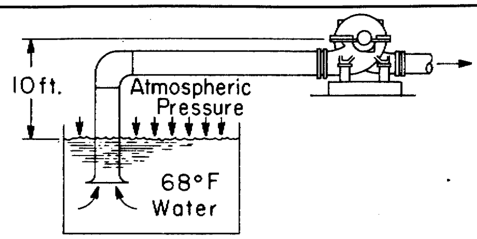
Verification Case 13 Problem Statement

Verification Case 13

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-13, 14, Example 2

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.



1

Fig. 4. (Example No 1)

Example No 1 (Fig 4)

Open system, source below pump; 68°F water at sea level. Atmospheric pressures 14.696 psia, 33.96 ft abs. Vapor pressure of liquid 0.339 psia = 0.783 ft abs.

NPSHA = 33.96 − 0.783 − 10.00 − 2.92 = 20.26 ft

Suction Lift = 10.00 + 2.92 = 12.92 ft—this is to be added to discharge head to obtain total head.

Note: No pump can actually lift water on the suction side. In this case, water is forced in by an excess of atmospheric pressure over the vapor pressure less 12.92 ft net static lift.

Example No 2 (Fig 5)

Open system, source above pump; 68°F water at sea level.

NPSHA = 33.96 − 0.783 + 10.00 − 2.92 = 40.26 ft.

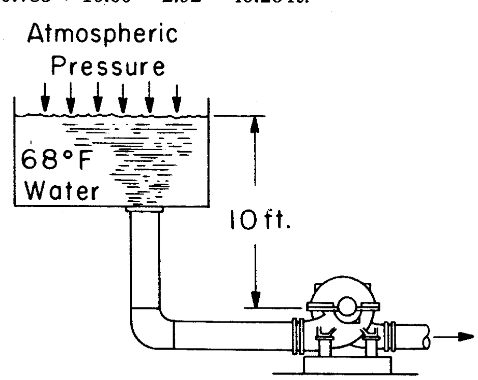


Fig. 5 (Example 2)

1-13

Ingersoll-Dresser Pumps Cameron Hydraulic Data

Suction Head—10.00 − 2.92 = 7.08 ft—this is to be subtracted from discharge head to obtain total head.

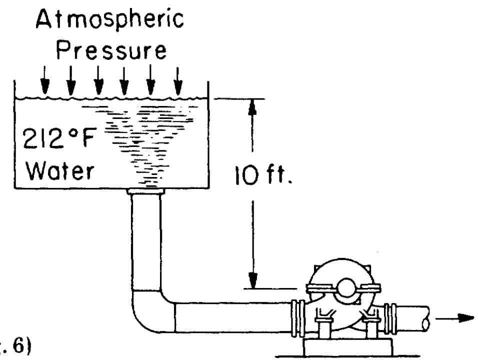


Fig. 6 (Example 3)

Example No. 3 (Fig. 6)

Open system, source above pump; 212°F water at sea level; vapor pressure same as atmospheric since liquid at boiling point. At 212°F, the vapor pressure of water—in feet of 212°F water—is 35.38 ft. (14.696 x 144 ÷ 59.81 = 35.38).

NPSHA = 35.38 − 35.38 + 10.00 − 2.92 = 7.08 ft. In this case, atmospheric pressure does not add to NPSHA since it is required to keep the water in liquid phase.

Suction Head = 10.00 − 2.92 = 7.08 ft—this is to be subtracted from discharge head to obtain total head.

Note: In this example it was assumed that pipe friction losses for 212°F water were the same as for 68°F water whereas actually they would be somewhat less, as will also be the case in Example 4.

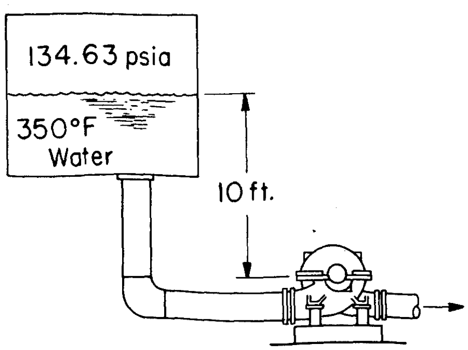
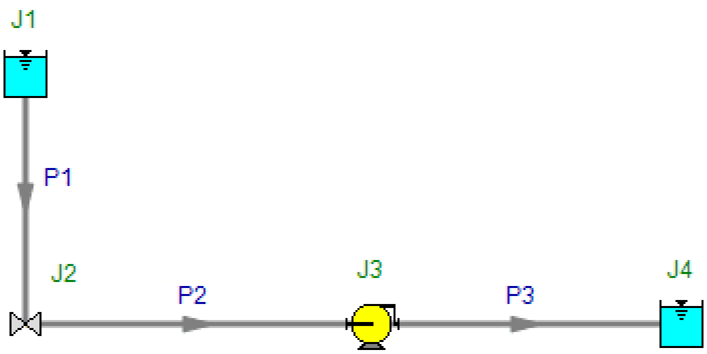


Fig. 7 (Example 4)

1-14

View Verification Case 13 Model

Verification Case 13



Verification Case 14

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify14.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-14, Example 3

FLUID: Water at 212 deg. F, 1 atm

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
NPSHA (feet)	7.08	7.048
Suction Head (feet)	7.08	7.080

DISCUSSION:

The example in the book does not actually perform a flow calculation. To demonstrate how AFT Fathom determines NPSHA, a few unusual steps were taken.

1. The head loss up the pump is given in Cameron as 2.92 feet. This was modeled in the valve J2, as a fixed pressure drop of 2.92 feet.
2. All three pipes were modeled as frictionless so that all pressure loss comes from valve J2.
3. A discharge pipe and reservoir were modeled to complete the system, even though the discharge piping does not affect the answer to the problem.
4. The pump was modeled as an assigned flow at 100 gpm. Since the pressure drop is fixed at J2 as 2.92 feet, the flow rate in the pump (and piping for that matter) does not affect the answer. In typical system analysis, the flow in the piping affects the results because as the flow changes the pressure drop changes. In this model this did not occur because all pipes were frictionless.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the EGL inlet to the pump, J3.

List of All Verification Models

Verification Case 14 Problem Statement

Verification Case 14

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-14, Example 3

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Suction Head—10.00 − 2.92 = 7.08 ft—this is to be subtracted from discharge head to obtain total head.

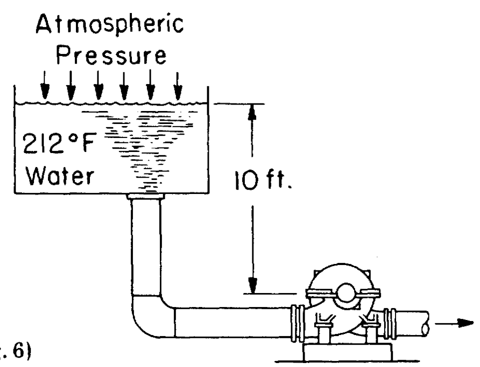


Fig. 6 (Example 3)

Example No. 3 (Fig. 6)

Open system, source above pump; 212°F water at sea level; vapor pressure same as atmospheric since liquid at boiling point. At 212°F, the vapor pressure of water—in feet of 212°F water—is 35.38 ft. (14.696 x 144 ÷ 59.81 = 35.38).

NPSHA = 35.38 − 35.38 + 10.00 − 2.92 = 7.08 ft. In this case, atmospheric pressure does not add to NPSHA since it is required to keep the water in liquid phase.

Suction Head = 10.00 − 2.92 = 7.08 ft—this is to be subtracted from discharge head to obtain total head.

Note: In this example it was assumed that pipe friction losses for 212°F water were the same as for 68°F water whereas actually they would be somewhat less, as will also be the case in Example 4.

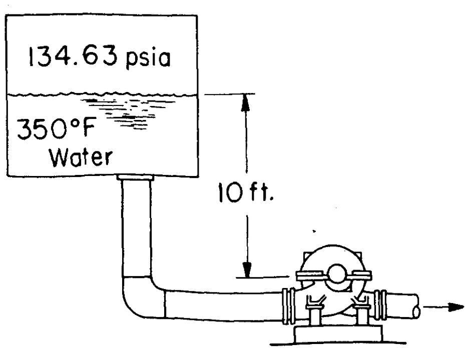
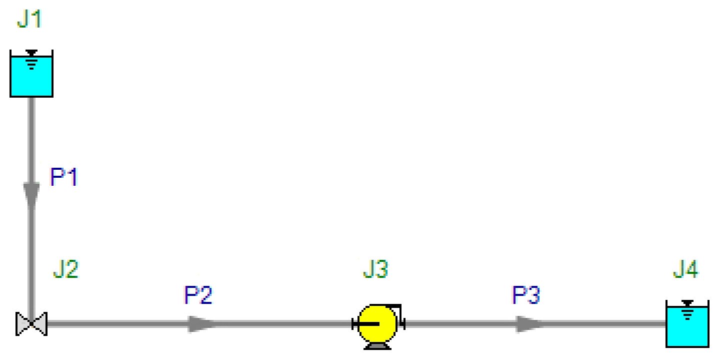


Fig. 7 (Example 4)

View Verification Case 14 Model

Verification Case 14



Verification Case 15

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify15.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data,AF 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-15, Example 4

FLUID: Water at 350 deg. F, 134.63 psia

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
NPSHA (feet)	7.08	7.158
Suction Head (feet)	317.69	317.71

DISCUSSION:

The example in the book does not actually perform a flow calculation. To demonstrate how AFT Fathom determines NPSHA, a few unusual steps were taken.

1. The head loss up the pump is given in Cameron as 2.92 feet. This was modeled in the valve J2, as a fixed pressure drop of 2.92 feet.
2. All three pipes were modeled as frictionless so that all pressure loss comes from valve J2.
3. A discharge pipe and reservoir were modeled to complete the system, even though the discharge piping does not affect the answer to the problem.
4. The pump was modeled as an assigned flow at 100 gpm. Since the pressure drop is fixed at J2 as 2.92 feet, the flow rate in the pump (and piping for that matter) does not affect the answer. In typical system analysis, the flow in the piping affects the results because as the flow changes the pressure drop changes. In this model this did not occur because all pipes were frictionless.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the EGL inlet to the pump, J3.

List of All Verification Models

Verification Case 15 Problem Statement

Verification Case 15

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-15, Example 4

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Example No 4 (Fig 7)

Closed system (under pressure as a feed water deareator) source above pump.
350°F water V.P. = 134.60 psia = 348.67 ft abs (at 350°F sp gr = 0.8904).

NPSHA = 348.67 − 348.67 + 10.00 − 2.92 = 7.08 ft.

Suction Head—(Figure basis gage pressures; i.e., 119.91 psig = 310.61 ft) = 310.61 + 10.00 − 2.92 = 317.69 ft—This is to be subtracted from the discharge head to obtain total system head. It is important to note that while the suction head is 317.69 ft (122.67 psig) the NPSHA is still only 7.08 ft.

Example No 5 (Fig 8)—Closed system (under vacuum as a condenser hotwell) liquid source above pump. Absolute pressure (h_a) = 1.50″ Hg × 1.1349 = 1.70 ft. Water at saturation point 91.72°F; therefore vapor pressure (h_{vpa}) = 1.50″ Hg × 1.1349 = 1.70 ft.

NPSHA = 1.70 − 1.70 + 10.00 − 2.92 = 7.08 ft.

Suction Condition—In this example the suction condition (head or lift) for the pump can best be visualized by the calculations listed below where it can be seen that we have a suction lift equal to the vacuum effect at the suction source less the net static submergence.

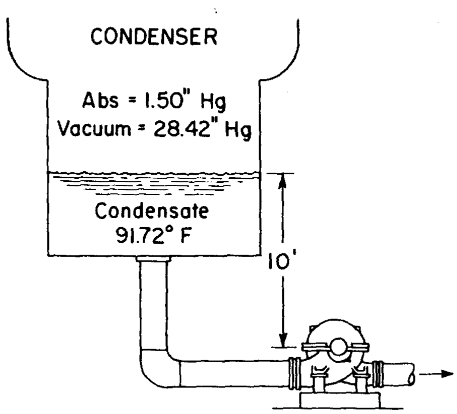
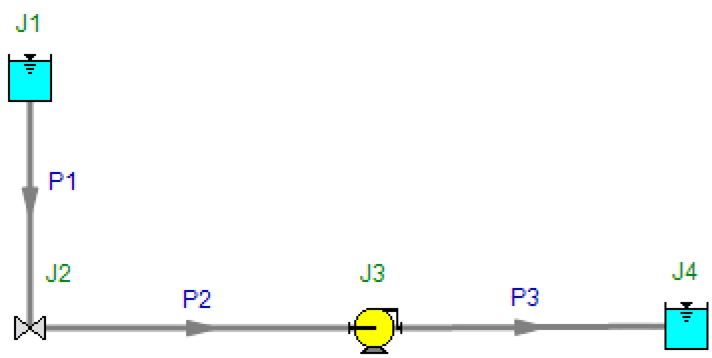


Fig. 8 (Example 5)

View Verification Case 15 Model

Verification Case 15



Verification Case 16

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify16.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-15, 16, Example 5

FLUID: Water at 91.72 deg. F, 1.5 in. Hg

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
NPSHA (feet)	7.08	7.08
Suction Head (feet)	-25.17	-25.30

DISCUSSION:

The example in the book does not actually perform a flow calculation. To demonstrate how AFT Fathom determines NPSHA, a few unusual steps were taken.

1. The head loss up the pump is given in Cameron as 2.92 feet. This was modeled in the valve J2, as a fixed pressure drop of 2.92 feet.
2. All three pipes were modeled as frictionless so that all pressure loss comes from valve J2.
3. A discharge pipe and reservoir were modeled to complete the system, even though the discharge piping does not affect the answer to the problem.
4. The pump was modeled as an assigned flow at 100 gpm. Since the pressure drop is fixed at J2 as 2.92 feet, the flow rate in the pump (and piping for that matter) does not affect the answer. In typical system analysis, the flow in the piping affects the results because as the flow changes the pressure drop changes. In this model this did not occur because all pipes were frictionless.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the EGL inlet to the pump, J3. It is the negative of the suction lift.

List of All Verification Models

Verification Case 16 Problem Statement

Verification Case 16

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 1-15, 16, Example 5

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Example No 4 (Fig 7)

Closed system (under pressure as a feed water deareator) source above pump.
350°F water V.P. = 134.60 psia = 348.67 ft abs (at 350°F sp gr = 0.8904).

NPSHA = 348.67 − 348.67 + 10.00 − 2.92 = 7.08 ft.

Suction Head—(Figure basis gage pressures; i.e., 119.91 psig = 310.61 ft) = 310.61 + 10.00 − 2.92 = 317.69 ft—This is to be subtracted from the discharge head to obtain total system head. It is important to note that while the suction head is 317.69 ft (122.67 psig) the NPSHA is still only 7.08 ft.

Example No 5 (Fig 8)—Closed system (under vacuum as a condenser hotwell) liquid source above pump. Absolute pressure (h_a) = 1.50″ Hg × 1.1349 = 1.70 ft. Water at saturation point 91.72°F; therefore vapor pressure (h_{vpa}) = 1.50″ Hg × 1.1349 = 1.70 ft.

NPSHA = 1.70 − 1.70 + 10.00 − 2.92 = 7.08 ft.

Suction Condition—In this example the suction condition (head or lift) for the pump can best be visualized by the calculations listed below where it can be seen that we have a suction lift equal to the vacuum effect at the suction source less the net static submergence.

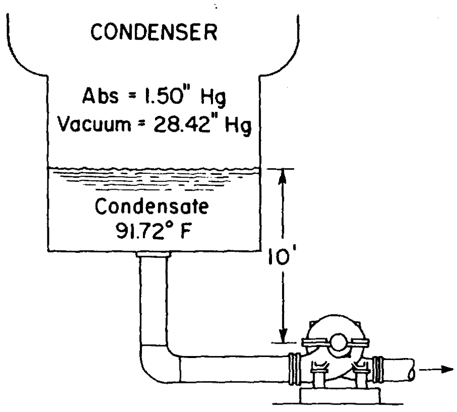


Fig. 8 (Example 5)

28.42" Hg Vacuum = 28.42 x 1.1349	=		32.25 ft
Static submergence		10.00 ft	
Friction and entrance loss		– 2.95 ft	
Net static submergence	=	7.08 ft	– 7.08 ft
Equivalent suction lift = vacuum effect less net submergence	=		25.17 ft

In this example it is noted that the NPSHA is equal to the static suction head less the friction and entrance losses. Also the equivalent suction lift must be added to the total discharge head to obtain the total system head.

In the foregoing examples standard sea level atmospheric conditions were assumed; for other locations where altitude is a factor proper corrections must be made. These examples (3, 4 and 5) illustrate that if the liquid is in equilibrium (vapor pressure corresponds to saturation temperature) then the NPSH is equal to the difference in elevation between the liquid supply level and the pump centerline elevation (or impeller eye) less the sum of the entrance loss and the friction losses in the suction line.

NPSH reductions—hydrocarbon liquids and hot water

The NPSH requirements of centrifugal pumps are normally determined on the basis of handling water at or near normal room temperatures. However, field experience and laboratory tests have confirmed that pumps handling certain gas free hydrocarbon fluids and water at elevated temperatures will operate satisfactorily with harmless cavitation and less NPSH available than would be required for cold water.

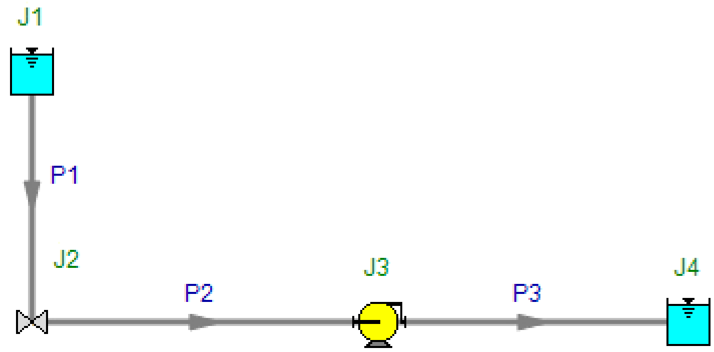
The figure on page 1-52 shows NPSH reductions that may be considered for hot water and certain gas free pure hydrocarbon liquids.

The use and application of this chart is subject to certain limitations some of which are summarized below:

1. The NPSH reductions shown are based on laboratory test data at steady state suction conditions and on the gas free pure hydrocarbon liquids shown; its application to other liquids must be considered experimental and is not recommended.
2. No NPSH reduction should exceed 50% of the NPSH required for cold water or ten feet whichever is smaller.

View Verification Case 16 Model

Verification Case 16



Verification Case 17

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify17.fth

REFERENCE: Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 3-9, 10

FLUID: Water at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Cameron	AFT Fathom
Suction head (feet)	-5.35	-5.35
Discharge head (feet)	290	289.7
Pump head (feet)	295	295.0
NPSHA (feet)	Not provided	27.80

DISCUSSION:

The losses at the pump suction are not explicitly provided in AFT Fathom's library, so the total K factor of 1.57 for the foot valve and long radius elbow is entered as a fitting and loss value in pipe P1. The losses in the discharge piping are in AFT Fathom's library, and are selected as fitting and loss values in pipe P2. The only loss not shown in pipe P2 is the entrance loss at the discharge tank, and that is modeled in the reservoir junction, J3.

The 200 gpm is modeled at the pump as an assigned flow pump.

The AFT Fathom NPSHA prediction is shown in the Pump Summary at the top of the Output window. The suction head is shown as the difference between the EGL inlet and the elevation (23.27 – 28.62) at the pump, J2. It is the negative of the suction lift. The discharge head is shown as the difference between the EGL outlet and the elevation (318.30 – 28.62) at the pump.

List of All Verification Models

Verification Case 17 Problem Statement

Verification Case 17

Ingersoll-Dresser Pumps, Cameron Hydraulic Data, 18th Ed., 1995, Published by Ingersoll-Dresser Pumps, Page 3-9, 10

Cameron Hydraulic Data Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Friction—head loss—sample calculation:

To illustrate the application of the friction and head loss data in calculating the total system head for a specific system the following example is offered:

Problem—referring to the accompanying figure, page 3-10, a pump takes water (68°F) from a sump and delivers it through 1250 feet of 4″ diameter schedule 40 steel pipe. The suction pipe is 4″ vertical 5 feet long and includes a foot valve and a long-radius elbow. The discharge line includes two standard 90 degree flanged elbows, a swing check valve and an open wedge—disc gate valve. It is required to find the suction lift (h_s) and the discharge head (h_d) when the rate of flow is 200 gpm.

3

Solution

(a) SUCTION LIFT—Data from table on page 3-20.

$$\text{Velocity head} = \frac{V^2}{2g} = 0.395 \text{ ft}$$

Pipe friction loss $h_f = 2.25$ ft per 100 ft of pipe.

The resistance coefficient for the foot valve (page 3-115) is K = 1.3 and for the long-radius elbow (page 3-112) is K = 0.27.

The head loss due to pipe friction will be:

$$h_f = 2.25 \times \frac{5}{100} = 0.11 \text{ ft}$$

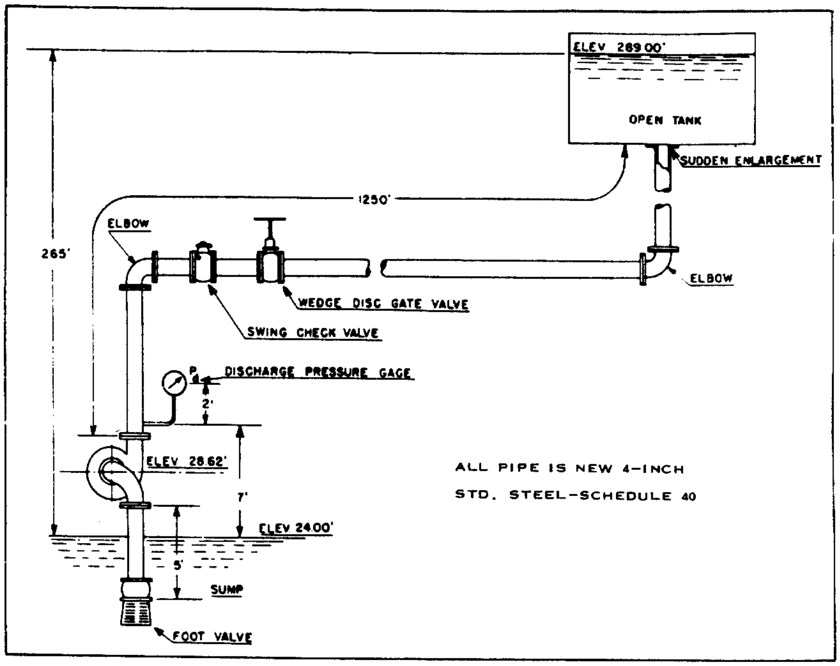
The head loss in the foot valve and long-radius elbow will be:

$$h_f = K\frac{V^2}{2g} = (1.3 + 0.27) \times 0.395 = 0.62 \text{ ft}$$

Total suction lift (h_s) = (28.62 − 24.00) + 0.62 + 0.11 = 5.35 ft

(b) DISCHARGE HEAD—The head loss due to pipe friction in the 4″ discharge line will be:

$$h_f = 2.25 \times \frac{1250}{100} = 28.13 \text{ ft}$$



The resistance coefficient for the various fittings as obtained from the tables will be:

Standard 90 degree flanged elbow (pg. 3-112)	K = 0.51
Swing check valve (pg. 3-115)	K = 1.70
Wedge-disc gate valve (pg. 3-111)	K = 0.14
Sudden enlargement (pg. 3-116 to 3-118)	K = 1.00

The total resistance coefficient for the fittings on the discharge side and sudden enlargement at exit will be:

$$K = 2 \times 0.51 + 1.70 + 0.14 + 1.0 = 3.86$$

Therefore the head loss due to the fittings on the discharge side and sudden enlargement will be:

$$h_f = K \frac{V^2}{2g} = 3.86 \times 0.395 = 1.52 \text{ ft}$$

The total discharge head (h_d) will be:

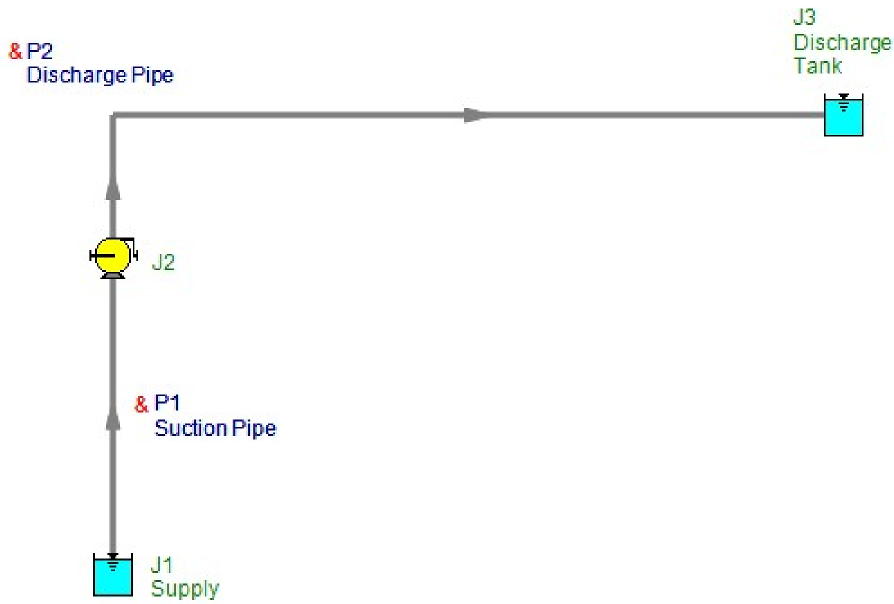
$$h_d = (289.00 - 28.62) + 1.52 + 28.13 = 290 \text{ ft}$$

Total system head (H) = $h_d + h_s = 290 + 5.35 = 295$ ft

Add a reasonable safety factor to allow for any abnormal condition of pipe's interior or surface (see page 3-5).

View Verification Case 17 Model

Verification Case 17



Verification Case 18

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify18.fth

REFERENCE: Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.9-11, Example 1

FLUID: Pumped system with fluid of 0.8 specific gravity

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Pump head (feet)	372	371.6

DISCUSSION:

The suction and discharge pipe head loss of 3 and 25 feet, respectively, was modeled as a fitting and loss K factor to obtain the proper head loss. Both pipes were modeled as frictionless. Thus the pipe lengths, which were not specified, do not affect the head loss. In addition, the viscosity of the fluid, which was not specified, does not affect the head loss since the pipe are frictionless. Thus a value of 1 lbm/hr-ft was assumed.

The 1000 gpm is modeled at the pump as an assigned flow pump.

The total pump head is shown in the AFT Fathom Pump Summary at the top of the Output window.

List of All Verification Models

Verification Case 18 Problem Statement

Verification Case 18

Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.9-11, Example 1

Karassik, Krutzsch, Fraser and Messina Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

8.1 PUMPING SYSTEMS AND SYSTEM-HEAD CURVES **8.9**

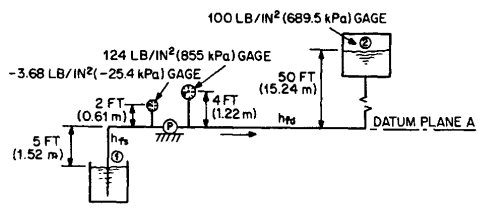


FIG. 8 Example 1.

expressed in feet (meters), the usual way of rating centrifugal pumps. For an explanation of positive displacement pump differential pressure, its use and relationship to pump total head, see Chap. 3.

Pump total head can be measured by installing gages at the pump suction and discharge connections and then substituting these gage readings into Eq. 4. Pump total head may also be found by measuring the energy difference between any two points in the pumping system, one on each side of the pump, providing all losses (other than pump losses) between these points are credited to the pump and added to the energy head difference. Therefore, between any two points in a pumping system where the energy is added only by the pump and the specific weight (force) of the liquid does not change (for example, as a result of temperature), the following general equation for determining pump total head applies:

$$TH = (H_2 - H_1) + \Sigma h_{f(1-2)} = \left(\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2\right) - \left(\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1\right) + \Sigma h_{f(1-2)} \quad (7)$$

where the subscripts 1 and 2 denote points in the pumping system anyplace upstream and downstream from the pump, respectively, and

H = total head of system, (+) or (−) ft (m) gage or (+) ft (m) abs
V = velocity, ft/s (m/s)
p = pressure, (+) or (−) lb/in² (N/m²) gage or (+) lb/in² (N/m²) abs
Z = elevation above (+) or below (−) datum plane, ft (m)
γ = specific weight (force) of liquid (assumed the same between points), lb/ft³ (N/m³)
g = acceleration of gravity, 32.17 ft/s² (9.807 m/s²)
Σh_f = sum of piping losses between points, ft (m)

When the specific gravity of the liquid is known, the pressure head may be calculated from the following relationships:

in feet $$\frac{p}{\gamma} = \frac{0.016\ \text{lb/ft}^2}{\text{sp. gr.}} \text{ or } \frac{2.3\ \text{lb/in}^2}{\text{sp. gr.}} \quad (8a)$$

in meters $$\frac{p}{\gamma} = \frac{0.102\ \text{kPa}}{\text{sp. gr.}} \text{ or } \frac{1.02 \times 10^{-3}\ \text{bar}}{\text{sp. gr.}} \quad (8b)$$

The velocity in a pipe may be calculated as follows:

in feet per second $$V = \frac{(\text{gm})(0.408)}{(\text{pipe ID in inches})^2} \quad (9a)$$

in meters per second $$V = \frac{(\text{m}^3/\text{h})(3.54)}{(\text{pipe ID in cm})^2} \text{ or } \frac{(\text{liters/s})(12.7)}{(\text{pipe ID in cm})^2} \quad (9b)$$

The following example illustrates the use of Eqs. 4 and 7 for determining pump total head.

EXAMPLE 1 A centrifugal pump delivers 1000 gpm (227 m³/s) of liquid of specific gravity 0.8 from the suction tank to the discharge tank through the piping shown in Fig. 8. (a) Calculate pump total head using gages and the datum plane selected. (b) Calculate total head using the pressures at points 1 and 2 and the same datum plane as (a).

Given: Suction pipe ID = 8 in (203 mm), discharge pipe ID = 6 in (152 mm), h_f = pipe valve, and fitting losses, h_{fs} = 3 ft (0.91 mm), h_{fd} = 25 ft (7.62 m).

in USCS units Calculated pipe velocity $= \dfrac{(\text{gpm})(0.408)}{(\text{ID in inches})^2}$

$$V_s = \frac{1000 \times 0.408}{8^2} = 6.38 \text{ ft/s}$$

$$V_d = \frac{1000 \times 0.408}{6^2} = 11.33 \text{ ft/s}$$

in SI units Calculated pipe velocity $= \dfrac{(\text{m}^3/\text{h})(3.54)}{(\text{ID in cm})^2}$

$$V_s = \frac{227 \times 3.54}{20.3^2} = 1.95 \text{ m/s}$$

$$V_d = \frac{227 \times 3.54}{15.2^2} = 3.48 \text{ m/s}$$

(a) From Eq. 4,

$$TH = \left(\frac{V_d^2}{2g} + \frac{p_d}{\gamma_d} + Z_d\right) - \left(\frac{V_s^2}{2g} + \frac{p_s}{\gamma_s} + Z_s\right)$$

and Eq. 8,

in USCS units $\dfrac{p}{\gamma} = \dfrac{2.31 \text{ lb/in}^2}{\text{sp. gr.}}$

in SI units $\dfrac{p}{\gamma} = \dfrac{0.102 \text{ kPa}}{\text{sp. gr.}}$

Therefore,

in USCS units

$$TH = \left(\frac{11.33^2}{2 \times 32.3} + \frac{2.31 \times 124}{0.8} + 4\right) - \left(\frac{6.38^2}{2 \times 32.2} + \frac{2.31(-3.68)}{0.8} + 2\right)$$

$$= 364 - (-8) = 372 \text{ ft·lb/lb, or ft}$$

in SI units

$$TH = \left(\frac{3.48^2}{2 \times 9.807} + \frac{0.102 \times 855}{0.8} + 1.22\right) - \left(\frac{1.95^2}{2 \times 9.807} + \frac{0.102(-25.4)}{0.8} + 2\right)$$

$$= 110.9 - (-2.43) = 113.3 \text{ N·m/N, or m}$$

(b) from Eq. 7,

$$TH = \left(\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2\right) - \left(\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1\right) + \Sigma h_{f(1-2)}$$

and Eq. 8,

in USCS units $\dfrac{p}{\gamma} = \dfrac{2.31 \text{ lb/in}^2}{\text{sp. gr.}}$

in SI units $\dfrac{p}{\gamma} = \dfrac{0.102 \text{ kPa}}{\text{sp. gr.}}$

Therefore

in USCS units $TH = \left(0 + \frac{2.31 \times 100}{0.8} + 50\right) - (0 + 0 - 5) + (3 + 25)$

$= 399 - (-5) + 28 = 372$ ft·lb/lb, or ft

in SI units $TH = \left(0 + \frac{0.102 \times 689.5}{0.8} + 15.24\right) - (0 + 0 - 1.52) + (0.91 + 7.62)$

$= 103.2 - (-1.52) + 8.53 = 113.3$ N·m/N, or m

ENERGY AND HYDRAULIC GRADIENTS

The total energy at any point in a pumping system may be calculated for a particular rate of flow using Bernoulli's equation (Eq. 1). If some convenient datum plane is selected and the total energy, or head, at various locations along the system is plotted to scale, the line drawn through these points is called the ***energy gradient***. Figure 9 shows the variation in total energy H measured in feet (meters) from the suction liquid surface point 3 to the discharge liquid surface point 4. A horizontal energy gradient indicates no loss of head.

The line drawn through the sum of the pressure and elevation heads at various points represents the pressure variation in flow measured above the datum plane. It also represents the height the liquid would rise in vertical columns relative to the datum plane when the columns are placed at various locations along pipes having positive pressure anywhere in the system. This line, shown dotted in Fig. 9, is called the ***hydraulic gradient***. The difference between the energy gradient line and the hydraulic gradient line is the velocity head in the pipe at that point.

The pump total head is the algebraic difference between the total energy at the pump discharge (point 2) and the total energy at the pump suction (point 1).

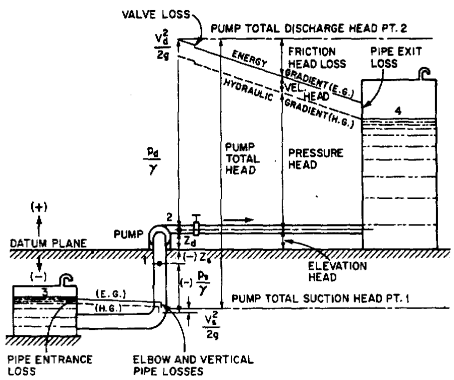


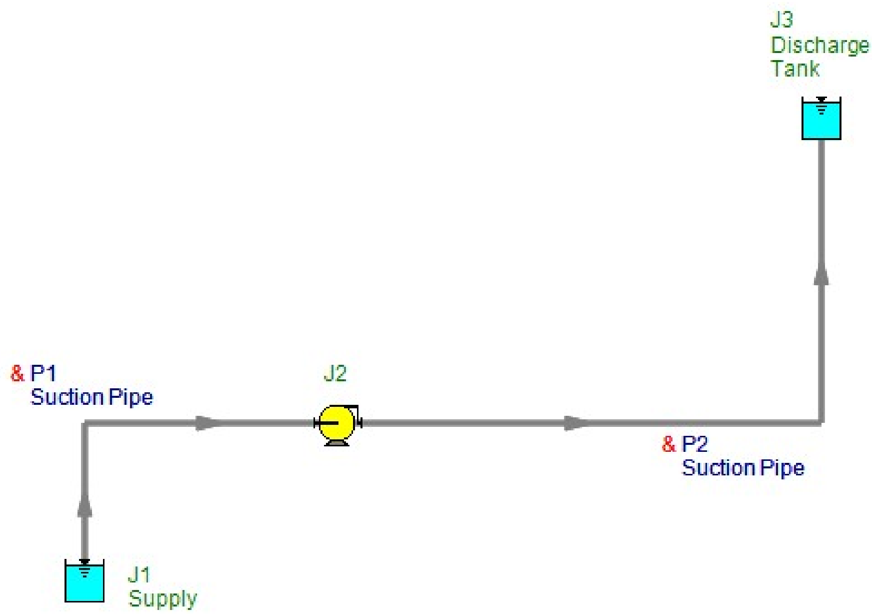
FIG. 9 Energy and hydraulic gradients.

SYSTEM-HEAD CURVES

A pumping system may consist of piping, valves, fittings, open channels, vessels, nozzles, weirs, meters, process equipment, and other liquid-handling conduits through which flow is required for various reasons. When a particular system is being analyzed for the purpose of selecting a pump or pumps, the resistance to flow of the liquid through these various components must be calculated. It will be explained in more detail later in this section that the resistance increases with flow at a rate approximately equal to the square of the flow through the system. In addition to over-

View Verification Case 18 Model

Verification Case 18



Verification Case 19

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify19.fth

REFERENCE: Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.35-37, Example 6

FLUID: Water at 109 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (feet)	1.82	1.874

DISCUSSION:

The handbook uses a chart to obtain the friction factor of 0.012. AFT Fathom uses a more accurate correlation based on Colebrook-White to obtain a friction factor of 0.1235, which is 3% higher. This is the reason the AFT Fathom head loss calculation differs slightly from the handbook. The AFT Fathom calculation is more accurate because of the higher accuracy friction factor.

List of All Verification Models

Verification Case 19 Problem Statement

Verification Case 19

Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.35-37, Example 6

Karassik, Krutzsch, Fraser and Messina Title Page

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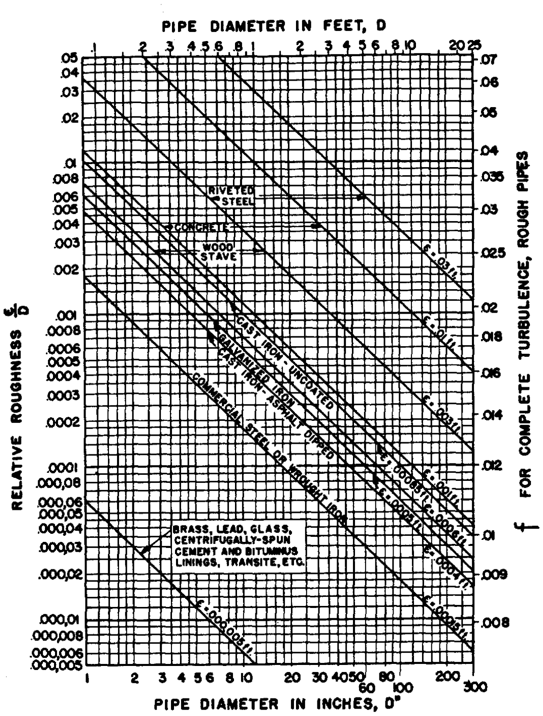


FIG. 32 Relative roughness and friction factors for new, clean pipes for flow of 60°F (15.6°C) water. (Ref. 5) (1 meter = 39.37 in = 3.28 ft)

EXAMPLE 6 Calculate the frictional head loss for 100 ft (30.48 m) of 20-in (50.8-cm), Schedule 20, 19.350-in (49.15-cm) ID, seamless steel pipe for 109°F (42.8°C) water flowing at a rate of 11,500 gpm (2612 m³/h). Use the Darcy-Weisbach formula.

In USCS units $V = \dfrac{\text{gpm}}{(\text{pipe ID in inches})^2} \times 0.408 = \dfrac{11{,}500}{19.35^2} \times 0.408 = 12.53 \text{ ft/s}$

$VD'' = 12.53 \times 19.35 = 242 \text{ ft/s} \times \text{in}$

In SI units $V = \dfrac{\text{m}^3/\text{h}}{(\text{pipe ID in cm})^2} \times 3.54 = \dfrac{2612}{49.15^2} \times 3.54 = 3.83 \text{ m/s}$

$VD = 3.83 \times 0.4915 = 1.88 \text{ m/s} \times \text{m} = 1.88 \times 129.2 = 242 \text{ ft/s} \times \text{in}$

From Fig. 33 $Re = 3 \times 10^6$

From Fig. 32 $\dfrac{\epsilon}{D} = 0.00009$

From Fig. 31 $f = 0.012$

TABLE 2 Values of Friction Factor *C* to Be Used with the Hazen-Williams Formula in Fig. 34.

Type of pipe	Age	Size, in[a]	*C*
Cast iron	New	All sizes	130
	5 years old	12 and over	120
		8	119
		4	118
	10 years old	24 and over	113
		12	111
		4	107
	20 years old	24 and over	100
		12	96
		4	89
	30 years old	30 and over	90
		16	87
		4	75
	40 years old	30 and over	83
		16	80
		4	64
		40 and over	77
		24	74
		4	55
Welded steel	Any age, any size		Same as for cast iron pipe 5 years older
Riveted steel	Any age, any size		Same as for cast iron pipe 10 years older
Wood-stave	Average value, regardless of age and size		120
Concrete or concrete-lined	Large sizes, goodworkmanship, steel forms		140
	Large sizes, good workmanship, wooden forms		120
	Centrifugally spun		135
Vitrified	In good condition		110

[a]In × 25.4 = mm.
SOURCE: Adapted from Ref. 15.

Using Eq. 16,

in USCS units

$$D = \frac{19.35}{12} = 1.61 \text{ ft}$$

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.012\frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.82 \text{ ft}$$

in SI units

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.012\frac{30.48}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 0.556 \text{ m}$$

EXAMPLE 7 The flow in Example 6 is increased until complete turbulence results. Determine the friction factor *f* and flow.

From Fig. 31, follow the relative roughness curve $\epsilon/D = 0.00009$ to the beginning of the zone marked "complete turbulence, rough pipes" and read

$$f = 0.0119 \qquad \text{at } Re = 2 \times 10^7$$

The problem may also be solved using Fig. 32. Enter relative roughness $\epsilon/D = 0.00009$ and read directly across to

$$f = 0.0119$$

An increase in Re from 3×10^6 to 2×10^7 would require an increase in flow to

in USCS units $$\frac{2 \times 10^7}{3 \times 10^6} \times 11{,}500 = 76{,}700 \text{ gpm}$$

in SI units $$\frac{2 \times 10^7}{3 \times 10^6} \times 2612 = 17{,}413 \text{ m}^3/\text{h}$$

EXAMPLE 8 The liquid in Example 6 is changed to water at 60°F (15.6°C). Determine Re, f, and the frictional head loss per 100 ft (100 m) of pipe.

$$VD'' = 242 \text{ ft/s} \times \text{in} \qquad \text{(as in Example 6)}$$

Since the liquid is 60°F (15.6°C) water, enter Fig. 31 and read directly downward from VD'' to

$$Re = 1.8 \times 10^6$$

Where the line VD'' to Re crosses $\epsilon/D = 0.00009$ in Fig. 31, read

$$f = 0.013$$

Water at 60°F (15.6°C) is more viscous than 109°F (42.8°C) water, and this accounts for the fact that Re decreases and f increases. Using Eq. 16, it can be calculated that the frictional head loss increases to

in USCS units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.97 \text{ ft}$$

in SI units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 1.97 \text{ m}$$

EXAMPLE 9 A 102-in (259-cm) ID welded steel pipe is to be used to convey water at a velocity of 11.9 ft/s (3.63 m/s). Calculate the expected loss of head due to friction per 1000 ft and per 1000 m of pipe after 20 years. Use the empirical Hazen-Williams formula.

From Table 2, $C = 100$.

in USCS units $$r = D/4 = 102/(4 \times 12) = 2.13 \text{ ft}$$
in SI units $$r = D/4 = 2.59/4 = 0.648 \text{ m}$$

Substituting in Eq. 17,

in USCS units $$S^{0.54} = \frac{V}{1.318Cr^{0.63}} = \frac{11.9}{1.318 \times 100 \times 2.13^{0.63}} = 0.0557$$
$$S = (0.0557)^{1/0.54} = 0.0048 \text{ ft/ft}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ ft}$$

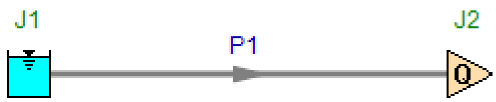
in SI units $$S^{0.54} = \frac{V}{0.8492Cr^{0.63}} = \frac{3.63}{0.8492 \times 100 \times 0.648^{0.63}} = 0.0562$$
$$S = (0.0562)^{1/0.54} = 0.0048 \text{ m/m}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ m}$$

The problem may also be solved by using Fig. 34, following the trace lines:

$$h_f \approx 5 \text{ ft (m)}$$

View Verification Case 19 Model

Verification Case 19



Verification Case 20

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify20.fth

REFERENCE: Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.38, Example 8

FLUID: Water at 60 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (feet)	1.97	1.937

DISCUSSION:

The handbook uses a chart to obtain the Reynolds number of 1.8x10^6 and friction factor of 0.013. AFT Fathom solves for the Reynolds number and uses a more accurate correlation based on Colebrook-White to obtain a friction factor of 0.1277, which is 2% lower. This is the reason the AFT Fathom head loss calculation differs slightly from the handbook. The AFT Fathom calculation is more accurate because of the higher accuracy Reynolds number and friction factor.

List of All Verification Models

Verification Case 20 Problem Statement

Verification Case 20

Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.38, Example 8

Karassik, Krutzsch, Fraser and Messina Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

From Fig. 31, follow the relative roughness curve $\epsilon/D = 0.00009$ to the beginning of the zone marked "complete turbulence, rough pipes" and read

$$f = 0.0119 \qquad \text{at } Re = 2 \times 10^7$$

The problem may also be solved using Fig. 32. Enter relative roughness $\epsilon/D = 0.00009$ and read directly across to

$$f = 0.0119$$

An increase in Re from 3×10^6 to 2×10^7 would require an increase in flow to

in USCS units $$\frac{2 \times 10^7}{3 \times 10^6} \times 11{,}500 = 76{,}700 \text{ gpm}$$

in SI units $$\frac{2 \times 10^7}{3 \times 10^6} \times 2612 = 17{,}413 \text{ m}^3/\text{h}$$

EXAMPLE 8 The liquid in Example 6 is changed to water at 60°F (15.6°C). Determine Re, f, and the frictional head loss per 100 ft (100 m) of pipe.

$$VD'' = 242 \text{ ft/s} \times \text{in} \qquad \text{(as in Example 6)}$$

Since the liquid is 60°F (15.6°C) water, enter Fig. 31 and read directly downward from VD'' to

$$Re = 1.8 \times 10^6$$

Where the line VD'' to Re crosses $\epsilon/D = 0.00009$ in Fig. 31, read

$$f = 0.013$$

Water at 60°F (15.6°C) is more viscous than 109°F (42.8°C) water, and this accounts for the fact that Re decreases and f increases. Using Eq. 16, it can be calculated that the frictional head loss increases to

in USCS units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.97 \text{ ft}$$

in SI units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 1.97 \text{ m}$$

EXAMPLE 9 A 102-in (259-cm) ID welded steel pipe is to be used to convey water at a velocity of 11.9 ft/s (3.63 m/s). Calculate the expected loss of head due to friction per 1000 ft and per 1000 m of pipe after 20 years. Use the empirical Hazen-Williams formula.

From Table 2, $C = 100$.

in USCS units $$r = D/4 = 102/(4 \times 12) = 2.13 \text{ ft}$$

in SI units $$r = D/4 = 2.59/4 = 0.648 \text{ m}$$

Substituting in Eq. 17,

in USCS units
$$S^{0.54} = \frac{V}{1.318Cr^{0.63}} = \frac{11.9}{1.318 \times 100 \times 2.13^{0.63}} = 0.0557$$
$$S = (0.0557)^{1/0.54} = 0.0048 \text{ ft/ft}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ ft}$$

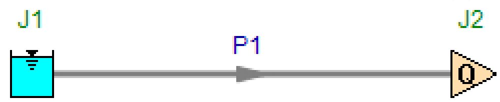
in SI units
$$S^{0.54} = \frac{V}{0.8492Cr^{0.63}} = \frac{3.63}{0.8492 \times 100 \times 0.648^{0.63}} = 0.0562$$
$$S = (0.0562)^{1/0.54} = 0.0048 \text{ m/m}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ m}$$

The problem may also be solved by using Fig. 34, following the trace lines:

$$h_f \approx 5 \text{ ft (m)}$$

View Verification Case 20 Model

Verification Case 20



Verification Case 21

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify21.fth

REFERENCE: Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.38, Example 9

FLUID: Water, using Hazen-Williams

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (feet)	4.8	4.8

DISCUSSION:

Results for AFT Fathom vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom help file.

List of All Verification Models

Verification Case 21 Problem Statement

Verification Case 21

Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.38, Example 9

Karassik, Krutzsch, Fraser and Messina Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

From Fig. 31, follow the relative roughness curve $\epsilon/D = 0.00009$ to the beginning of the zone marked "complete turbulence, rough pipes" and read

$$f = 0.0119 \qquad \text{at } Re = 2 \times 10^7$$

The problem may also be solved using Fig. 32. Enter relative roughness $\epsilon/D = 0.00009$ and read directly across to

$$f = 0.0119$$

An increase in Re from 3×10^6 to 2×10^7 would require an increase in flow to

in USCS units $$\frac{2 \times 10^7}{3 \times 10^6} \times 11{,}500 = 76{,}700 \text{ gpm}$$

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$$VD'' = 242 \text{ ft/s} \times \text{in} \qquad \text{(as in Example 6)}$$

Since the liquid is 60°F (15.6°C) water, enter Fig. 31 and read directly downward from VD'' to

$$Re = 1.8 \times 10^6$$

Where the line VD'' to Re crosses $\epsilon/D = 0.00009$ in Fig. 31, read

$$f = 0.013$$

Water at 60°F (15.6°C) is more viscous than 109°F (42.8°C) water, and this accounts for the fact that Re decreases and f increases. Using Eq. 16, it can be calculated that the frictional head loss increases to

in USCS units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.97 \text{ ft}$$

in SI units $$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.013\frac{100}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 1.97 \text{ m}$$

EXAMPLE 9 A 102-in (259-cm) ID welded steel pipe is to be used to convey water at a velocity of 11.9 ft/s (3.63 m/s). Calculate the expected loss of head due to friction per 1000 ft and per 1000 m of pipe after 20 years. Use the empirical Hazen-Williams formula.

From Table 2, $C = 100$.

in USCS units $$r = D/4 = 102/(4 \times 12) = 2.13 \text{ ft}$$

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Substituting in Eq. 17,

in USCS units
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$$S = (0.0557)^{1/0.54} = 0.0048 \text{ ft/ft}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ ft}$$

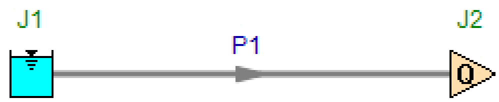
in SI units
$$S^{0.54} = \frac{V}{0.8492Cr^{0.63}} = \frac{3.63}{0.8492 \times 100 \times 0.648^{0.63}} = 0.0562$$
$$S = (0.0562)^{1/0.54} = 0.0048 \text{ m/m}$$
$$h_f = 1000 \times 0.0048 = 4.8 \text{ m}$$

The problem may also be solved by using Fig. 34, following the trace lines:

$$h_f \approx 5 \text{ ft (m)}$$

View Verification Case 21 Model

Verification Case 21



Verification Case 22

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify22.fth

REFERENCE: Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.61-62, Example 13

FLUID: Oil

ASSUMPTIONS: N/A

RESULTS:

Parameter	Handbook	AFT Fathom
Head loss (feet)	98.34	98.58

DISCUSSION:

The viscosity was not provided, so the friction factors determined in the handbook were used directly for pipes 1 and 2. In addition, a loss factor for the 1 ½ inch gate valve was not in the AFT Fathom library, so was entered in pipe 2 as a separate loss factor on the Fittings & Losses tab.

The head loss is given in the Junction Deltas table at the top of the Output window. The junction delta was setup in the Output Control window. Or it can be calculated by adding up the pipe head losses.

List of All Verification Models

Verification Case 22 Problem Statement

Verification Case 22

Igor Karassik, William Krutzsch, Warren Fraser, Joseph Messina, Pump Handbook, 2nd Ed., 1986, McGraw-Hill, Page 8.61-62, Example 13

Karassik, Krutzsch, Fraser and Messina Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

in SI units
$$\Sigma h_f = h_{fs} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5}$$
$$= 0.56 + 28.39 + 0.0078 + 0.044 + 0.0250 + 0.0803 + 1.06 = 30.17 \text{ m}$$
$$\text{Total variation} = \pm(0.044 + 0.0062 + 0.02 + 0.32) = \pm 0.39 \text{ m}$$

EXAMPLE 13 Solve Example 12 using resistance coefficients from Tables 6.

Suction pipe:

in USCS units $h_{fs} = 1.84$ ft (same as in Example 12)

in SI units $h_{fs} = 0.56$ m (same as in Example 12)

Discharge pipe:

in USCS units $h_{fd} = 92.9$ ft (same as in Example 12)

in SI units $h_{fd} = 28.39$ m (same as in Example 12)

Valve and fitting losses from Tables 6 and Eq. 20: 2-in (51-mm) bellmouth, $K = 0.04$

in USCS units
$$h_{f1} = 0.04 \frac{5.73^2}{2 \times 32.17} = 0.020 \text{ ft}$$

in SI units
$$h_{f1} = 0.04 \frac{1.75^2}{2 \times 9.807} = 0.0062 \text{ m}$$

2-in (51-mm) SR 90° elbow, $K = 30\,f_T$
$$f_T = 0.019 \quad \text{(from Table 5a)}$$
$$K = 30 \times 0.019 = 0.57$$

in USCS units
$$h_{f2} = 0.57 \frac{5.73^2}{2 \times 32.17} = 0.29 \text{ ft}$$

in SI units
$$h_{f2} = 0.57 \frac{1.57^2}{2 \times 9.807} = 0.089 \text{ m}$$

2-in (51-mm) gate valve, $\beta = 1$, $\theta = 0$, $K = 8f_T$
$$K = 8 \times 0.019 = 0.15$$

in USCS units
$$h_{f3} = 0.15 \frac{5.73^2}{2 \times 32.17} = 0.077 \text{ ft}$$

in SI units
$$h_{f3} = 0.15 \frac{1.75^2}{2 \times 9.807} = 0.023 \text{ m}$$

1½-in (38-mm) gate valve, $\beta = 1$, $\theta = 0$, $K = 8f_T$ for Schedule 80
$$K = 8\,f_T(1.10/1.500)^4 = 10.62 f_T \text{ for Schedule 40}$$
$$f_T = 0.021 \quad \text{(from Table 6a)}$$
$$K = 10.62 \times 0.021 = 0.22$$

in USCS units
$$h_{f4} = 0.22 \frac{9.44^2}{2 \times 32.17} = 0.30 \text{ ft}$$

in SI units
$$h_{f4} = 0.22 \frac{2.88^2}{2 \times 9.807} = 0.093 \text{ m}$$

1½-in (38-mm) swing check valve, $K = 100f_T$ (from Table 6b)

Minimum pipe velocity for full disk lift $= 35\sqrt{\overline{V}} = 35\sqrt{0.0189} = 4.81$ ft/s < 9.44 ft/s

$$K = 100 \times 0.021 = 2.1$$

in USCS units $$h_{f5} = 2.1 \frac{9.44^2}{2 \times 32.17} = 2.91 \text{ ft}$$

in SI units $$h_{f5} = 2.1 \frac{2.88^2}{2 \times 9.807} = 0.89 \text{ m}$$

Total pipe, valve, and fitting losses:

in USCS units $$\Sigma h_f = h_{fs} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5}$$
$$= 1.84 + 92.9 + 0.020 + 0.29 + 0.077 + 0.30 + 2.91$$
$$= 98.34 \text{ ft}$$

in SI units $$\Sigma h_f = h_{fs} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5}$$
$$= 0.56 + 28.39 + 0.062 + 0.089 + 0.023 + 0.093 + 0.89$$
$$= 30.05 \text{ m}$$

INCREASERS The head lost when there is a sudden increase in pipe diameter, with velocity changing from V_1 to V_2 in the direction of flow, can be calculated analytically. Computed results have been confirmed experimentally to be true to within ±3%. The head loss is expressed as shown below, with K computed to be equal to unity:

$$h = K\frac{(V_1 - V_2)^2}{2g} = K\left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 \frac{V_1^2}{2g} = K\left[\left(\frac{D_2}{D_1}\right)^2 - 1\right]^2 \frac{V_2^2}{2g} \qquad (23)$$

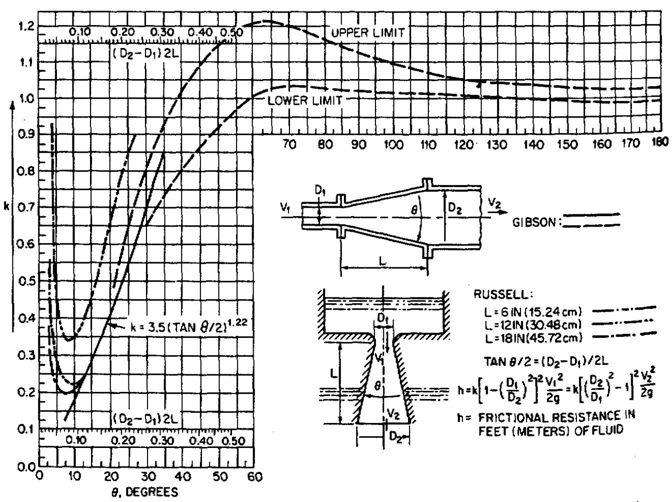
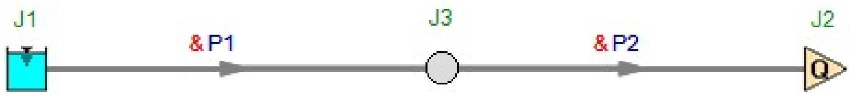


FIG. 39 Resistance coefficients for increasers and diffusers. (D = in or ft [m]; V = ft/s [m/s]). (Ref. 5)

View Verification Case 22 Model

Verification Case 22



Verification Case 23

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify23.fth

REFERENCE: Hydraulic Institute, Effects of Liquid Viscosity on Rotodynamic (Centrifugal and Vertical) Pump Performance, ANSI/HI 9.6.7-2010, 2010, Hydraulic Institute, Page 13

FLUID: Viscous fluid through a pump

ASSUMPTIONS: N/A

RESULTS:

Parameter	HI Standard	AFT Fathom
CQ	0.938	0.9381
CH	0.938	0.9380
CE	0.739	0.7388
CH at 60% Flow	0.958	0.9577

DISCUSSION:

The kinematic viscosity in centistokes (120 cSt) was converted to dynamic viscosity for use in AFT Fathom assuming a specific gravity of 0.9. This results in a viscosity of 108 cp. Discharge and suction pressures and pipes were used, with the pipe diameters made very large to minimize pressure loss.

The water flow rate and pressure difference from the problem statement were used to find an approximate CQ of 0.9395, which provides a viscous flow rate of 413.38 gal/min for the 100 percent flow case and 248.028 gal/min for the 60 percent flow case. These were used in assigned flow junctions which replaced the inlet assigned pressures for the final calculation. The pumps were redefined with the water pump curve from the problem statement, shown below, to find the final values of CQ, CH, and CE.

Water Flow Rate (gal/min)	Water Head (ft)	Water Efficiency (Decimal)
264	340	0.602
352	323	0.660
440	300	0.680
528	272	0.660

The correction factors, CQ, CH and CE, all correspond closely to those in the handbook. The value of CE given in the reference problem was shown as 0.738 in the calculation, but 0.739 in Table 9.6.7.4.5b. In addition, CE is calculated as 0.739 if calculated using the same numbers as given in Step 4, so it is assumed that the difference in the value given in the reference includes round-off errors.

The CQ, CH, and CE values in AFT Fathom are displayed in the Pump Summary and within the labels on the Workspace.

List of All Verification Models

Verification Case 23 Problem Statement

Verification Case 23

Hydraulic Institute, Effects of Liquid Viscosity on Rotodynamic (Centrifugal and Vertical) Pump Performance, ANSI/HI 9.6.7-2010, 2010, Hydraulic Institute, Page 13

Hydraulic Institute ANSI HI 9.6.7-2010 Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

EXAMPLE (US customary units): Refer to Figure 9.6.7.4.5d, Example performance chart of a single-stage pump, and Table 9.6.7.4.5b, Example calculations. The given single-stage pump has a water performance best efficiency flow of 440 gpm at 300 ft total head at 3550 rpm and has a pump efficiency of 0.68. The procedure below illustrates how to correct the pump performance characteristics for a viscous liquid of 120 cSt and specific gravity of 0.90.

Step 1. Calculate parameter B based on the water performance best efficiency flow conditions using Equation 3. If the pump is a multistage configuration, calculate parameter B using the head per stage.

Given units of $Q_{BEP\text{-}W}$ in gpm, H_W in ft, N in rpm, and V_{vis} in cSt:

$$B = 26.6 \times \frac{(120)^{0.50} \times (300)^{0.0625}}{(440)^{0.375} \times (3550)^{0.25}} = 5.50$$

Step 2. Calculate correction factor for flow (C_Q) using Equation 4 and correct the flows corresponding to ratios of water best efficiency flow ($Q_W / Q_{BEP\text{-}W}$).

$$C_Q = (2.71)^{-0.165 \times (\log 5.50)^{3.15}} = 0.938$$

At: $\frac{Q_W}{Q_{BEP-W}} = 1.00$

$$Q_{vis} = 0.938 \times 440 \times 1.00 = 413 \text{ gpm}$$

At: $\frac{Q_W}{Q_{BEP-W}} = 0.60$

$$Q_{vis} = 0.938 \times 440 \times 0.60 = 248 \text{ gpm}$$

From Equation 5, the correction factor for head ($C_{BEP\text{-}H}$) is equal to (C_Q) at $Q_{BEP\text{-}W}$

$$Q_{BEP\text{-}H} = C_Q = 0.938$$

At $Q_{BEP\text{-}W}$, the corresponding viscous head ($H_{BEP\text{-}vis}$) is:

$$H_{BEP\text{-}vis} = 0.938 \times 300 = 281 \text{ ft}$$

Step 3. Calculate head correction factors (C_H) and corresponding values of viscous head (H_{vis}) for flows (Q_W) greater than or less than the water best efficiency flow ($Q_{BEP\text{-}W}$).

At 60% of $Q_{BEP\text{-}W}$, the corresponding head correction factor (C_H) and viscous head (H_{vis}) are calculated using Equation 6:

$$C_H = 1 - (1 - 0.938) \times (0.60)^{0.75} = 0.958$$

$$H_{vis} = 0.958 \times 340 = 326 \text{ ft}$$

Step 4. Calculate the correction factor for efficiency (C_η) and the corresponding values of viscous pump efficiency (η_{vis}) for flows (Q_W) greater than, less than, and equal to the water best efficiency flow $Q_{BEP\text{-}W}$. Equation 7 is used to calculate C_η as the value of parameter B = 5.50 calculated in Step 1 falls within the range of 1 to 40:

$$C_\eta = (5.50)^{-[0.0547 \times (5.50)^{0.69}]} = 0.738$$

At: $\frac{Q_W}{Q_{BEP-W}} = 1.00$

Where: $\eta_W = 0.680$

$\eta_{vis} = 0.738 \times 0.680 = 0.502$

At: $\frac{Q_W}{Q_{BEP-W}} = 0.60$

Where: $\eta_W = 0.602$

$\eta_{vis} = 0.738 \times 0.602 = 0.444$

Step 5. Calculate the values for viscous pump shaft input power (P_{vis}) for flows (Q_W) greater than, less than, or equal to the water best efficiency flow Q_{BEP-W} using Equation 9:

At: $\frac{Q_W}{Q_{BEP-W}} = 1.00$

$$P_{vis} = \frac{413 \times 281 \times 0.90}{3960 \times 0.502} = 52.5 \text{ hp}$$

At: $\frac{Q_W}{Q_{BEP-W}} = 0.60$

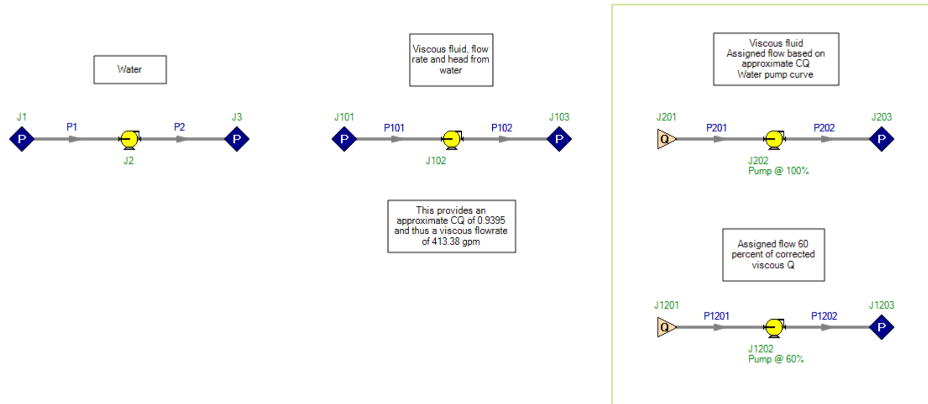
$$P_{vis} = \frac{248 \times 326 \times 0.90}{3960 \times 0.444} = 41.4 \text{ hp}$$

Table 9.6.7.4.5b — Example calculations (US customary units)

Viscosity of liquid to be pumped (V_{vis}) — cSt	120			
Specific gravity of viscous liquid (s)	0.90			
Pump shaft speed (N) — rpm	3550			
Ratio of water best efficiency flow Q_W / Q_{BEP-W}	0.60	0.80	1.00	1.20
Water rate of flow (Q_W or Q_{BEP-W}) — gpm	264	352	440	528
Water head per stage (H_W) — ft	340	323	300	272
Water pump efficiency (η_W)	0.602	0.66	0.680	0.66
Parameter B	5.50			
Correction factor for flow (C_Q)	0.938			
Correction factors for head (C_H or C_{BEP-H})	0.958	0.948	0.938	0.929
Correction factor for efficiency (C_η)	0.739			
Corrected flow (Q_{vis}) — gpm	248	330	413	495
Corrected head per stage (H_{vis} or $H_{BEP-vis}$) — ft	326	306	281	252
Corrected efficiency (η_{vis})	0.44	0.49	0.50	0.49
Viscous shaft input power (P_{vis}) — bhp	41.4	46.7	52.5	57.9

View Verification Case 23 Model

Verification Case 23



Verification Case 24

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify24.fth

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 3-19, 21, Example 3.19

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Lindenburg	AFT Fathom
Head loss (feet)	51.6	51.53

DISCUSSION:

All input was provided explicitly by the problem statement.

Lindeburg pulls the friction factor directly from the Moody chart, while AFT Fathom uses the Colebrook-White equation to iterate for friction factor. Both methods result in a friction factor of 0.0195 if the friction factor is rounded to 4 digits.

List of All Verification Models

Verification Case 24 Problem Statement

Verification Case 24

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 3-19, 21, Example 3.19

Lindeburg Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

PART 4: Fluid Dynamics

1 FLUID CONSERVATION LAWS

Many fluid flow problems can be solved by using the principles of conservation of mass and energy.

When applied to fluid flow, the principle of mass conservation is known as the *continuity equation*:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad 3.66$$

$$\dot{m}_1 = \dot{m}_2 \quad 3.67$$

If the fluid is incompressible, $\rho_1 = \rho_2$, so

$$A_1 v_1 = A_2 v_2 \quad 3.68$$

$$\dot{V}_1 = \dot{V}_2 \quad 3.69$$

The energy conservation principle is based on the Bernoulli equation. However, terms for friction loss and hydraulic machines must be included.

$$\left(\frac{p_1}{\rho} + \frac{v_1^2}{2g_c} + z_1\right) + h_A = \left(\frac{p_2}{\rho} + \frac{v_2^2}{2g_c} + z_2\right) + h_E + h_f \quad 3.70$$

2 HEAD LOSS DUE TO FRICTION

The most common expression for calculating head loss due to friction (h_f) is the *Darcy formula*:

$$h_f = \frac{fLv^2}{2Dg_c} \quad 3.71$$

The *Moody friction factor chart* (figure 3.13) probably is the most convenient method of determining the friction factor, f.

The basic parameter required to use the Moody friction factor chart is the Reynolds number. If the Reynolds number is less than 2000, the friction factor is given by equation 3.72.

$$f = \frac{64}{N_{Re}} \quad 3.72$$

For turbulent flow ($N_{Re} > 2000$), the friction factor depends on the relative roughness of the pipe. This roughness is expressed by the ratio $\frac{\epsilon}{D}$, where ϵ is the specific surface roughness and D is the inside diameter. Values of ϵ for various types of pipe are found in table 3.8.

Another method for finding the friction head loss is the *Hazen-Williams formula*. The Hazen-Williams formula gives good results for liquids that have kinematic viscosities around 1.2 EE−5 ft^2/sec (corresponding to 60°F water). At extremely high and low temperatures, the Hazen-Williams formula can be as much as 20% in error for water. The Hazen-Williams formula should be used only for turbulent flow.

The Hazen-Williams head loss is

$$h_f = \frac{(3.022)(v)^{1.85}L}{(C)^{1.85}(D)^{1.165}} \quad 3.73$$

Or, in terms of other units,

$$h_f = (10.44)(L)\frac{(gpm)^{1.85}}{(C)^{1.85}(d_{\text{inches}})^{4.8655}} \quad 3.74$$

Use of these formulas requires a knowledge of the Hazen-Williams coefficient, C, which is assumed to be independent of the Reynolds number. Table 3.8 gives values of C for various types of pipe.

Values of f and h_f are appropriate for clean, new pipe. As some pipes age, it is not uncommon for scale build-up to decrease the equivalent flow diameter. This diameter decrease produces a dramatic increase in the friction loss.

$$\frac{h_{f,\text{scaled}}}{h_{f,\text{new}}} = \left(\frac{D_{\text{new}}}{D_{\text{scaled}}}\right)^5 \quad 3.75$$

Because of this scale effect, an uprating factor of 10-30% is commonly applied to f or h_f in anticipation of future service conditions.

Example 3.19

50°F water is pumped through 4″ schedule 40 welded steel pipe ($\epsilon = .0002$) at the rate of 300 gpm. What is the friction head loss calculated by the Darcy formula for 1000 feet of pipe?

First, it is necessary to collect data on the pipe and water. The fluid viscosity and pipe dimensions can be found from tables at the end of the chapter.

kinematic viscosity = 1.41 EE − 5 ft^2/sec
inside diameter = .3355 ft
flow area = .0884 ft^2

The flow quantity is

$$(300)(.002228) = .6684 \text{ cfs}$$

The velocity is

$$v = \frac{\dot{V}}{A} = \frac{.6684}{.0884} = 7.56 \text{ fps}$$

The Reynolds number is

$$N_{Re} = \frac{(.3355)(7.56)}{1.41 \text{ EE} - 5} = 1.8 \text{ EE5}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{.0002}{.3355} = .0006$$

From the Moody friction factor chart, $f = .0195$.

From equation 3.71,

$$h_f = \frac{(.0195)(1000)(7.56)^2}{(2)(.3355)(32.2)} = 51.6 \text{ ft}$$

Example 3.20

Repeat example 3.19 using the Hazen-Williams formula. Assume $C = 100$.

Using equation 3.73,

$$h_f = \frac{(3.012)(7.56)^{1.85}(1000)}{(100)^{1.85}(.3355)^{1.165}} = 90.5 \text{ ft}$$

Using equation 3.74,

$$h_f = (10.44)(1000)\frac{(300)^{1.85}}{(100)^{1.85}(4.026)^{4.8655}} = 90.9 \text{ ft}$$

3 MINOR LOSSES

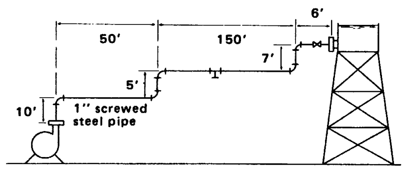
In addition to the head loss caused by friction between the fluid and the pipe wall, losses also are caused by obstructions in the line, changes in direction, and changes in flow area. These losses are named ***minor losses*** because they are much smaller in magnitude than the h_f term. Two methods are used to determine these losses: the method of equivalent lengths and the method of loss coefficients.

The method of ***equivalent lengths*** uses a table to convert each valve and fitting into an equivalent length of straight pipe. This length is added to the actual pipeline length and substituted into the Darcy equation for L_e.

$$h_f = \frac{fL_e v^2}{2Dg_c} \qquad 3.76$$

Example 3.21

Using table 3.9, determine the equivalent length of the piping network shown.



The line consists of:

1 gate valve	.84
5 90° standard elbows	5.2×5
1 tee run	3.2
straight pipe	228
L_e =	258 feet

The alternative is to use a loss coefficient, K. This loss coefficient, when multiplied by the velocity head, will give the head loss in feet. This method must be used to find exit and entrance losses.

$$h_f = K\frac{v^2}{2g_c} \qquad 3.77$$

Table 3.9
Typical Equivalent Lengths of Schedule 40 Straight Pipe For Steel Fittings and Valves
(For any fluid in turbulent flow)

Equivalent Length, ft

Fitting Type	1"	Pipe Size* 2"	4"	(flanged pipe) 6"	8"
Short Radius 90° Elbow	5.2	8.5	13.0	8.9	12.0
Long Radius 90° Elbow	2.7	3.6	4.6	5.7	7.0
Regular 45° Elbow	1.3	2.7	5.5	5.6	7.7
Tee, flow through line (run)	3.2	7.7	17.0	3.8	4.7
Tee, flow through stem	6.6	12.0	21.0	18.0	24.0
180° Return Bend	5.2	8.5	13.0	8.9	12.0
Globe Valve	29.0	54.0	110.0	190.0	260.0
Gate Valve	.84	1.5	2.5	3.2	3.2
Angle Valve	17.0	18.0	18.0	63.0	90.0
Swing Check Valve	11.0	19.0	38.0	63.0	90.0
Coupling or Union	.29	.45	.65	—	—

*Screwed pipe and fittings unless flanged indicated.

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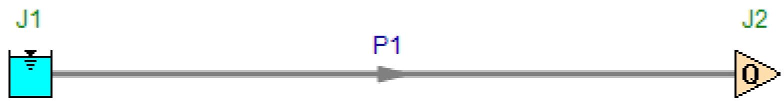
View Verification Case 24 Model

Verification Case 24



Verification Case 25

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify25.fth

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 3-21, Example 3.20

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Lindenburg	AFT Fathom
Head loss (feet)	90.5	90.66

DISCUSSION:

Results for AFT Fathom vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom help file.

List of All Verification Models

Verification Case 25 Problem Statement

Verification Case 25

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 3-21, Example 3.20

Lindeburg Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

The relative roughness is

$$\frac{\epsilon}{D} = \frac{.0002}{.3355} = .0006$$

From the Moody friction factor chart, $f = .0195$.

From equation 3.71,

$$h_f = \frac{(.0195)(1000)(7.56)^2}{(2)(.3355)(32.2)} = 51.6 \text{ ft}$$

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Repeat example 3.19 using the Hazen-Williams formula. Assume $C = 100$.

Using equation 3.73,

$$h_f = \frac{(3.012)(7.56)^{1.85}(1000)}{(100)^{1.85}(.3355)^{1.165}} = 90.5 \text{ ft}$$

Using equation 3.74,

$$h_f = (10.44)(1000)\frac{(300)^{1.85}}{(100)^{1.85}(4.026)^{4.8655}} = 90.9 \text{ ft}$$

3 MINOR LOSSES

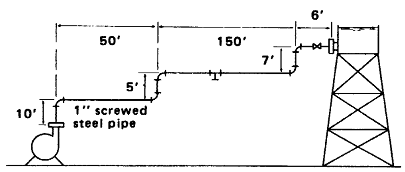
In addition to the head loss caused by friction between the fluid and the pipe wall, losses also are caused by obstructions in the line, changes in direction, and changes in flow area. These losses are named *minor losses* because they are much smaller in magnitude than the h_f term. Two methods are used to determine these losses: the method of equivalent lengths and the method of loss coefficients.

The method of *equivalent lengths* uses a table to convert each valve and fitting into an equivalent length of straight pipe. This length is added to the actual pipeline length and substituted into the Darcy equation for L_e.

$$h_f = \frac{fL_ev^2}{2Dg_c} \qquad 3.76$$

Example 3.21

Using table 3.9, determine the equivalent length of the piping network shown.



The line consists of:

1 gate valve	.84
5 90° standard elbows	5.2×5
1 tee run	3.2
straight pipe	228
L_e =	258 feet

The alternative is to use a loss coefficient, K. This loss coefficient, when multiplied by the velocity head, will give the head loss in feet. This method must be used to find exit and entrance losses.

$$h_f = K\frac{v^2}{2g_c} \qquad 3.77$$

Table 3.9
Typical Equivalent Lengths of Schedule 40 Straight Pipe For Steel Fittings and Valves
(For any fluid in turbulent flow)

Equivalent Length, ft

Fitting Type	Pipe Size* 1"	2"	4"	6"	(flanged pipe) 8"
Short Radius 90° Elbow	5.2	8.5	13.0	8.9	12.0
Long Radius 90° Elbow	2.7	3.6	4.6	5.7	7.0
Regular 45° Elbow	1.3	2.7	5.5	5.6	7.7
Tee, flow through line (run)	3.2	7.7	17.0	3.8	4.7
Tee, flow through stem	6.6	12.0	21.0	18.0	24.0
180° Return Bend	5.2	8.5	13.0	8.9	12.0
Globe Valve	29.0	54.0	110.0	190.0	260.0
Gate Valve	.84	1.5	2.5	3.2	3.2
Angle Valve	17.0	18.0	18.0	63.0	90.0
Swing Check Valve	11.0	19.0	38.0	63.0	90.0
Coupling or Union	.29	.45	.65	—	—

*Screwed pipe and fittings unless flanged indicated.

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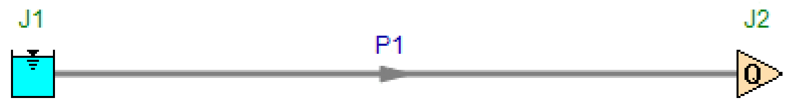
View Verification Case 25 Model

Verification Case 25



Verification Case 26

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify26.fth

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 4-6, Example 4.1

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

Parameter	Lindenburg	AFT Fathom
NPSHA (feet)	50.7	50.69
Total suction head (feet)	Not given	18.38

DISCUSSION:

The NPSHA is given in the AFT Fathom Pump Summary of the Output window.

List of All Verification Models

Verification Case 26 Problem Statement

Verification Case 26

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 4-6, Example 4.1

Lindeburg Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

step 2: Calculate NPSHA from either equation 4.12 or 4.13.

step 3: If NPSHA is greater than NPSHR, cavitation will not occur. A good safety margin is 2–3 feet of fluid. If NPSHA is insufficient, it should be increased or the NPSHR should be decreased. NPSHA can be increased by:

a. increasing the height of the free fluid level of the supply tank
b. reducing the distance and minor losses between the supply tank and the pump, or by using a larger pipe size
c. reducing the temperature of the fluid
d. pressurizing the supply tank
e. **reducing the flow rate or velocity to reduce friction in the suction line**

NPSHR can be reduced by:

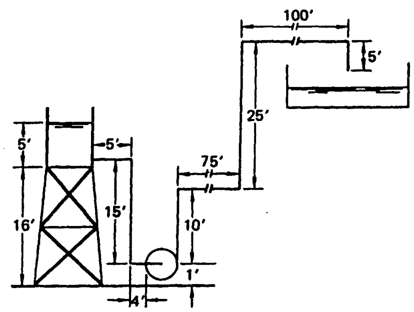
a. placing a throttling valve in the discharge line (i.e., this will increase the total head, thereby reducing the capacity of the pump and driving its operating point into a region of lower NPSHR)
b. using a double suction pump

Applications which require very high NPSHR, such as boiler feed pumps needing 150 to 250 feet, should use booster pumps in front of the high NPSHR pumps. Such booster pumps are typically single stage, double suction pumps running at low speed. Their NPSHR can be 25 feet or less.

It is important to note that throttling the input line to a pump and venting or evacuating the receiving tank both work to increase cavitation. Throttling the input line increases the friction head term and decreases NPSHA. Evacuating the receiving tank increases the flow rate, which also increases the friction head term.

Example 4.1

2 cfs of water are pumped from a feed tank mounted on a platform to an open reservoir through 6″ schedule 40 (ϵ/D = .000293) steel pipe. Determine the static suction head, total suction head, and NPSHA.



step 1: Assume 60°F and 14.7 psia. From equation 4.4,

$$h_a = \frac{(14.7)(144)}{62.4} = 33.9 \text{ ft}$$

step 2: For 6″ schedule 40 steel pipe, D = .505 ft, A = .201 ft^2.

step 3:

$$v = \frac{Q}{A} = \frac{2}{.201} = 9.95 \text{ ft/sec}$$

step 4: The equivalent lengths of the pipe and flanged fittings are:

square entrance loss	1 × 16 = 16
90° long radius elbows	2 × 5.7 = 11.4
pipe run (5 + 15 + 4)	24
	51.4 ft

step 5: At 60°, the kinematic viscosity of water is 1.217 EE−5 ft^2/sec. The vapor pressure is .6 feet of water.

step 6: The Reynolds number is

$$N_{Re} = \frac{(.505)(9.95)}{1.217 \text{ EE} - 5} = 4.13 \text{ EE5}$$

step 7: From the Moody friction factor chart, f = .0165, so

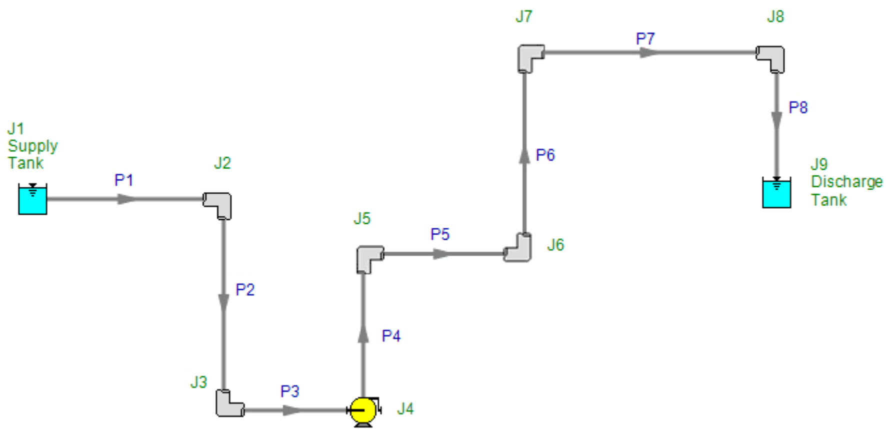
$$h_f = \frac{(.0165)(51.4)(9.95)^2}{(2)(.505)(32.2)} = 2.6 \text{ ft}$$

step 8: From equation 4.12,

$$\text{NPSHA} = 33.9 + 20 - 2.6 - .6 = 50.7 \text{ ft}$$

View Verification Case 26 Model

Verification Case 26



Verification Case 27

View Model Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify27.fth

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 5-6, Example 5.6

FLUID: Air

ASSUMPTIONS: Air is at 70 deg. F, inlet pressure is 5 inches of water, gauge, wall roughness is 0.00538 inches

RESULTS:

Parameter	Lindenburg	AFT Fathom
Velocity (feet/min)	2200	2170
Pressure loss(in. water per 100 feet)	0.5	0.4867

DISCUSSION:

The reference solves the problem using a chart. No roughness data for the duct is given. Using the chart on page 5-7 of the reference, a representative roughness was determined as follows: for a 12 inch duct at 3000 feet/min, the pressure drop is 1 inch of water per 100 feet. This information was entered into the AFT Pipe Sizing Utility (PSU, part of the Engineering Utility Suite) to obtain an effective duct roughness. A value of 0.00538 inches was obtained, and this was used in AFT Fathom. Results from PSU are given below.

Slight differences in predictions above result from the reference's round-off errors and reading the solution from a chart.

- PSU Results
  - Friction Factor = 0.01788
  - Epsilon = 0.00538 inches
  - Hazen-Williams Factor = 110.390
- Input Parameters
  - Delta Pressure = 1.000 in. H2O std.
    - Velocity = 3000 feet/min
    - Vol. Flow Rate = 39.270 ft3/sec
    - Mass Flow Rate = 2.941 lbm/sec
    - Reynolds Number = 306548
- Geometry: Cylindrical Pipe
  - Diameter = 12 inches
  - Area = 0.78540 ft2

    - Wetted Perimeter = 37.699 inches
    - Length = 100 feet
- Fluid = Air @ 1 atm (vapor)
    - Temperature = 70 deg. F
    - Density = 0.0749 lbm/ft3
    - Dynamic Viscosity = 0.04398 lbm/hr-ft

List of All Verification Models

Verification Case 27 Problem Statement

Verification Case 27

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Page 5-6, Example 5.6

Lindeburg Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

PART 2: Duct Design

1 FRICTION LOSSES IN DUCTS

Friction losses can be calculated from the Moody equation.

$$p_{loss} = \frac{fLp_v}{D} \approx (.0270)\frac{L}{(d)^{1.22}}\left(\frac{v}{1000}\right)^{1.82} \quad 5.27$$

$$\approx (3.9\ \text{EE}-9)(v)^{2.43}\frac{L}{(Q)^{.61}} \quad 5.28$$

In practice, equation 5.27 seldom is used. Figure 5.4 is based on equation 5.27 with a value of specific roughness equal to .0005, a standard density of .075 lbm/ft^3, clean round galvanized metal ductwork, and approximately 40 joints per 100 feet. The chart can be used for temperatures between 50 °F and 90 °F. For operation outside this range, the pressure loss should be corrected with equation 5.29. K_v is usually taken as 1.0.

$$(p_{loss})_{actual} = p_{loss,\,fig.\,5.4} \times \frac{K_v}{K_d} \quad 5.29$$

Figure 5.4 is for use with standard round, galvanized ducts. Multiply the friction losses by the factors in table 5.2 for other materials. (Actual values are velocity dependent. Tables and charts exist for this purpose.)

Table 5.2
Multiplicative Factors for Non-Standard Ducts

Smooth ducts—no joints	.6-.95
Smooth concrete	1.1–1.4
Rough concrete/good brick	1.2–1.8

The *equivalent diameter* of a rectangular air duct with dimensions a and b, and aspect ratio less than 8.0 is

$$D_e = 1.3\frac{(ab)^{.625}}{(a+b)^{.25}} \quad 5.30$$

A round duct with diameter D_e will have the same friction and capacity as a square duct with dimensions a and b. Figure 5.4 can be used with D_e and flow rate to find the friction loss.

If the aspect ratio of the rectangular duct is known, a round duct can be converted to a rectangular duct of equal friction. The *aspect ratio*, which should be kept below 8.0 for ease of manufacturing, is

$$R = \frac{\text{longest side}}{\text{short side}} \quad 5.31$$

The short side, a, is given by equation 5.32.

$$a = \frac{D_e(R+1)^{\frac{1}{4}}}{1.3(R)^{.625}} \quad 5.32$$

Example 5.6

2000 cfm of air flow in a 13″ diameter duct. What is the velocity and the friction loss per 100 feet of duct?

Although the $Q = Av$ relationship could be used to find the velocity, it is expedient to use figure 5.4. By locating the intersection of the 2000 cfm and the 13″ lines, the velocity is found to be 2200 fpm.

Dropping straight down from the intersection point to the horizontal scale gives the friction loss as approximately .5″ w.g. per 100 feet.

Example 5.7

What size duct is required to carry 2000 cfm at 1600 fpm?

Figure 5.4 shows that a 15″ diameter duct is required.[4] The friction loss is approximately 0.23″ w.g. per 100 feet.

2 MINOR AND DYNAMIC LOSSES

Minor losses are fairly independent of air velocity and roughness. In the *loss coefficient method*, the losses are calculated as a percentage of the velocity pressure.

$$p = c\left(\frac{v}{4005}\right)^2 = cp_v \quad 5.33$$

Typical values of c are given in table 5.3. Subscripts 1 and 2 refer to upstream and downstream, respectively. The coefficient c always should be used with the velocity at the point corresponding to its subscript.

The *equivalent length method* also can be used to calculate the friction of a bend or an elbow. As with equivalent lengths used in liquid flow problems, each obstruction produces a frictional loss equivalent to some length of duct. These lengths are given in multiples of the duct diameter in table 5.3.

[4] Any size duct can be manufactured. However, there are standard sizes available, and these sizes should be chosen to minimize cost. Generally, every whole-inch size up to 30″ diameter is available, although some odd-number sizes may be premium-priced. After 30″, sizes are available in 2″ increments.

View Verification Case 27 Model

Verification Case 27

Verification Case 28

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify28.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-1, Example 4-1

FLUID: Water at 80 deg. F

ASSUMPTIONS: Supply pressure is from a 10 feet tank

RESULTS:

Parameter	Crane	AFT Fathom
Reynolds number	89,600	88,780
Friction factor	0.0182	0.01845

List of All Verification Models

Verification Case 28 Problem Statement

Verification Case 28

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-1, Example 4-1

Crane Title Page

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Examples of Flow Problems

CHAPTER 4

Theory and answers to questions regarding proper application of formulas to flow problems can be presented to good advantage by the solution of practical problems. A few simple flow problems were presented in Chapter 3 to illustrate the use of the nomographs. Other problems, both simple and complex, are presented in this chapter.

Many of the examples given in this chapter employ the basic formulas of Chapters 1 and 2; these formulas were rewritten in more commonly used terms for Chapter 3. Use of nomographs, when applicable, are indicated in the solution of these problems.

The controversial subject regarding the selection of a formula most applicable to the flow of gas through long pipe lines is analyzed in Chapter 1. It is shown that the three commonly used formulas are basically identical, the only difference being in the selection of friction factors. A comparison of results obtained, using the three formulas, is presented in this chapter.

An original method has been developed for the solution of problems involving the discharge of compressible fluids from pipe systems. Illustrative examples applying this method demonstrate the simplicity of handling these, heretofore complex, problems.

Reynolds Number and Friction Factor For Pipe Other Than Steel

The example below shows the procedure in obtaining the Reynolds number and friction factor for smooth pipe (plastic). The same procedure applies for any pipe other than steel such as concrete, wood stave, riveted steel, etc. For relative roughness of these and other piping materials, see page A-23.

Example 4-1 . . . Smooth Pipe (Plastic)

Given: Water at 80 F is flowing through 70 feet of 2-inch standard wall plastic pipe (smooth wall) at a rate of 50 gallons per minute.

Find: The Reynolds number and friction factor.

Solution:

1. $R_e = \frac{50.6\ Q\rho}{d\mu}$ . . . . page 3-2
2. $\rho = 62.220$ . . . . page A-6
3. $d = 2.067$ . . . . page B-16
4. $\mu = 0.85$ . . . . page A-3
5. $R_e = \frac{50.6 \times 50 \times 62.220}{2.067 \times 0.85}$

   $R_e =$ **89 600** or **8.96×10^4**
6. $f =$ **0.0182** for smooth pipe . . . page A-24

View Verification Case 28 Model

Verification Case 28

Verification Case 29

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify29.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-3, Example 4-6

FLUID: Water at 60 deg. F

ASSUMPTIONS: Friction factor in pipe is 0.018.

RESULTS:

Parameter	Crane	AFT Fathom
Velocity (feet/sec)	8.5	8.545
Flow Rate (gal/min)	196	196.9

DISCUSSION:

The loss factor for the flanged ball valve was not in the AFT Fathom library, and the resulting K factor of 0.58 was entered directly as a fitting and loss value for pipe 1.

Slight differences in the predictions result from round-off.

List of All Verification Models

Verification Case 29 Problem Statement

Verification Case 29

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-3, Example 4-6

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Check Valves
Determination of Size

Example 4-5 . . . Lift Check Valves

Given: A globe type lift check valve with a wing-guided disc is required in a 3-inch Schedule 40 horizontal pipe carrying 70 F water at the rate of 80 gallons per minute.

Find: The proper size check valve and the pressure drop. The valve should be sized so that the disc is fully lifted at the specified flow; see page 2-7 for discussion.

Solution:

1. $v_{min} = 40\sqrt{\overline{V}}$ . . . . page A-27

$$v = \frac{0.408\,Q}{d^2}$$ . . . . page 3-2

$$\Delta P = \frac{18 \times 10^{-6}\,K\rho Q^2}{d^4}$$ . . . . page 3-4

$K_1 = 600\,f_T$ . . . . page A-27

$$K_2 = \frac{K_1 + \beta\,[0.5\,(1-\beta^2) + (1-\beta^2)^2]}{\beta^4}$$ . . page A-27

$$\beta = \frac{d_1}{d_2}$$ . . . . page A-26

2. $d_1 = 2.469$ . . for 2½" Sched. 40 pipe; page B-16

$d_2 = 3.068$ . . . for 3" Sched. 40 pipe; page B-16

$\overline{V} = 0.01605$ . . . . 70 F water; page A-6

$\rho = 62.305$ . . . . 70 F water; page A-6

$f_T = 0.018$ . . . . . for 2½" or 3" size; page A-26

3. $v_{min} = 40\sqrt{0.01605} = 5.1$

$$v = \frac{0.408 \times 80}{3.068^2} = 3.5$$ . . . . for 3" valve

Inasmuch as v is less than v_{min}, a 3-inch valve will be too large. Try a 2½-inch size.

$$v = \frac{0.408 \times 80}{2.469^2} = 5.35$$ . . . . . . for 2½" valve

Based on above, a 2½-inch valve installed in 3-inch Schedule 40 pipe with reducers is advisable.

4. $\beta = \frac{2.469}{3.068} = 0.80$

$\beta^2 = 0.64$

$\beta^4 = 0.41$

5. $$K_2 = \frac{600 \times .018 + .8\,[0.5\,(1-0.64) + (1-0.64)^2]}{0.41}$$

$K_2 = 27$

6. $$\Delta P = \frac{18 \times 10^{-6} \times 27 \times 62.305 \times 80^2}{3.068^4} = \mathbf{2.2}$$

Reduced Port Valves
Velocity and Rate of Discharge

Example 4-6 . . . Reduced Port Ball Valve

Given: Water at 60 F discharges from a tank with 22-feet average head to atmosphere through:

200 feet—3" Schedule 40 pipe
6—3" standard 90° threaded elbows
1—3" flanged ball valve having a 2⅜" diameter seat, 16° conical inlet, and 30° conical outlet end. Sharp-edged entrance is flush with the inside of the tank.

Find: Velocity of flow in the pipe and rate of discharge in gallons per minute.

Solution:

1. $$h_L = K\frac{v^2}{2g} \text{ or } v = \sqrt{\frac{2gh_L}{K}}$$ . . . . page 3-4

$$v = 0.408\frac{Q}{d^2} \text{ or } Q = 2.451\,vd^2$$ . . . . page 3-2

2. $K = 0.5$ . . . . entrance; page A-29

$K = 1.0$ . . . . exit; page A-29

$f_T = 0.018$ . . . . page A-26

3. For K (ball valve), page A-28 indicates use of Formula 5. However, when inlet and outlet angles (θ) differ, Formula 5 must be expanded to:

$$K_2 = \frac{K_1 + .8\sin\frac{\theta}{2}(1-\beta^2) + 2.6\sin\frac{\theta}{2}(1-\beta^2)^2}{\beta^4}$$

4. $\beta = \frac{d_1}{d_2} = \frac{2.375}{3.068} = 0.77$ . . . . page A-26

5. $\sin\theta/2 = \sin 8° = 0.14$ . . . . valve inlet

6. $\sin\theta/2 = \sin 15° = 0.26$ . . . . valve outlet

7. $$K_2 = \frac{3 \times .018 + 0.8 \times 0.14\,(1-0.77^2)}{0.77^4} + \frac{2.6 \times 0.26\,(1-0.77^2)^2}{0.77^4} = 0.58$$ . . . . valve

$K = 6 \times 30 f_T = 180 \times 0.018 = 3.24$ . . . 6 elbows; p. A-29

$$K = f\frac{L}{D} = \frac{0.018 \times 200 \times 12}{3.068} = 14.08$$ pipe; p. 3-4

8. Then, for entire system (entrance, pipe, ball valve, six elbows, and exit),

$K = 0.5 + 14.08 + 0.58 + 3.24 + 1.0 = 19.4$

9. $v = \sqrt{(64.4 \times 22) \div 19.4} = \mathbf{8.5}$

$Q = 2.451 \times 8.5 \times 3.068^2 = \mathbf{196}$

10. Calculate Reynolds number to verify that friction factor of 0.018 (zone of complete turbulence) is correct for flow condition . . . or, use "vd" scale at top of Friction Factor chart on page A-25.

$vd = 8.5 \times 3.068 = 26$

11. Enter chart on page A-25 at $vd = 26$. Note f for 3-inch pipe is less than 0.02. Therefore, flow is in the transition zone (slightly less than fully turbulent) but the difference is small enough to forego any correction of K for the pipe.

View Verification Case 29 Model

Verification Case 29

Verification Case 30

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify30.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-4, Example 4-7

FLUID: SAE 10 Lube Oil at 60F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Crane	AFT Fathom
Volumetric flow rate (gal/min)	118	121.4
Velocity (feet/sec)	5.13	5.267
Reynolds number	1040	1095

DISCUSSION:

Because of the high oil viscosity, there is laminar flow in the pipe. As discussed in Crane, this is an iterative problem. Crane performs one iteration, while AFT Fathom performs numerous iterations. The difference in results is Crane's single iteration.

List of All Verification Models

Verification Case 30 Problem Statement

Verification Case 30

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-4, Example 4-7

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Laminar Flow in Valves, Fittings, and Pipe

In flow problems where viscosity is high, calculate the Reynolds Number to determine whether the flow is laminar or turbulent.

Example 4-7

Given: S.A.E. 10 Lube Oil at 60 F flows through the system described in Example 4-6 at the same differential head.

Find: The velocity in the pipe and rate of flow in gallons per minute.

Solution:

1. $h_L = K\frac{v^2}{2g}$ .... page 3-4

$v = \sqrt{\frac{2gh_L}{K}}$

$v = 0.408\frac{Q}{d^2}$ .... page 3-2

$Q = 2.451\, vd^2$

$R_e = 124\frac{dv\rho}{\mu}$ .... page 3-2

$f = \frac{64}{R_e}$ .... pipe, laminar flow; page 3-2

$K = f\frac{L}{D}$ .... pipe; page 3-4

2. $K_2 = 0.58$ .... valve; Example 4-6

$K = 3.24$ .... 6 elbows; Example 4-6

$K = 0.5$ .... entrance; Example 4-6

$K = 1.0$ .... exit; Example 4-6

$\rho = 54.64$ .... page A-7

$\mu = 100$ .... page A-3

$h_L = 22$ .... Example 4-6

3. *Assume laminar flow with $v = 5$.

$R_e = \frac{124 \times 3.068 \times 5 \times 54.64}{100} = 1040$

$f = 64 \div 1040 = 0.062$ .... pipe

$K = \frac{0.062 \times 200 \times 12}{3.068} = 48.5$ .... pipe

$K = 48.5 + 0.58 + 3.24 + 0.5 + 1.0$

$K = 53.8$ .... entire system

4. $v = \sqrt{\frac{64.4 \times 22}{53.8}} = \mathbf{5.13}$

5. $Q = 2.451 \times 5.13 \times 3.068^2 = \mathbf{118}$

**Note:* This problem has two unknowns and, therefore, requires a trial-and-error solution. Two or three trial assumptions will usually bring the solution and final assumption into agreement within desired limits.

Example 4-8

Given: S.A.E. 70 Lube Oil at 100 F is flowing at the rate of 600 barrels per hour through 200 feet of 8-inch Schedule 40 pipe, in which an 8-inch conventional globe valve with full area seat is installed.

Find: The pressure drop due to flow through the pipe and valve.

Solution:

1. $\Delta P = \frac{8.82 \times 10^{-6} K\rho B^2}{d^4}$ .... page 3-4

$R_e = \frac{35.4\, \rho B}{d\mu}$ .... page 3-2

$K_1 = 340\, f_T$ .... valve; page A-27

$K = f\frac{L}{D}$ .... pipe; page 3-4

$f = \frac{64}{R_e}$ .... pipe

2. $S = 0.916$ at 60 F .... page A-7

$S = 0.90$ at 100 F .... page A-7

$d = 7.981$ .... 8" Sched. 40 pipe; page B-17

$\mu = 470$ .... page A-3

$f_T = 0.014$ .... page A-26

3. $\rho = 62.371 \times 0.90 = 56.1$

$R_e = \frac{35.4 \times 600 \times 56.1}{7.981 \times 470} = 318$

$R_e < 2000$; therefore flow is laminar.

4. $f = \frac{64}{318} = 0.20$ .... pipe

$K_1 = 340 \times 0.014 = 4.76$ .... valve

$K = \frac{0.20 \times 200 \times 12}{7.981} = 60.14$ .... pipe

$K = 4.76 + 60.14 = 64.9$ .... total system

5. $\Delta P = \frac{8.82 \times 10^{-6} \times 64.9 \times 56.1 \times 600^2}{7.981^4}$

$\Delta P = \mathbf{2.85}$

View Verification Case 30 Model

Verification Case 30

Verification Case 31

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify31.fthFthVerify31.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-4, Example 4-8

FLUID: SAE 70 Lube Oil at 100 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Crane	AFT Fathom
Pressure drop (psid)	2.85	2.866

DISCUSSION:

Because of the high oil viscosity, there is laminar flow in the pipe.

List of All Verification Models

Verification Case 31 Problem Statement

Verification Case 31

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-4, Example 4-8

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Laminar Flow in Valves, Fittings, and Pipe

In flow problems where viscosity is high, calculate the Reynolds Number to determine whether the flow is laminar or turbulent.

Example 4-7

Given: S.A.E. 10 Lube Oil at 60 F flows through the system described in Example 4-6 at the same differential head.

Find: The velocity in the pipe and rate of flow in gallons per minute.

Solution:

1. $h_L = K \frac{v^2}{2g}$ .... page 3-4

   $v = \sqrt{\frac{2gh_L}{K}}$

   $v = 0.408 \frac{Q}{d^2}$ .... page 3-2

   $Q = 2.451\ vd^2$

   $R_e = 124 \frac{dv\rho}{\mu}$ .... page 3-2

   $f = \frac{64}{R_e}$ .... pipe, laminar flow; page 3-2

   $K = f \frac{L}{D}$ .... pipe; page 3-4

2. $K_2 = 0.58$ .... valve; Example 4-6

   $K = 3.24$ .... 6 elbows; Example 4-6

   $K = 0.5$ .... entrance; Example 4-6

   $K = 1.0$ .... exit; Example 4-6

   $\rho = 54.64$ .... page A-7

   $\mu = 100$ .... page A-3

   $h_L = 22$ .... Example 4-6

3. *Assume laminar flow with $v = 5$.

   $R_e = \frac{124 \times 3.068 \times 5 \times 54.64}{100} = 1040$

   $f = 64 \div 1040 = 0.062$ .... pipe

   $K = \frac{0.062 \times 200 \times 12}{3.068} = 48.5$ .... pipe

   $K = 48.5 + 0.58 + 3.24 + 0.5 + 1.0$

   $K = 53.8$ .... entire system

4. $v = \sqrt{\frac{64.4 \times 22}{53.8}} = \mathbf{5.13}$

5. $Q = 2.451 \times 5.13 \times 3.068^2 = \mathbf{118}$

*Note: This problem has two unknowns and, therefore, requires a trial-and-error solution. Two or three trial assumptions will usually bring the solution and final assumption into agreement within desired limits.

Example 4-8

Given: S.A.E. 70 Lube Oil at 100 F is flowing at the rate of 600 barrels per hour through 200 feet of 8-inch Schedule 40 pipe, in which an 8-inch conventional globe valve with full area seat is installed.

Find: The pressure drop due to flow through the pipe and valve.

Solution:

1. $\Delta P = \frac{8.82 \times 10^{-6} K \rho B^2}{d^4}$ .... page 3-4

   $R_e = \frac{35.4\ \rho B}{d\mu}$ .... page 3-2

   $K_1 = 340\ f_T$ .... valve; page A-27

   $K = f \frac{L}{D}$ .... pipe; page 3-4

   $f = \frac{64}{R_e}$ .... pipe

2. $S = 0.916$ at 60 F .... page A-7

   $S = 0.90$ at 100 F .... page A-7

   $d = 7.981$ .... 8" Sched. 40 pipe; page B-17

   $\mu = 470$ .... page A-3

   $f_T = 0.014$ .... page A-26

3. $\rho = 62.371 \times 0.90 = 56.1$

   $R_e = \frac{35.4 \times 600 \times 56.1}{7.981 \times 470} = 318$

   $R_e < 2000$; therefore flow is laminar.

4. $f = \frac{64}{318} = 0.20$ .... pipe

   $K_1 = 340 \times 0.014 = 4.76$ .... valve

   $K = \frac{0.20 \times 200 \times 12}{7.981} = 60.14$ .... pipe

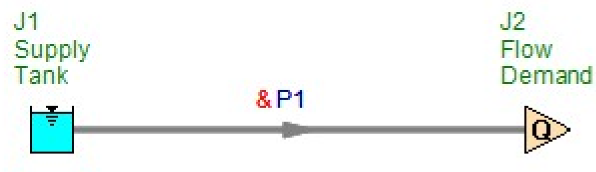
   $K = 4.76 + 60.14 = 64.9$ .... total system

5. $\Delta P = \frac{8.82 \times 10^{-6} \times 64.9 \times 56.1 \times 600^2}{7.981^4}$

   $\Delta P = \mathbf{2.85}$

View Verification Case 31 Model

Verification Case 31



Verification Case 32

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify32.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-5, Example 4-9

FLUID: SAE 70 Lube Oil at 100 deg. F

ASSUMPTIONS: Inlet pressure is 200 psig, and the gate valve is located 100 feet from the inlet (this does not affect the answer)

RESULTS:

Parameter	Crane	AFT Fathom
Pressure drop (psid)	56.6	56.67

DISCUSSION:

Because of the high oil viscosity, there is laminar flow in the pipes. The total pressure change in AFT Fathom can be found in two places. First, the discharge presssure (143.3 psig) can be subtracted from the inlet pressure (200 psig). Second, the pressure difference is given in the Junction Deltas table at the top of the Output window. The junction delta was setup in the Output Control window.

List of All Verification Models

Verification Case 32 Problem Statement

Verification Case 32

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-5, Example 4-9

Crane Title Page

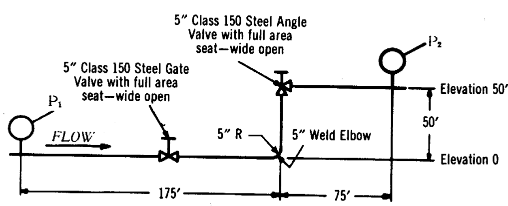
Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Laminar Flow in Valves, Fittings, and Pipe — continued

In flow problems where viscosity is high, calculate the Reynolds Number to determine whether the flow is laminar or turbulent.

Example 4-9

Given: S.A.E. 70 Lube Oil at 100 F is flowing through 5-inch Schedule 40 pipe at a rate of 600 gallons per minute, as shown in the following sketch.



Find: The velocity in feet per second and pressure difference between gauges P_1 and P_2.

Solution:

1. $v = \frac{0.408Q}{d^2}$ .... page 3-2

   $R_e = \frac{50.6Q\rho}{d\mu}$ .... page 3-2

   $\Delta P = \frac{18 \times 10^{-6} K \rho Q^2}{d^4}$ loss due to flow; page 3-4

   $\Delta P = \frac{h_L \rho}{144}$ loss due to elevation change; page 3-5

2. $K_1 = 8 f_T$ .... gate valve; page A-27

   $K_1 = 150 f_T$ .... angle valve; page A-27

   $K = 20 f_T$ .... elbow; page A-29

   $K = f \frac{L}{D}$ .... pipe; page 3-4

   $f = \frac{64}{R_e}$ .... pipe; page 3-2

3. $d = 5.047$ ...... 5″ Sched. 40 pipe; page B-17

   $S = 0.916$ at 60 F .... page A-7

   $S = 0.90$ at 100 F .... page A-7

   $\mu = 470$ .... page A-3

   $\rho = 62.371 \times 0.90 = 56.1$

   $f_T = 0.016$

4. $R_e = \frac{50.6 \times 600 \times 56.1}{5.047 \times 470} = 718$

   $R_e < 2000$; therefore flow is laminar.

5. $f = \frac{64}{718} = 0.089$

6. Summarizing K for the entire system (gate valve, angle valve, elbow, and pipe),

   $K = (8 \times 0.016) + (150 \times 0.016) + (20 \times 0.016) + \frac{(0.089 \times 300 \times 12)}{5.047} = 66.3$

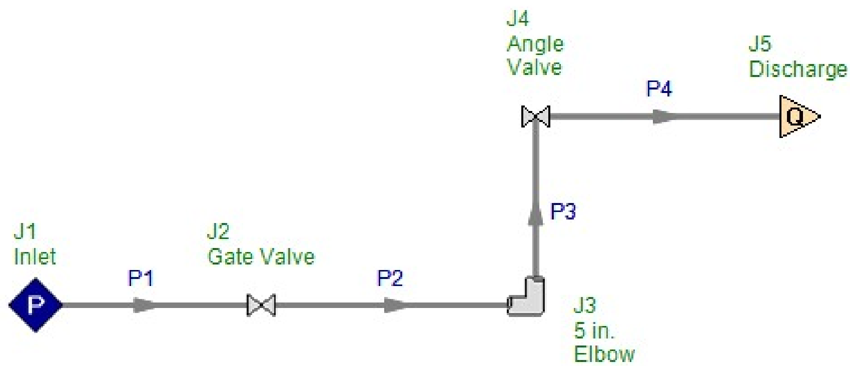
7. $v = \frac{0.408 \times 600}{5.047^2} = \mathbf{9.6}$

8. $\Delta P = \frac{18 \times 10^{-6} \times 66.3 \times 56.1 \times 600^2}{5.047^4} + \frac{50 \times 56.1}{144}$

   $\Delta P = \mathbf{56.6}$ .... total

View Verification Case 32 Model

Verification Case 32



Verification Case 33

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify33.fthFthVerify33.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-6, Example 4-11

FLUID: Water at 180 deg. F

ASSUMPTIONS: Inlet pressure is 100 psig.

RESULTS:

Parameter	Crane	AFT Fathom
Pressure drop (psid)	1.91	1.91

DISCUSSION:

The total pressure change in AFT Fathom can be found in two places. First, the discharge presssure (98.09 psig) can be subtracted from the inlet pressure (100 psig). Second, the pressure difference is given in the Junction Deltas table at the top of the Output window. The junction delta was setup in the Output Control window.

List of All Verification Models

Verification Case 33 Problem Statement

Verification Case 33

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-6, Example 4-11

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

4-6 CHAPTER 4 – EXAMPLES OF FLOW PROBLEMS CRANE

Pressure Drop and Velocity in Piping Systems

Example 4-10 . . . Piping Systems—Steam

Given: 600 psig steam at 850 F flows through 400 feet of horizontal 6-inch Schedule 80 pipe at a rate of 90,000 pounds per hour.

The system contains three 90 degree weld elbows having a relative radius of 1.5, one fully-open 6 x 4-inch Class 600 venturi gate valve as described in Example 4-4, and one 6-inch Class 600 y-pattern globe valve. Latter has a seat diameter equal to 0.9 of the inside diameter of Schedule 80 pipe, disc fully lifted.

Find: The pressure drop through the system.

Solution:

1. $\Delta P = \frac{28 \times 10^{-8} K W^2 \bar{V}}{d^4}$ . . . . page 3-4

2. For globe valve (see page A-27),

$K_2 = \frac{K_1 + \beta \left[0.5 (1 - \beta^2) + (1 - \beta^2)^2\right]}{\beta^4}$

$K_1 = 55 f_T$

$\beta = 0.9$

3. $K = 14 f_T$ . . . . 90° weld elbows; page A-29

$K = f \frac{L}{D}$ . . . . pipe; page 3-4

$R_e = 6.31 \frac{W}{d\mu}$ . . . . page 3-2

4. $d = 5.761$ . . . . . . 6" Sched. 80 pipe; page B-17

$\bar{V} = 1.216$ . . . . 600 psi steam, 850 F; page A-17

$\mu = 0.027$ . . . . page A-2

$f_T = 0.015$ . . . . page A-26

5. For globe valve,

$K_2 = \frac{55 \times .015 + .9 \left[.5 (1 - .9^2) + (1 - .9^2)^2\right]}{.9^4}$

$K_2 = 1.44$

6. $R_e = \frac{6.31 \times 90\,000}{5.761 \times 0.027} = 3.65 \times 10^6$

$f = 0.015$ . . . . pipe; page A-25

$K = \frac{0.015 \times 400 \times 12}{5.761} = 12.5$ . . . . pipe

$K = 3 \times 14 \times 0.015 = 0.63$ 3 elbows; page A-29

$K_2 = 1.44$ . . . . . . 6 x 4" gate valve; Example 4-4

7. Summarizing K for the entire system (globe valve, pipe, venturi gate valve, and elbows),

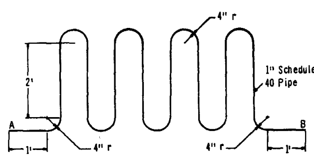
$K = 1.44 + 12.5 + 0.63 + 1.44 = 16$

8. $\Delta P = \frac{28 \times 10^{-8} \times 16 \times 9^2 \times 10^8 \times 1.216}{5.761^4}$

$\Delta P = \mathbf{40.1}$

Example 4-11 . . . Flat Heating Coils—Water

Given: Water at 180 F is flowing through a flat heating coil, shown in the sketch below, at a rate of 15 gallons per minute.



Find: The pressure drop from Point A to B.

Solution:

1. $\Delta P = \frac{18 \times 10^{-6} K \rho Q^2}{d^4}$ . . . . page 3-4

$R_e = \frac{50.6\, Q \rho}{d \mu}$ . . . . page 3-2

$K = f \frac{L}{D}$ . . . . straight pipe; page 3-4

$r/d = 4$ . . . . pipe bends

$K_{90} = 14 f_T$ . . . . . . 90° bends; page A-29

$K_B = (n-1) \left(.25 \pi f_T \frac{r}{d} + .5 K_{90}\right) + K_{90}$

. . . . 180° bends; page A-29

2. $\rho = 60.57$ . . . . water, 180 F; page A-6

$\mu = 0.34$ . . . . water, 180 F; page A-3

$d = 1.049$ . . . . . . 1" Sched. 40 pipe; page B-16

$f_T = 0.023$ . . . . . . 1" Sched. 40 pipe; page A-26

3. $R_e = \frac{50.6 \times 15 \times 60.57}{1.049 \times 0.34} = 1.3 \times 10^5$

$f = 0.024$ . . . . pipe

$K = \frac{0.024 \times 18 \times 12}{1.049} = 4.94$ 18' straight pipe

$K = 2 \times 14 \times 0.023 = 0.64$ . . . two 90° bends

4. For seven 180° bends,

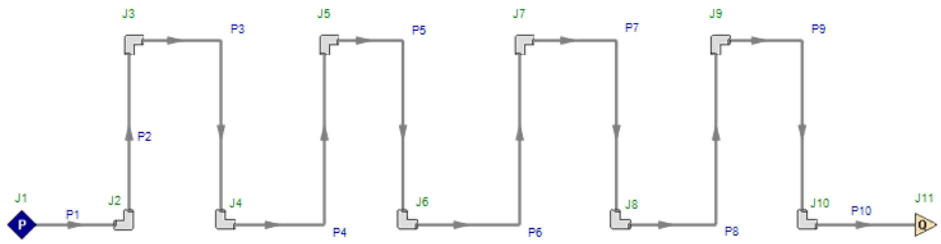
$K_B = 7[(2-1)(0.25\pi \times 0.023 \times 4) + (0.5 \times 0.32) + 0.32] = 3.87$

5. $K_{TOTAL} = 4.94 + 0.64 + 3.87 = 9.45$

6. $\Delta P = \frac{18 \times 10^{-6} \times 9.45 \times 60.57 \times 15^2}{1.049^4} = \mathbf{1.91}$

View Verification Case 33 Model

Verification Case 33



Verification Case 34

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify34.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-7, Example 4-13

FLUID: Fuel Oil

ASSUMPTIONS: Inlet pressure is 2 bars

RESULTS:

Parameter	Crane	AFT Fathom
Head loss (meters)	8.95	8.91
Pressure drop (kg/cm2)	0.729	0.7262
Pressure drop (bar)	0.715	0.7121
Pressure drop (MPa)	0.0715	0.07121

DISCUSSION:

The viscosity is given as kinematic. Converting to dynamic viscosity yields 2.20E-03 kg/m-s.

The head loss in AFT Fathom can be found in three places. First, it is given in the pipe output table. Second, it can be obtained by subtracting the discharge EGL from the supply. Third, it can be viewed in the Junction Deltas summary at the top of the Output window. The junction delta was setup in the Output Control window.

The pressure loss in AFT Fathom can be found in the same three places. Note that three junction deltas are setup for pressure, with different units for each. This allows a more straightforward comparison with the Crane results.

List of All Verification Models

Verification Case 34 Problem Statement

Verification Case 34

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-7, Example 4-13

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pressure Drop and Velocity in Piping Systems — continued

Example 4-12 . . . Orifice Size for Given Pressure Drop and Velocity

Given: A 12-inch Schedule 40 steel pipe 60 feet long, containing a standard gate valve 10 feet from the entrance, discharges 60 F water to atmosphere from a reservoir. The entrance projects inward into the reservoir and its center line is 12 feet below the water level in the reservoir.

Find: The diameter of thin-plate orifice that must be centrally installed in the pipe to restrict the velocity of flow to 10 feet per second when the gate valve is wide open.

Solution:

1. $h_L = K\frac{v^2}{2g}$ or System $K = \frac{2gh_L}{v^2}$ . . . . page 3-4

   $R_e = \frac{123.9\, d v \rho}{\mu}$ . . . . page 3-2

2. $K = 0.78$ . . . . entrance; page A-29

   $K = 1.0$ . . . . exit; page A-29

   $K_1 = 8 f_T$ . . . . gate valve; page A-27

   $K = f\frac{L}{D}$ . . . . pipe; page 3-4

3. $d = 11.938$ . . . . pipe; page B-17

   $f_T = 0.013$ . . . . page A-26

   $\rho = 62.371$ . . . . page A-6

   $\mu = 1.1$ . . . . page A-3

4. $R_e = \frac{123.9 \times 11.938 \times 10 \times 62.371}{1.1} = 8.4 \times 10^5$

   $f = 0.014$ . . . . page A-25

5. Total K required $= 64.4 \times 12 \div 10^2 = 7.72$

   $K_1 = 8 \times 0.013 = 0.10$ . . . . gate valve

   $K = 60 \times 0.014 = 0.84$ . . . . pipe

   Then, exclusive of orifice,

   $K_{total} = 0.78 + 1.0 + 0.1 + 0.84 = 2.72$

6. $K_{orifice} = 7.72 - 2.72 = 5$

7. $K_{orifice} \simeq \frac{1-\beta^2}{C^2\beta^4}$ . . . . page A-20

8. Assume $\beta = 0.7$ $\therefore C = 0.7$ page A-20
   then $K \simeq 4.3$ $\therefore \beta$ is too large

9. Assume $\beta = 0.65$ $\therefore C = 0.67$ page A-20
   then $K \simeq 7.1$ $\therefore \beta$ is too small

10. Assume $\beta = 0.67$ $\therefore C = 0.682$ page A-20
    then $K \simeq 5.8$ $\therefore$ use $\beta = 0.68$

11. Orifice size $\simeq 11.938 \times 0.68 =$ **8.1″**

Example 4-13 . . . Flow Given in International Metric System (SI) Units—Oil

Given: Fuel oil with a density of 0.815 grams per cubic centimeter and a kinematic viscosity of 2.7 centistokes is flowing through 50 millimeter I.D. steel pipe (30 meters long) at a rate of 7.0 liters per second.

Find: Head loss in meters of fluid and pressure drop in kg/cm², bar, and megapascal (MPa).

Solution: 1. Define symbols in SI units as follows:

A . . . cross-sectional area of pipe, in meters²
D . . . internal diameter of pipe, in meters
g . . . acceleration of gravity = 9.8 meters/sec/sec
h_L . . . head loss, in meters of fluid
L . . . length of pipe, in meters
q . . . rate of flow, in meters³/second
v . . . mean velocity of flow, in meters/second
ρ . . . fluid density, in grams/centimeter³
ΔP (kg/cm²) . . . pressure drop, in kilograms/centimeter²
ΔP (bar) . . . . . pressure drop, in bars
ΔP (MPa) . . . . pressure drop, in megapascals

2. Use metric-imperial equivalents as indicated below and on pages B-10 and B-11.

   meter (1) = 3.28 feet = 39.37 inches
   bar = 0.98067 × kg/cm²
   megapascal = 0.098067 × kg/cm²

   A column of fluid one square centimeter in cross-sectional area and one meter high is equal to a pressure of 0.1 ρ kg/cm²; therefore:

   ΔP (kg/cm²) equals . . . $0.1\rho\, h_L$
   ΔP (bar) equals . . . . . 0.98067 ΔP (kg/cm²)
   ΔP (MPa) equals . . . . 0.098067 ΔP (kg/cm²)

3. $v = \frac{q}{A} = \frac{7 \times 10^{-3}}{(\pi \div 4) \times 50^2 \times 10^{-6}} = 3.566$ . . . page 3-2

   $R_e = \frac{7740 \times 39.37 D \times 3.28 v}{\nu} = \frac{Dv \times 10^6}{\nu}$ . . . . . . page 3-2

   $R_e = \frac{0.050 \times 3.566 \times 10^6}{2.7} = 6.6 \times 10^4$

   $f = 0.023$ . . . . page A-25

4. $h_L = f\frac{L}{D}\frac{v^2}{2g} = \frac{0.023 \times 30 \times 3.566^2}{0.050 \times 2 \times 9.8} = \mathbf{8.95}$ . . . . page 3-4

   ΔP (kg/cm²) $= 0.1 \times 0.815 \times 8.95 =$ **0.729**
   ΔP (bar) $= 0.98067 \times 0.729 =$ **0.715**
   ΔP (MPa) $= 0.098067 \times 0.729 =$ **0.0715**

View Verification Case 34 Model

Verification Case 34

Verification Case 35

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify35.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-8, Example 4-14

FLUID: Water at 60 deg. F

ASSUMPTIONS: Inlet pressure is 100 psig

RESULTS:

Parameter	Crane	AFT Fathom
Static Head loss (feet)	15.8	15.5
Static Pressure drop (psid)	39.0	39.22

DISCUSSION:

The head loss in AFT Fathom can be found in three places. First, it is given in the pipe output table. Second, it can be obtained by subtracting the discharge HGL from the supply. Third, it can be viewed in the Junction Deltas summary at the top of the Output window. The junction delta was setup in the Output Control window.

The pressure loss in AFT Fathom can be found by subtracting the discharge pressure from the supply. It also can be found in the Junction Deltas on the Output window.

List of All Verification Models

Verification Case 35 Problem Statement

Verification Case 35

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-8, Example 4-14

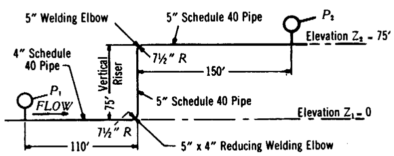
Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pressure Drop and Velocity in Piping Systems — continued

Example 4-14 . . . Bernoulli's Theorem—Water

Given: Water at 60 F is flowing through the piping system, shown in the sketch below, at a rate of 400 gallons per minute.



Find: The velocity in both the 4 and 5-inch pipe sizes and the pressure differential between gauges P_1 and P_2.

Solution:

1. Use Bernoulli's theorem (see page 3-2):

$$Z_1 + \frac{144\,P_1}{\rho_1} + \frac{v_1^2}{2g} = Z_2 + \frac{144\,P_2}{\rho_2} + \frac{v_2^2}{2g} + h_L$$

Since, $\rho_1 = \rho_2$

$$P_1 - P_2 = \frac{\rho}{144}\left((Z_2 - Z_1) + \frac{v_2^2 - v_1^2}{2g} + h_L\right)$$

2. $h_L = \dfrac{0.00259KQ^2}{d^4}$ . . . . page 3-4

$R_e = \dfrac{50.6Q\rho}{d\mu}$ . . . . page 3-2

$K = f\dfrac{L}{D}$ . . . . page 3-4

$K = \dfrac{fL}{D\beta^4}$ . . . . {small pipe, in terms of larger pipe; page 2-11

$K = 14\,f_T$ . . . . 90° elbow; page A-29

$K = 14\,f_T + \dfrac{(1-\beta^2)^2}{\beta^4}$ . . . . . {reducing 90° elbow; page A-26

Note: In the absence of test data for increasing elbows, the resistance is conservatively estimated to be equal to the summation of the resistance due to a straight size elbow and a sudden enlargement.

$\beta = \dfrac{d_1}{d_2}$ . . . . page A-26

3. $\rho = 62.371$ . . . . page A-6

$\mu = 1.1$ . . . . page A-3

$d_1 = 4.026$ . . . . . . 4" Sched. 40 pipe; page B-17

$d_2 = 5.047$ . . . . . . 5" Sched. 40 pipe; page B-17

$f_T = 0.016$ . . . . 5" size; page A-26

4. $\beta = \dfrac{4.026}{5.047} = 0.80$

$Z_2 - Z_1 = 75 - 0 = 75$ feet

$v_1 = \mathbf{10.08}$ . . . . 4" pipe, page B-14

$v_2 = \mathbf{6.42}$ . . . . 5" pipe, page B-14

$$\frac{v_2^2 - v_1^2}{2g} = \frac{6.42^2 - 10.08^2}{2 \times 32.2} = -0.94 \text{ feet}$$

5. For Schedule 40 pipe,

$R_e = \dfrac{50.6 \times 400 \times 62.371}{4.026 \times 1.1} = 2.85 \times 10^5$ . . . . 4" pipe

$R_e = \dfrac{50.6 \times 400 \times 62.371}{5.047 \times 1.1} = 2.27 \times 10^5$ . . . . 5" pipe

$f = 0.018$ . . . . 4 or 5" pipe; page A-25

6. $K = \dfrac{0.018 \times 225 \times 12}{5.047}$, or

$K = 9.6$ . . . . for 225' of 5" Sched. 40 pipe

$K = \dfrac{0.018 \times 110 \times 12}{4.026}$, or

$K = 5.9$ . . . . for 110' of 4" Sched. 40 pipe

With reference to velocity in 5" pipe,

$K_2 = 5.9 \div 0.8^4 = 14.4$

$K = 14 \times 0.016 = 0.22$ . . . . 5" 90° elbow

$K = 0.22 + \dfrac{0.36^2}{0.8^4} = 0.54$ . . . . 5x4" 90° elbow

7. Then, in terms of 5-inch pipe,

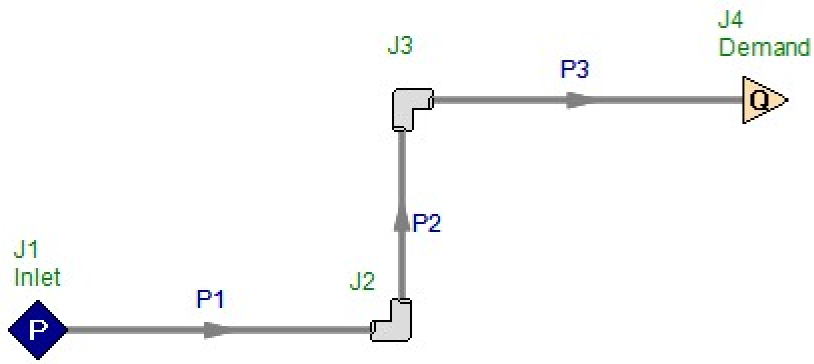
$K_{TOTAL} = 9.6 + 14.4 + 0.22 + 0.54 = 24.8$

8. $h_L = \dfrac{0.00259 \times 24.8 \times 400^2}{5.047^4} = \mathbf{15.8}$

9. $P_1 - P_2 = \dfrac{62.371}{144}(75 - 0.94 + 15.8) = \mathbf{39.0}$

View Verification Case 35 Model

Verification Case 35



Verification Case 36

[View Model](#) [Problem Statement](#)

PRODUCT: AFT Fathom

TITLE: FthVerify36.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-9, Example 4-15

FLUID: Water at 70 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Crane	AFT Fathom
Head loss (feet)	21	20.5
Pump added head (feet)	421	420.5
Pump power usage @ 70% efficiency (hp)	15.2	15.19

DISCUSSION:

AFT Fathom did not have the globe valve with reducers in its library, so the K factor of 27 was entered directly as a fitting and loss value in pipe 2. The standard gate valve was in the library, and was included in pipe 2. No information was provided about the pump suction, so a reservoir with zero elevation was used as the inlet with a frictionless suction pipe. Also, the discharge at 400 feet was to a reservoir connected to the last elbow by a frictionless pipe. The pump was modeled as an assigned flow.

The head loss from the pump discharge to the discharge tank can be obtained in AFT Fathom by subtracting the reservoir EGL from the pump outlet EGL. It also can be viewed in the Junction Deltas summary at the top of the Output window. The junction delta was setup in the Output Control window.

The added head in AFT Fathom is given in the Pump Summary on the Output window. The ideal power usage is also given in the Pump Summary as 10.62 hp. Dividing by 0.7 for the efficiency obtains 15.18 as shown in the table above.

[List of All Verification Models](#)

Verification Case 36 Problem Statement

Verification Case 36

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-9, Example 4-15

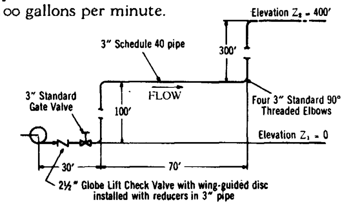
Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pressure Drop and Velocity in Piping Systems — continued

Example 4-15... Power Required for Pumping

Given: Water at 70 F is pumped through the piping system below at a rate of 100 gallons per minute.



Find: The total discharge head (H) at flowing conditions and the brake horsepower (bhp) required for a pump having an efficiency (e_p) of 70 per cent.

Solution: 1. Use Bernoulli's theorem (see page 3-2):

$$Z_1 + \frac{144 P_1}{\rho_1} + \frac{v_1^2}{2g} = Z_2 + \frac{144 P_2}{\rho_2} + \frac{v_2^2}{2g} + h_L$$

2. Since $\rho_1 = \rho_2$ and $v_1 = v_2$, the equation can be rewritten to establish the pump head, H:

$$\frac{144}{\rho}(P_1 - P_2) = (Z_2 - Z_1) + h_L$$

3. $h_L = \dfrac{0.00259\, KQ^2}{d^4}$ .... page 3-4

$R_e = 123.9 \dfrac{d v \rho}{\mu}$ .... page 3-2

$v = \dfrac{0.408\, Q}{d^2}$ .... page 3-2

$bhp = \dfrac{Q H \rho}{247\,000\, e_p}$ .... page B-9

4. $K = 30 f_T$ .... 90° elbow; page A-29

$K_1 = 8 f_T$ .... gate valve; page A-27

$K = f \dfrac{L}{D}$ .... straight pipe; page 3-4

$K = 1.0$ .... exit; page A-29

5. $d = 3.068$ ...... 3″ Sched. 40 pipe; page B-16

$\rho = 62.305$ .... page A-6

$\mu = 0.95$ .... page A-3

$f_T = 0.018$ .... page A-26

6. $v = \dfrac{0.408 \times 100}{3.068^2} = 4.33$

$R_e = \dfrac{123.9 \times 3.068 \times 4.33 \times 62.305}{0.95} = 1.1 \times 10^5$

$f = 0.021$ .... page A-25

7. $K = 4 \times 30 \times 0.018 = 2.16$ .. four 90° elbows

$K_1 = 8 \times 0.018 = 0.14$ .... gate valve

$K = 27.0$ .... lift check valve with reducers; Example 4-5

For 500 feet of 3-inch Schedule 40 pipe,

$$K = \frac{0.021 \times 500 \times 12}{3.068} = 41.06$$

And,

$$K_{TOTAL} = 2.16 + 0.14 + 27.0 + 41.06 + 1 = 71.4$$

8. $h_L = \dfrac{0.00259 \times 71.4 \times 100^2}{3.068^4} = 21$

9. $H = 400 + 21 = \mathbf{421}$

$bhp = \dfrac{100 \times 421 \times 62.305}{24700 \times 0.70} = \mathbf{15.2}$

Example 4-16... Air Lines

Given: Air at 65 psig and 110 F is flowing through 75 feet of 1-inch Schedule 40 pipe at a rate of 100 standard cubic feet per minute (scfm).

Find: The pressure drop in pounds per square inch and the velocity in feet per minute at both upstream and downstream gauges.

Solution: 1. Referring to the table on page B-15, read pressure drop of 2.21 psi for 100 psi, 60 F air at a flow rate of 100 scfm through 100 feet of 1-inch Schedule 40 pipe.

2. Correction for length, pressure, and temperature (page B-15):

$$\Delta P = 2.21 \left(\frac{75}{100}\right)\left(\frac{100 + 14.7}{65 + 14.7}\right)\left(\frac{460 + 110}{520}\right)$$

$\Delta P = \mathbf{2.61}$

3. To find the velocity, the rate of flow in cubic feet per minute at flowing conditions must be determined from page B-15.

$$q_m = q'_m \left(\frac{14.7}{14.7 + P}\right)\left(\frac{460 + t}{520}\right)$$

At upstream gauge:

$$q_m = 100\left(\frac{14.7}{14.7 + 65}\right)\left(\frac{460 + 110}{520}\right) = 20.2$$

At downstream gauge:

$$q_m = 100\left[\frac{14.7}{14.7 + (65 - 2.61)}\right]\left(\frac{460 + 110}{520}\right) = 20.9$$

4. $V = \dfrac{q_m}{A}$ .... page 3-2

5. $A = 0.006$ .... page B-16

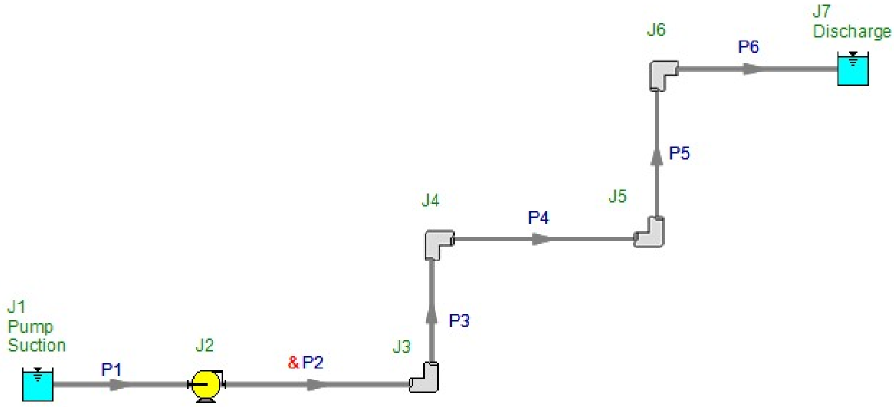
6. $V = \dfrac{20.2}{0.006} = \mathbf{3367}$ ..... at upstream gauge

$V = \dfrac{20.9}{0.006} = \mathbf{3483}$ ... at downstream gauge

Note: Example 4-16 may also be solved by use of the pressure drop formula and nomograph shown on pages 3-2 and 3-21 respectively or the velocity formula and nomograph shown on pages 3-2 and 3-17 respectively.

View Verification Case 36 Model

Verification Case 36



Verification Case 37

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify37.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-10, Example 4-17

FLUID: Crude Oil 30 degree API

ASSUMPTIONS: Assumed a rated pump speed of 1800 rpm for the HI viscosity correction calculation.

RESULTS:

Parameter	Crane	AFT Fathom
Head loss (feet)	1,405	1,409
Pump added head (feet)	3,405	3,409
Pump power usage @ 67% efficiency (hp)	1,496	1,498

DISCUSSION:

No information was provided about the pump suction, so a reservoir with zero elevation was used as the inlet with a frictionless suction pipe. The pump was modeled as an assigned flow.

The head loss from the pump discharge to the discharge tank can be obtained in AFT Fathom by subtracting the reservoir EGL from the pump outlet EGL. It also can be viewed in the Junction Deltas summary at the top of the Output window. The junction delta was setup in the Output Control window.

The added head in AFT Fathom is given in the Pump Summary on the Output window. The ideal power usage is also given in the Pump Summary as 1004 hp. Dividing by 0.67 for the efficiency obtains 1498 as shown in the table above.

List of All Verification Models

Verification Case 37 Problem Statement

Verification Case 37

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-10, Example 4-17

Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pipe Line Flow Problems

Example 4-17...Sizing of Pump for Oil Pipe Lines

Given: Crude oil 30 degree API at 15.6 C with a viscosity of 75 Universal Saybolt seconds is flowing through a 12-inch Schedule 30 steel pipe at a rate of 1900 barrels per hour. The pipe line is 50 miles long with discharge at an elevation of 2000 feet above the pump inlet. Assume the pump has an efficiency of 67 per cent.

Find: The brake horsepower of the pump.

Solution:

1. $\Delta P = 0.000\,1058 \frac{fL\rho B^2}{d^5}$ ... Equation 3-5 on page 3-2 or, after converting B to Q, use nomograph on page 3-11

   $t = 1.8\,t_c + 32$ .... page B-10

   $R_e = 35.4 \frac{B\rho}{d\mu}$ .... page 3-2 or 3-8

   $h_L = \frac{144\Delta P}{\rho}$ .... page 3-5

   brake horsepower $= \frac{QH\rho}{247\,000\,e_p}$ .... page B-9

2. $t = (1.8 \times 15.6) + 32 = 60$ F

3. $\rho = 54.64$ .... page B-7

   $S = 0.8762$ .... page B-7

4. $d = 12.09$ .... page B-17

   $d^5 = 258\,304$

5. 75 USS = 12.5 centipoise .... page B-5

6. $R_e = \frac{35.4 \times 1900 \times 54.64}{12.09 \times 12.5} = 24\,300$

7. $f = 0.025$ .... page A-25

8. $\Delta P = \frac{0.000\,1058 \times 0.025 \times 50 \times 5280 \times 54.64 \times 1900^2}{258\,304}$

   $\Delta P = 533$

9. $h_L = \frac{144 \times 533}{54.64} = 1405$

10. The total discharge head at the pump is:

    $H = 1405 + 2000 = 3405$

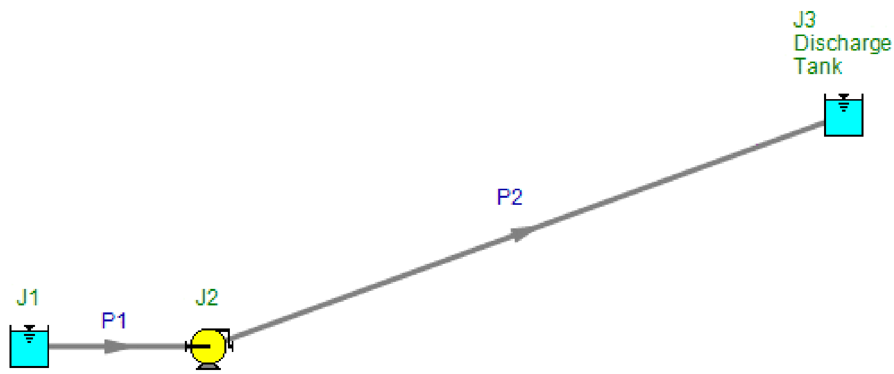
11. $Q = \left(\frac{1900\ \text{bbl}}{\text{hr}}\right)\left(\frac{42\ \text{gal}}{\text{bbl}}\right)\left(\frac{\text{hr}}{60\ \text{min}}\right) = 1330$

12. Then, the brake horsepower is:

    $\frac{1330 \times 3405 \times 54.64}{247\,000 \times 0.67} = 1496$, or say **1500**

View Verification Case 37 Model

Verification Case 37



Verification Case 38

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify38.fth

REFERENCE: Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-12, Example 4-19

FLUID: Water at 60 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Crane	AFT Fathom
Flow rate (gpm)	137	139

DISCUSSION:

Crane used friction factors of 0.021 and 0.020 based on a chart lookup. AFT Fathom calculated friction factors of 0.0201 and 0.0206 for the two pipes based on the Colebrook-White correlation. Crane's slightly larger friction factor from the chart lookup is the reason the results differ slightly.

List of All Verification Models

Verification Case 38 Problem Statement

Verification Case 38

Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988, Page 4-12, Example 4-19

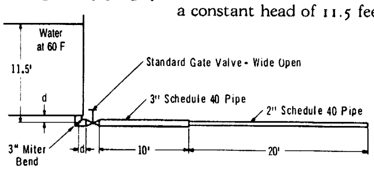
Crane Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Discharge of Fluids from Piping Systems

Example 4-19...Water

Given: Water at 60 F is flowing from a reservoir through the piping system below. The reservoir has a constant head of 11.5 feet.



Find: The flow rate in gallons per minute.

Solution: 1. $Q = 19.65\, d^2 \sqrt{\frac{h_L}{K}}$ ..page 3-4

$R_e = \frac{50.6\, Q\rho}{d\mu}$ ....page 3-2

$\beta = d_1/d_2$ ....page A-26

2. $K = 0.5$ ....entrance; page A-29

$K = 60 f_T$ ....mitre bend; page A-29

$K_1 = 8 f_T$ ....gate valve; page A-27

$K = f \frac{L}{D}$ ....straight pipe; page 3-4

$K_2 = \frac{0.5\,(1-\beta^2)\sqrt{\sin\frac{\theta}{2}}}{\beta^4}$ ....sudden contraction; page A-26

$K = \frac{fL}{D\beta^4}$ ....small pipe, in terms of larger pipe; page 2-5

$K = \frac{1}{\beta^4}$ ....exit from small pipe in terms of larger pipe

3. $d = 2.067$ ....2" Sched. 40 pipe; page B-16

$d = 3.068$ ....3" Sched. 40 pipe; page B-16

$\mu = 1.1$ ....page A-3

$\rho = 62.371$ ....page A-6

$f_T = 0.019$ ....2" pipe; page A-26

$f_T = 0.018$ ....3" pipe; page A-26

4. $\beta = 2.067 \div 3.068 = 0.67$

$K = 0.5$ ....3" entrance

$K = 60 \times 0.018 = 1.08$ ....3" mitre bend

$K_1 = 8 \times 0.018 = 0.14$ ....3" gate valve

$K = \frac{0.018 \times 10 \times 12}{3.068} = 0.70$ ..10 feet, 3" pipe

For 20 feet of 2-inch pipe, in terms of 3-inch pipe,

$K = \frac{0.019 \times 20 \times 12}{2.067 \times 0.67^4} = 10.9$

For 2-inch exit, in terms of 3-inch pipe,
$K = 1 \div 0.67^4 = 5.0$

For sudden contraction,
$K_2 = \frac{0.5\,(1-0.67^2)\,(1)}{0.67^4} = 1.37$

and, $K_{TOTAL} = 0.5 + 1.08 + 0.14 + 0.70 + 10.9 + 5.0 + 1.37 = 19.7$

5. $Q = 19.65 \times 3.068^2 \sqrt{11.5 \div 19.7} = 141$

(this solution assumes flow in fully turbulent zone)

6. Calculate Reynolds numbers and check friction factors for flow in straight pipe of the 2-inch size:

$R_e = \frac{50.6 \times 141 \times 62.371}{2.067 \times 1.1} = 1.96 \times 10^5$

$f = 0.021$ ....page A-25

and for flow in straight pipe of the 3-inch size:

$R_e = \frac{50.6 \times 141 \times 62.371}{3.068 \times 1.1} = 1.32 \times 10^5$

$f = 0.020$ ....page A-25

7. Since assumed friction factors used for straight pipe in Step 4 are not in agreement with those based on the approximate flow rate, the K factors for these items and the total system should be corrected accordingly.

$K = \frac{0.020 \times 10 \times 12}{3.068} = 0.78$ ..10 feet, 3" pipe

For 20 feet of 2-inch pipe, in terms of 3-inch pipe,

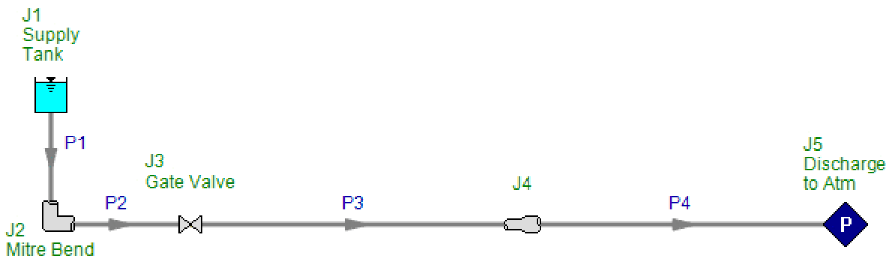
$K = \frac{0.021 \times 20 \times 12}{2.067 \times 0.67^4} = 12.1$

and, $K_{TOTAL} = 0.5 + 1.08 + 0.14 + 0.78 + 12.1 + 5.0 + 1.37 = 21.0$

8. $Q = 19.65 \times 3.068^2 \sqrt{11.5 \div 21} = \mathbf{137}$

View Verification Case 38 Model

Verification Case 38



Verification Case 39

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify39.fth

REFERENCE: Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 373-374, example 8.5

FLUID: Water at 20 deg. C

ASSUMPTIONS: N/A

RESULTS:

Parameter	Fox & McDonald	AFT Fathom
EGL inlet (meters)	44.6	44.702

DISCUSSION:

The supply reservoir is modeled as an Assigned Flow junction at 0.03 m^3/sec. The solution for inlet EGL is the reservoir height required.

List of All Verification Models

Verification Case 39 Problem Statement

Verification Case 39

Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 373-374, example 8.5

Fox and McDonald Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

friction factor is obtained from Fig. 8.14. Then the head loss is computed from Eqs. 8.32 and 8.38, and Eq. 8.28 is solved for pressure drop. The resulting trial value is compared to the system requirement.

If the trial value of Δp is too large, calculations are repeated for a larger assumed value of D. If the trial value of Δp is less than the criterion, a smaller assumed value of D should be checked.

In choosing a pipe diameter, it is logical to work with values that are available commercially. Pipe is manufactured in a limited number of standard sizes. Some data for standard pipe sizes are given in Table 8.4. For data on extra strong or double extra strong pipes, consult a handbook, e.g. [7]. Pipe larger than 12 in. nominal diameter is produced in multiples of 2 in. up to a nominal diameter of 36 in. and in multiples of 6 in. for still larger sizes.

Table 8.4 Standard Sizes for Carbon Steel, Alloy Steel, and Stainless Steel Pipe (Data from [7])

Nominal Pipe Size (in.)	Inside Diameter (in.)	Nominal Pipe Size (in.)	Inside Diameter (in.)
$\frac{1}{8}$	0.269	$2\frac{1}{2}$	2.469
$\frac{1}{4}$	0.364	3	3.068
$\frac{3}{8}$	0.493	4	4.026
$\frac{1}{2}$	0.622	5	5.047
$\frac{3}{4}$	0.824	6	6.065
1	1.049	8	7.981
$1\frac{1}{2}$	1.610	10	10.020
2	2.067	12	12.000

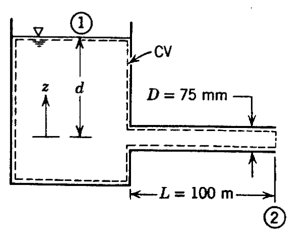
Example 8.5

A 100 m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of 0.03 m^3/sec of water? The inside diameter of the smooth pipe is 75 mm. The loss coefficient, K, for the square-edged inlet is 0.5. The water discharges to the atmosphere.

EXAMPLE PROBLEM 8.5

GIVEN: Water flow at 0.03 m^3/sec through a 75 mm diameter pipe, with $L = 100$ m, attached to a constant-level reservoir. Inlet loss coefficient, $K = 0.5$.

FIND: Reservoir depth, d, to maintain the flow.



SOLUTION:
Computing equation:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} = h_l + h_{l_m} \quad (8.28)$$

where

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{and} \quad h_{l_m} = K\frac{\bar{V}^2}{2}$$

For the given problem, $p_1 = p_2 = p_{atm}$, $\bar{V}_1 \simeq 0$, $\bar{V}_2 = \bar{V}$, and $\alpha_2 \simeq 1.0$. If it is assumed that $z_2 = 0$, then $z_1 = d$. Simplifying Eq. 8.28 gives

$$gd - \frac{\bar{V}^2}{2} = f\frac{L}{D}\frac{\bar{V}^2}{2} + K\frac{\bar{V}^2}{2}$$

Then

$$d = \frac{1}{g}\left[f\frac{L}{D}\frac{\bar{V}^2}{2} + K\frac{\bar{V}^2}{2} + \frac{\bar{V}^2}{2}\right] = \frac{\bar{V}^2}{2g}\left[f\frac{L}{D} + K + 1\right]$$

Since $\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2}$, then

$$d = \frac{8Q^2}{\pi^2 D^4 g}\left[f\frac{L}{D} + K + 1\right]$$

Assuming water at 20 C, $\rho = 999\ \text{kg/m}^3$, and $\mu = 1.0 \times 10^{-3}\ \text{kg/m}\cdot\text{sec}$. Thus

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{4\rho Q}{\pi\mu D}$$

$$Re = \frac{4}{\pi} \times \frac{999\ \text{kg}}{\text{m}^3} \times \frac{0.03\ \text{m}^3}{\text{sec}} \times \frac{\text{m}\cdot\text{sec}}{1.0 \times 10^{-3}\ \text{kg}} \times \frac{1}{0.075\ \text{m}} = 5.09 \times 10^5$$

For smooth pipe, from Fig. 8.14, $f = 0.0131$. Then

$$d = \frac{8Q^2}{\pi^2 D^4 g}\left[f\frac{L}{D} + K + 1\right]$$

$$= \frac{8}{\pi^2} \times \frac{(0.03)^2\ \text{m}^6}{\text{sec}^2} \times \frac{1}{(0.075)^4\ \text{m}^4} \times \frac{\text{sec}^2}{9.81\ \text{m}}\left[(0.0131)\frac{100\ \text{m}}{0.075\ \text{m}} + 0.5 + 1\right]$$

$d = 44.6$ m d

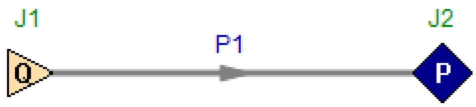
{This problem illustrates the method for calculating the total head loss.}

Example 8.6

A compressed air drill requires an air supply of 0.25 kg/sec at a gage pressure of 650 kPa at the drill. The hose from the air compressor to the drill is 40 mm inside diameter. The maximum compressor discharge gage pressure is 690 kPa. Neglect changes in density and any effects due to hose curvature. Air leaves the compressor at 40 C. Calculate the longest hose that may be used.

View Verification Case 39 Model

Verification Case 39



Verification Case 40

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify40.fth

REFERENCE: Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 376-377, example 8.7

FLUID: Water at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Fox & McDonald	AFT Fathom
Flow rate (gpm)	350	351.8

DISCUSSION:

There is a slight difference in flow rates because Fox & McDonald use a chart for friction factor, which is less precise than AFT Fathom's correlation based method.

List of All Verification Models

Verification Case 40 Problem Statement

Verification Case 40

Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 376-377, example 8.7

Fox and McDonald Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

376 8/INTERNAL INCOMPRESSIBLE VISCOUS FLOW

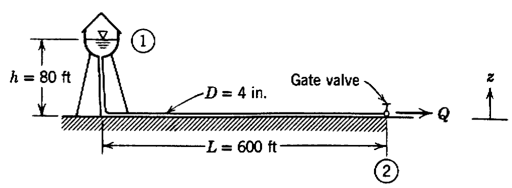
Example 8.7

A fire protection system is supplied from a water tower and standpipe 80 ft tall. The longest pipe in the system is 600 ft long, and is made of cast iron about 20 years old. The pipe contains one gate valve; other minor losses may be neglected. The pipe diameter is 4 in. Determine the maximum rate of flow through this pipe, in gallons per minute.

EXAMPLE PROBLEM 8.7

GIVEN: Fire protection system, as shown.

FIND: Q, gpm.



SOLUTION:

Computing equations: $$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} \quad (8.28)$$

($\bar{V}_1^2 \approx 0$ (2))

$$h_{l_T} = f\frac{L}{D}\frac{\bar{V}_2^2}{2} + h_{l_m} = f\frac{(L + L_e)}{D}\frac{\bar{V}_2^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$

(2) $\bar{V}_1 \simeq 0$, and $\alpha_2 \simeq 1.0$

For a fully open gate valve, from Table 8.3, $L_e/D = 8$. Then

$$h_{l_T} = f\frac{L}{D}\frac{\bar{V}_2^2}{2} + 8f\frac{\bar{V}_2^2}{2} = g(z_1 - z_2) - \frac{\bar{V}_2^2}{2}$$

or

$$\frac{\bar{V}_2^2}{2}\left[f\left(\frac{L}{D} + 8\right) + 1\right] = g(z_1 - z_2)$$

Solving for $\bar{V}_2$, we obtain

$$\bar{V}_2 = \left[\frac{2g(z_1 - z_2)}{f(L/D + 8) + 1}\right]^{1/2}$$

To be conservative, assume the standpipe is the same diameter as the horizontal pipe. Then

$$\frac{L}{D} = \frac{600 \text{ ft} + 80 \text{ ft}}{4 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} = 2040$$

Also

$$z_1 - z_2 = h = 80 \text{ ft}$$

Since $\bar{V}_2$ is not known, we cannot compute Re. But we can assume a value of friction factor in the fully rough flow region. From Fig. 8.15, $e/D \simeq 0.0025$ for cast iron pipe. Since the pipe is quite old, choose $e/D = 0.005$. Then, from Fig. 8.14, guess $f \simeq 0.03$. Then a first approximation to $\bar{V}_2$ is

$$\bar{V}_2 = \left[2 \times \frac{32.2}{} \frac{\text{ft}}{\text{sec}^2} \times 80\,\text{ft} \times \frac{1}{0.03(2040 + 8) + 1}\right]^{1/2} = 9.08\ \text{ft/sec}$$

Now check the value assumed for f.

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{9.08}{} \frac{\text{ft}}{\text{sec}} \times \frac{\text{ft}}{3} \times \frac{\text{sec}}{1.2 \times 10^{-5}\,\text{ft}^2} = 2.52 \times 10^5$$

For $e/D = 0.005$, $f = 0.031$ from Fig. 8.14. Using this value, we obtain

$$\bar{V}_2 = \left[2 \times \frac{32.2}{} \frac{\text{ft}}{\text{sec}^2} \times 80\,\text{ft} \times \frac{1}{0.031(2040 + 8) + 1}\right]^{1/2} = 8.94\ \text{ft/sec}$$

Thus convergence is satisfactory. The volume flow rate is

$$Q = \bar{V}_2 A = \bar{V}_2 \frac{\pi D^2}{4} = \frac{8.94\,\text{ft}}{\text{sec}} \times \frac{\pi}{4}\left(\frac{1}{3}\right)^2 \text{ft}^2 \times \frac{7.48\,\text{gal}}{\text{ft}^3} \times \frac{60\,\text{sec}}{\text{min}}$$

$Q = 350$ gpm $\leftarrow$ Q

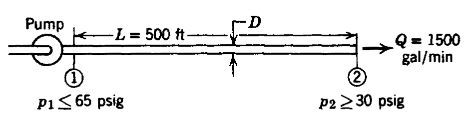
This problem illustrates the procedure for solving pipe flow problems in which the flow rate is unknown. Note that the velocity and, hence, the flow rate, is essentially proportional to $1/\sqrt{f}$. Doubling the value of e/D to account for aging reduced the flow rate by about 10 percent.

Example 8.8

Spray heads in an agricultural spraying system are to be supplied with water through 500 ft of drawn aluminum tubing from an engine-driven pump. In its most efficient operating range, the pump output is 1500 gpm at a discharge pressure not exceeding 65 psig. For satisfactory operation, the sprinklers must operate at 30 psig or higher pressure. Minor losses and elevation changes may be neglected. Determine the smallest standard pipe size that can be used.

EXAMPLE PROBLEM 8.8

GIVEN: Water supply system, as shown.



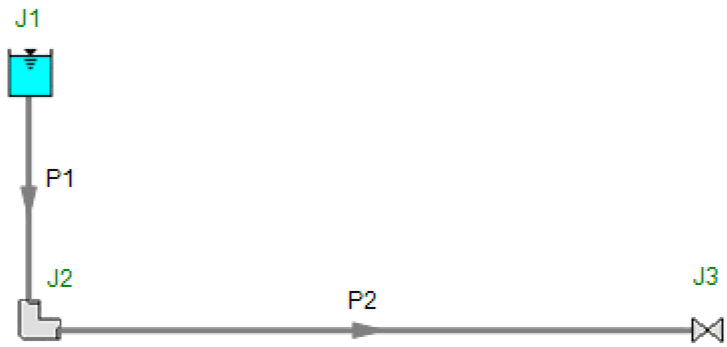
FIND: Smallest standard D.

SOLUTION:

Δp, L, and Q are known. D is unknown, so iteration will be required to determine the minimum standard diameter that satisfies the pressure drop constraint at the given flow rate.

View Verification Case 40 Model

Verification Case 40



Verification Case 41

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify41.fth

REFERENCE: Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 385-389, example 8.11

FLUID: Water at 60 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Fox & McDonald	AFT Fathom
Flow rate to Branch 1 (gal/min)	493	493.48
Flow rate to Branch 2 (gal/min)	557	557.44
Flow rate to Branch 3 (gal/min)	449	449.09
Pressure at inlet (psig)	91.2	91.52

DISCUSSION:

Fox and McDonald use equivalent lengths to model the elbows. These were converted to K factors of 0.4 in AFT Fathom. The loss factors at the three sprays (modeled as assigned pressure junctions) were specified as 16. This allows for the area ratio of 2 to the 4th power to account for the energy lost at the discharge.

List of All Verification Models

Verification Case 41 Problem Statement

Verification Case 41

Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Page 385-389, example 8.11

Fox and McDonald Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

SOLUTION OF PIPE FLOW PROBLEMS/8-8 **385**

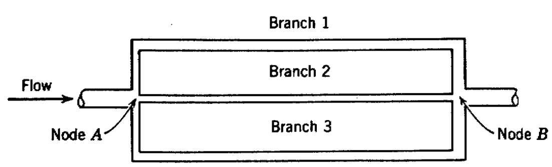
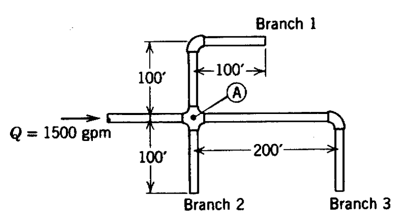


Fig. 8.20 Simple multiple-path pipe flow system.

The fluid flow rate and pressure drop are, respectively, analogous to the current and voltage in an electric circuit. However, the simple linear relation between voltage and current given by Ohm's law does not apply to the fluid flow system. Instead, the flow pressure drop is approximately proportional to the square of the flow rate. This nonlinearity makes iterative solutions necessary, and the resulting calculations can be quite lengthy and tedious. A number of schemes have been developed for use with digital computers. For an example, see [20].

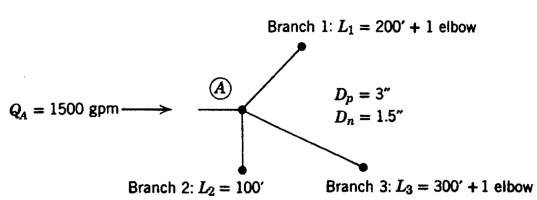
Example 8.11

The irrigation system of Example 8.8 is to be extended, holding the total flow rate constant at 1500 gpm, by adding three branches, each made from 3 in. pipe, as shown in the sketch. The sprinkler at the end of each branch has a nozzle of 1.5 in. minimum diameter. Minor losses at elbows should be included, but elevation terms may be neglected. Determine the flow rate in each branch, the pressure required at Ⓐ, and the pressure applied to each sprinkler nozzle.



EXAMPLE PROBLEM 8.11

GIVEN: Pipe network shown schematically in the following figure.



FIND: (a) Q_1, Q_2, Q_3.
(b) p_A.
(c) p_1, p_2, p_3 at nozzle inlets.

SOLUTION:
Apply Eq. 8.28 to each branch. Choose subscript n for nozzle exit, subscript p for pipe.

Computing equations:

$$\frac{p_A}{\rho} + \alpha_A \underbrace{\frac{\bar{V}_A^2}{2}}_{\approx 0(1)} + g z_A = \frac{p_n}{\rho} + \alpha_n \frac{\bar{V}_n^2}{2} + g z_n + h_{l_T} \qquad h_{l_T} = f \frac{L}{D}\frac{\bar{V}_p^2}{2} + f \frac{L_e}{D}\frac{\bar{V}_p^2}{2}$$

Assumptions: (1) $\bar{V}_A^2 \ll \bar{V}_n^2$; $\alpha_n \simeq 1.0$
(2) $z_A \simeq z_n$
(3) $p_n = p_{\text{atmosphere}}$
(4) Neglect loss due to flow split at Ⓐ.

Then, for each branch of the flow system, using gage pressure at Ⓐ, we obtain

$$\frac{p_A}{\rho} = \frac{\bar{V}_n^2}{2} + f \frac{L}{D}\frac{\bar{V}_p^2}{2} + f \frac{L_e}{D}\frac{\bar{V}_p^2}{2}$$

But for incompressible flow, $\bar{V}_p A_p = \bar{V}_n A_n$, so $\bar{V}_n^2 = \bar{V}_p^2 (A_p/A_n)^2 = \bar{V}_p^2 (D_p/D_n)^4$, and

$$\frac{p_A}{\rho} = \frac{\bar{V}_p^2}{2}\left[\left(\frac{D_p}{D_n}\right)^4 + f\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]$$

or

$$\bar{V}_p = \sqrt{\frac{2p_A}{\rho}}\left[\frac{1}{\left(\frac{D_p}{D_n}\right)^4 + f\left(\frac{L}{D} + \frac{L_e}{D}\right)}\right]^{1/2}$$

The corresponding flow rate for each branch is

$$Q_p = \bar{V}_p A_p = A_p \sqrt{\frac{2p_A}{\rho}}\left[\frac{1}{\left(\frac{D_p}{D_n}\right)^4 + f\left(\frac{L}{D} + \frac{L_e}{D}\right)}\right]^{1/2} \qquad (1)$$

To find f, we must determine the pipe Reynolds number. If flow is equally split as a first approximation, then $Q_p \simeq 500$ gpm per branch, and

$$Re = \frac{\bar{V}_p D_p}{\nu} = \frac{4 Q_p}{\pi \nu D_p}$$

$$= \frac{4}{\pi} \times \frac{500\ \text{gal}}{\text{min}} \times \frac{\text{sec}}{1.2 \times 10^{-5}\ \text{ft}^2} \times \frac{1}{3\ \text{in.}} \times \frac{\text{min}}{60\ \text{sec}} \times \frac{\text{ft}^3}{7.48\ \text{gal}} \times \frac{12\ \text{in.}}{\text{ft}}$$

$$Re = 4.73 \times 10^5$$

From Fig. 8.14, for smooth pipe, $f \simeq 0.0133$. For an elbow, $L_e/D = 30$, from Table 8.3. Using these values, we can obtain a first approximation for Q_p for each branch. Substituting into

Eq. 1 yields

$$Q_{p_1} \simeq A_p\sqrt{\frac{2p_A}{\rho}}\left[\frac{1}{\left(\frac{3.0}{1.5}\right)^4 + 0.0133\left(\frac{200\text{ ft}}{} \times \frac{1}{3\text{ in.}} \times \frac{12\text{ in.}}{\text{ft}} + 30\right)}\right]^{1/2}$$

$$Q_{p_1} \simeq 0.192A_p\sqrt{\frac{2p_A}{\rho}}$$

$$Q_{p_2} \simeq A_p\sqrt{\frac{2p_A}{\rho}}\left[\frac{1}{\left(\frac{3.0}{1.5}\right)^4 + 0.0133\left(\frac{100\text{ ft}}{} \times \frac{1}{3\text{ in.}} \times \frac{12\text{ in.}}{\text{ft}}\right)}\right]^{1/2}$$

$$Q_{p_2} \simeq 0.217A_p\sqrt{\frac{2p_A}{\rho}}$$

and

$$Q_{p_3} \simeq A_p\sqrt{\frac{2p_A}{\rho}}\left[\frac{1}{\left(\frac{3.0}{1.5}\right)^4 + 0.0133\left(\frac{300\text{ ft}}{} \times \frac{1}{3\text{ in.}} \times \frac{12\text{ in.}}{\text{ft}} + 30\right)}\right]^{1/2}$$

$$Q_{p_3} \simeq 0.176A_p\sqrt{\frac{2p_A}{\rho}}$$

From continuity,

$$Q_A = Q_{p_1} + Q_{p_2} + Q_{p_3} \simeq (0.192 + 0.217 + 0.176)A_p\sqrt{\frac{2p_A}{\rho}} \simeq 0.585A_p\sqrt{\frac{2p_A}{\rho}}$$

Thus

$$\frac{Q_{p_1}}{Q_A} \simeq \frac{0.192A_p\sqrt{\frac{2p_A}{\rho}}}{0.585A_p\sqrt{\frac{2p_A}{\rho}}} = 0.328$$

or

$$Q_{p_1} \simeq (0.328)1500\text{ gpm} = 492\text{ gpm}$$

Similarly

$$\frac{Q_{p_2}}{Q_A} \simeq \frac{0.217}{0.585} = 0.371; \quad Q_{p_2} \simeq 557\text{ gpm}$$

$$\frac{Q_{p_3}}{Q_A} \simeq \frac{0.176}{0.585} = 0.301; \quad Q_{p_3} \simeq 452\text{ gpm}$$

Better approximations for the Reynolds number and friction factor for each branch now may be calculated.

$$Re_1 = \frac{Q_{p_1}}{500\text{ gpm}} \times 4.73 \times 10^5 \simeq \frac{492}{500} \times 4.73 \times 10^5 = 4.65 \times 10^5$$

From Fig. 8.14, $f_1 \simeq 0.0133$. Similarly,

$$Re_2 \simeq \frac{557}{500} \times 4.73 \times 10^5 = 5.27 \times 10^5; \quad f_2 \simeq 0.0130$$

and

$$Re_3 \simeq \frac{452}{500} \times 4.73 \times 10^5 = 4.28 \times 10^5; \quad f_3 \simeq 0.0136$$

Substituting these values into Eq. 1, we obtain

$$Q_{p_1} \simeq 0.192 A_p \sqrt{\frac{2p_A}{\rho}}$$

$$Q_{p_2} \simeq 0.217 A_p \sqrt{\frac{2p_A}{\rho}}$$

and

$$Q_{p_3} \simeq 0.175 A_p \sqrt{\frac{2p_A}{\rho}}$$

From continuity,

$$Q_A = Q_{p_1} + Q_{p_2} + Q_{p_3} \simeq (0.192 + 0.217 + 0.175) A_p \sqrt{\frac{2p_A}{\rho}} \simeq 0.584 A_p \sqrt{\frac{2p_A}{\rho}}$$

Solving for the individual flow rates,

$$Q_{p_1} \simeq \frac{0.192}{0.584} \times 1500 \text{ gpm} = 493 \text{ gpm}$$

$$Q_{p_2} \simeq \frac{0.217}{0.584} \times 1500 \text{ gpm} = 557 \text{ gpm}$$

and

$$Q_{p_3} \simeq \frac{0.175}{0.584} \times 1500 \text{ gpm} = 449 \text{ gpm}$$ ← Q_1, Q_2, Q_3

Solving Eq. 1 for p_A gives

$$p_A = \frac{\rho}{2}\left(\frac{Q_p}{A_p}\right)^2\left[\left(\frac{D_p}{D_n}\right)^4 + f\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]$$

Substituting values for Branch ①,

$$p_A = \frac{1}{2} \times 1.94 \frac{\text{slug}}{\text{ft}^3}\left(493 \frac{\text{gal}}{\text{min}} \times \frac{4}{\pi}\frac{1}{(3)^2 \text{ in.}^2} \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times \frac{\text{min}}{60 \text{ sec}} \times 144 \frac{\text{in.}^2}{\text{ft}^2}\right)^2$$

$$\times \left[\left(\frac{3.0}{1.5}\right)^4 + 0.0133\left(200 \text{ ft} \times \frac{1}{3 \text{ in.}} \times 12 \frac{\text{in.}}{\text{ft}} + 30\right)\right] \frac{\text{lbf} \cdot \text{sec}^2}{\text{slug} \cdot \text{ft}} \times \frac{\text{ft}^2}{144 \text{ in.}^2}$$

$p_A = 91.2 \text{ lbf/in.}^2$

The values for Branches ② and ③ by similar calculations are 91.4 and 90.5 psig, respectively. (The slight differences are due to rounding the flow rate values.) Thus

$$p_A \approx 91.0 \text{ psig} \qquad p_A$$

The pressure at the inlet to each sprinkler nozzle may be calculated from the energy equation. The equation between Ⓐ and the nozzle inlet, section ⓘ, is

$$\frac{p_A}{\rho} + \frac{\bar{V}_A^2}{2} + gz_A = \frac{p_i}{\rho} + \frac{\bar{V}_p^2}{2} + gz_i + h_{l_T} \qquad h_{l_T} = f\frac{L}{D}\frac{\bar{V}_p^2}{2} + f\frac{L_e}{D}\frac{\bar{V}_p^2}{2}$$

From continuity, $\bar{V}_A = Q_A/A_A$ and $\bar{V}_p = Q_p/A_p$, so

$$p_i = p_A + \frac{\rho}{2}\left[\left(\frac{Q_A}{A_A}\right)^2 - \left(f\frac{L}{D} + f\frac{L_e}{D} + 1\right)\left(\frac{Q_p}{A_p}\right)^2\right]$$

or

$$p_i = p_A + \frac{\rho}{2}\left(\frac{Q_A}{A_A}\right)^2\left\{1 - \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + 1\right]\left(\frac{Q_p}{Q_A}\right)^2\left(\frac{A_A}{A_p}\right)^2\right\}$$

Substituting values for Branch ① gives

$$p_{i_1} = \frac{91.0 \text{ lbf}}{\text{in.}^2} + \frac{1}{2} \times \frac{1.94 \text{ slug}}{\text{ft}^3}\left(\frac{1500 \text{ gal}}{\text{min}} \times \frac{4}{\pi}\frac{1}{(6)^2 \text{ in.}^2} \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times \frac{\text{min}}{60 \text{ sec}} \times \frac{144 \text{ in.}^2}{\text{ft}^2}\right)^2$$

$$\times \left\{1 - \left[0.0133\left(\frac{200 \text{ ft}}{} \times \frac{1}{3 \text{ in.}} \times \frac{12 \text{ in.}}{\text{ft}} + 30\right) + 1\right]\left(\frac{493}{1500}\right)^2\left(\frac{6}{3}\right)^4\right\}\frac{\text{lbf}\cdot\text{sec}^2}{\text{slug}\cdot\text{ft}}$$

$$\times \frac{\text{ft}^2}{144 \text{ in.}^2}$$

$$p_{i_1} = 52.3 \text{ lbf/in.}^2$$

Similar calculations for Branches ② and ③ give

$$p_{i_2} = 66.3 \text{ lbf/in.}^2 \quad \text{and} \quad p_{i_3} = 43.3 \text{ lbf/in.}^2 \qquad p_i$$

{This problem illustrates the general method used to solve multiple-path pipe flow problems.}

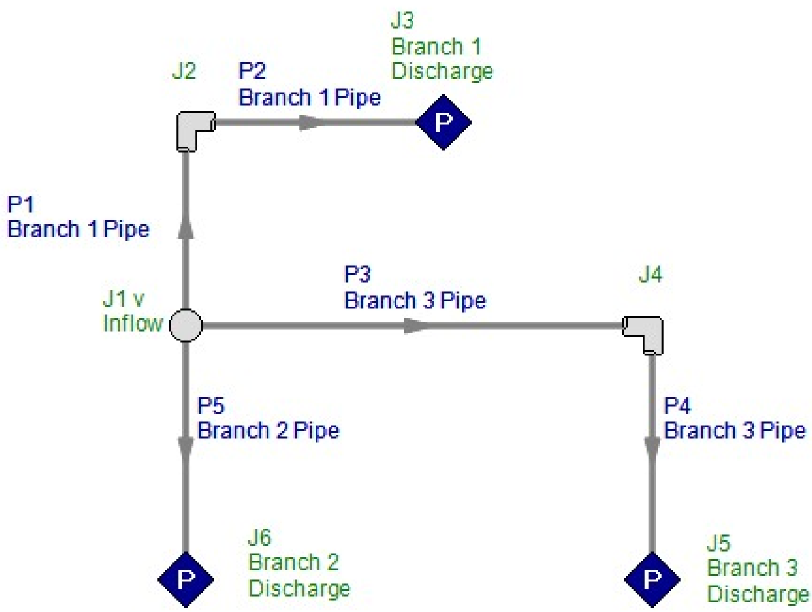
**8-8.3 Noncircular Ducts

The empirical correlations for pipe flow also may be used for computations involving noncircular ducts, provided their cross sections are not too exaggerated. Thus ducts of square or rectangular cross section may be treated if the ratio of height to width is less than about 3 or 4.

** This section may be omitted without loss of continuity in the text material.

View Verification Case 41 Model

Verification Case 41



Verification Case 42

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify42.fth

REFERENCE: Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-58

FLUID: Water at 68 deg. F

ASSUMPTIONS: Area change elevation is 130 feet, tanks are 10 feet deep

RESULTS:

Parameter	Marks'	AFT Fathom
Flow rate (ft3/sec)	0.4143	0.4144

DISCUSSION:

The fitting K factors were included in the pipes as a fitting and loss value.

List of All Verification Models

Verification Case 42 Problem Statement

Verification Case 42

Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-58

Marks' Handbook Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Table 11. Representative Equivalent Length in Pipe Diameters (L/D) of Various Valves and Fittings

Globe valves, fully open	450
Angle valves, fully open	200
Gate valves, fully open	13
¾ open	35
½ open	160
¼ open	900
Swing check valves, fully open	135
In line, ball check valves, fully open	150
Butterfly valves, 6 in and larger, fully open	20
90° standard elbow	30
45° standard elbow	16
90° long-radius elbow	20
90° street elbow	50
45° street elbow	26
Standard tee:	
Flow through run	20
Flow through branch	60

Compiled from data given in "Flow of Fluids," Crane Company Technical Paper 410, ASME, 1971.

sizes, the practice is to group all of one size together and apply the continuity equation, as shown in the following example.

Example. Water at 68°F (20°C) leaves an open tank whose surface elevation is 180 ft and enters a 2-in schedule 40 steel pipe via a sharp-edged entrance. After 50 ft of straight 2-in pipe that contains a 2-in globe valve, the line enlarges suddenly to an 8-in schedule 40 steel pipe which consists of 100 ft of straight 8-in pipe, two standard 90° elbows, and one 8-in angle valve. The 8-in line discharges below the surface of another open tank whose surface elevation is 100 ft. Determine the volumetric flow rate.

$$D_1 = 2.067/12 = 0.1723 \quad \text{and} \quad D_2 = 7.981/12 = 0.6651$$
$$\epsilon/D_1 = 150 \times 10^{-6}/0.1723 = 8.706 \times 10^{-4}$$
$$\epsilon/D_2 = 150 \times 10^{-6}/0.6651 = 2.255 \times 10^{-4}$$

For turbulent flow,

$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{\epsilon/D}{3.7}\right)$$
$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7}\right) \qquad f_1 = 0.01899$$
$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{2.255 \times 10^{-4}}{3.7}\right) \qquad f_2 = 0.01407$$

1. 2-in components

	K
Entrance loss, sharp-edged entrance	= 0.5
50 ft straight pipe = f_1 (50/0.1723)	= 290.2 f_1
Globe valve = f_1 (L/D)	= 450.0 f_1
Sudden enlargement $K = [1 - (D_1/D_2)^2]^2$	
$= [1 - (2.067/7.981)^2]^2$	= 0.87
	$\Sigma K_1 = 1.37 + 740.2 f_1$

2. 8-in components

100 ft of straight pipe f_2 (100/0.6651)	= 150.4 f_2
2 standard 90° elbows 2 × 30 f_2	= 60 f_2
1 angle valve 200 f_2	= 200 f_2
Exit loss	= 1
	$\Sigma K_2 = 1 + 410.4 f_2$

3. Apply equation of motion

$$h_{1/2} = z_1 - z_2 = (\Sigma K_1)\frac{V_1^2}{2g} + (\Sigma K_2)\frac{V_2^2}{2g}$$

From continuity, $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ for $\rho_1 = \rho_2$

$$V_2 = V_1(A_1/A_2) = V_1(D_1/D_2)^2$$
$$h_{1/2} = z_1 - z_2 = [\Sigma K_1 + \Sigma K_2 (D_1/D_2)^4] V_1^2/2g$$
$$V_1 = \{[2g(z_1 - z_2)]/[\Sigma K_1 + \Sigma K_2(D_1/D_2)^4]\}^{1/2}$$
$$V_1 = \left[\frac{2 \times 32.17 \times (180 - 100)}{(1.37 + 740.2 f_1) + (1 + 410.4 f_2)(2.067/7.981)^4}\right]^{1/2}$$
$$V_1 = \frac{71.74}{(1.374 + 740.2 f_1 + 1.846 f_2)^{1/2}}$$

4. For first trial assume f_1 and f_2 for complete turbulence

$$V_1 = \frac{71.74}{(1.374 + 740.2 \times 0.01899 + 1.846 \times 0.01407)^{1/2}}$$
$$V_1 = 18.25 \text{ ft/s}$$
$$V_2 = 18.25\,(2.067/7.981)^2 = 1.224 \text{ ft/s}$$
$$\mathbf{R}_1 = \rho_1 V_1 D_1/\mu = (1.937)(18.25)(0.1723)/(20.92 \times 10^{-6})$$
$$\mathbf{R}_1 = 291{,}100 > 4{,}000 \therefore \text{flow is turbulent}$$
$$\mathbf{R}_2 = \rho_2 V_2 D_2/\mu_2 = (1.937)(1.224)(0.6651)/(20.92 \times 10^{-6})$$ 1 pt.
$$\mathbf{R}_2 = 75{,}420 > 4{,}000 \therefore \text{flow is turbulent}$$

5. For second trial use first trial V_1 and V_2. From Fig. 24 and the Colebrook equation,

$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7} + \frac{2.51}{291{,}100\sqrt{0.020}}\right)$$
$$f_1 = 0.02008$$
$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{2.255 \times 10^{-4}}{3.7} + \frac{2.51}{75{,}420\sqrt{0.020}}\right)$$
$$f_2 = 0.02008$$
$$V_1 = \frac{71.74}{(1.374 + 740.2 \times 0.02008 + 1.864 \times 0.02008)^{1/2}}$$
$$V_1 = 17.78$$

A third trial results in $V = 17.77$ ft/s or $Q = A_1V_1 = (\pi/4)(0.1723)^2(17.77) = 0.4143$ ft³/s (1.173×10^{-2} m³/s).

Parallel Systems In solution of problems involving two or more parallel pipes, the head loss for each of the pipes is the same as shown in the following example:

Example. Benzene at 68°F (20°C) flows at a rate of 0.5 ft³/s through two parallel straight, horizontal pipes connecting two pressurized tanks. The pipes are both schedule 40 steel, one being 1 in, the other 2 in. They both are 100 ft long and have connections that project inwardly in the supply tank. If the pressure in the supply tank is maintained at 100 lbf/in², what pressure should be maintained on the receiving tank?

$$D_1 = 1.049/12 = 0.08742 \text{ ft} \quad \text{and} \quad D_2 = 2.067/12 = 0.1723 \text{ ft}$$
$$\epsilon/D_1 = 150 \times 10^{-6}/0.08742 = 1.716 \times 10^{-3}$$
$$\epsilon/D_2 = 150 \times 10^{-6}/0.1723 = 8.706 \times 10^{-4}$$

For turbulent flow,

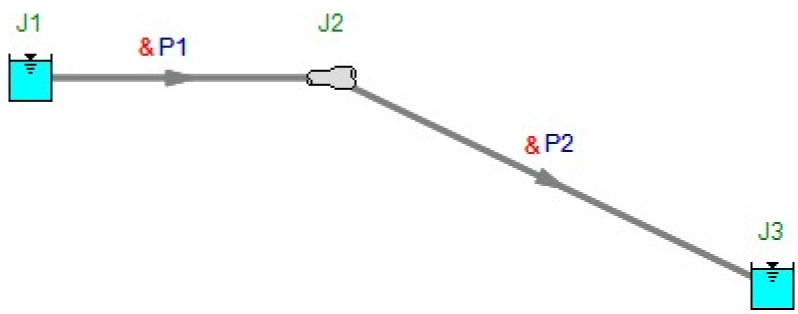
$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{\epsilon/D}{3.7}\right)$$
$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{1.716 \times 10^{-3}}{3.7}\right) \qquad f_1 = 0.02249$$
$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7}\right) \qquad f_2 = 0.01899$$

1. 1-in components

	K
Entrance loss, inward projection	= 1.0
100 ft straight pipe f_1 (100/0.08742)	= 1,144 f_1
Exit loss	= 1.0
	$\Sigma K_1 = 2.0 + 1{,}144 f_1$

View Verification Case 42 Model

Verification Case 42



Verification Case 43

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify43.fth

REFERENCE: Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-58

FLUID: Benzene at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Marks'	AFT Fathom
Flow rate in 1 inch pipe (ft3/sec)	0.0750	0.0744
Flow rate in 2 inch pipe (ft3/sec)	0.4250	0.4256
Pressure at discharge (psig)	73.40	73.28

DISCUSSION:

In order to fix the flowrate of 0.5 ft3/sec given in the problem statement the second pressurized tank is defined as a branch junction with a flow sink. In AFT Fathom the convention for flow out of the system is to use a negative sign, so the flow sink is entered as -0.5 ft3/sec.

List of All Verification Models

Verification Case 43 Problem Statement

Verification Case 43

Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-58

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Compiled from data given in "Flow of Fluids," Crane Company Technical Paper 410, ASME, 1971.

sizes, the practice is to group all of one size together and apply the continuity equation, as shown in the following example.

EXAMPLE. Water at 68°F (20°C) leaves an open tank whose surface elevation is 180 ft and enters a 2-in schedule 40 steel pipe via a sharp-edged entrance. After 50 ft of straight 2-in pipe that contains a 2-in globe valve, the line enlarges suddenly to an 8-in schedule 40 steel pipe which consists of 100 ft of straight 8-in pipe, two standard 90° elbows, and one 8-in angle valve. The 8-in line discharges below the surface of another open tank whose surface elevation is 100 ft. Determine the volumetric flow rate.

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$$\epsilon/D_2 = 150 \times 10^{-6}/0.6651 = 2.255 \times 10^{-4}$$

For turbulent flow,

$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{\epsilon/D}{3.7}\right)$$
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$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{2.255 \times 10^{-4}}{3.7}\right) \qquad f_2 = 0.01407$$

1. 2-in components

	K
Entrance loss, sharp-edged entrance	= 0.5
50 ft straight pipe = f_1 (50/0.1723)	= 290.2 f_1
Globe valve = f_1 (L/D)	= 450.0 f_1
Sudden enlargement $K = [1 - (D_1/D_2)^2]^2$ = $[1 - (2.067/7.981)^2]^2$	= 0.87
	$\Sigma K_1 = 1.37 + 740.2 f_1$

2. 8-in components

100 ft of straight pipe f_2 (100/0.6651)	= 150.4 f_2
2 standard 90° elbows 2 × 30 f_2	= 60 f_2
1 angle valve 200 f_2	= 200 f_2
Exit loss	= 1
	$\Sigma K_2 = 1 + 410.4 f_2$

3. Apply equation of motion

$$h_{1/2} = z_1 - z_2 = (\Sigma K_1)\frac{V_1^2}{2g} + (\Sigma K_2)\frac{V_2^2}{2g}$$

From continuity, $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ for $\rho_1 = \rho_2$

$$V_2 = V_1(A_1/A_2) = V_1(D_1/D_2)^2$$
$$h_{1/2} = z_1 - z_2 = [\Sigma K_1 + \Sigma K_2 (D_1/D_2)^4] V_1^2/2g$$
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$$V_1 = \left[\frac{2 \times 32.17 \times (180 - 100)}{(1.37 + 740.2 f_1) + (1 + 410.4 f_2)(2.067/7.981)^4}\right]^{1/2}$$
$$V_1 = \frac{71.74}{(1.374 + 740.2 f_1 + 1.846 f_2)^{1/2}}$$

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$$V_1 = \frac{71.74}{(1.374 + 740.2 \times 0.01899 + 1.846 \times 0.01407)^{1/2}}$$
$$V_1 = 18.25 \text{ ft/s}$$
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5. For second trial use first trial V_1 and V_2. From Fig. 24 and the Colebrook equation,

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$$f_1 = 0.02008$$
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EXAMPLE. Benzene at 68°F (20°C) flows at a rate of 0.5 ft³/s through two parallel straight, horizontal pipes connecting two pressurized tanks. The pipes are both schedule 40 steel, one being 1 in, the other 2 in. They both are 100 ft long and have connections that project inwardly in the supply tank. If the pressure in the supply tank is maintained at 100 lbf/in², what pressure should be maintained on the receiving tank?

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For turbulent flow,

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$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7}\right) \qquad f_2 = 0.01899$$

1. 1-in components

	K
Entrance loss, inward projection	= 1.0
100 ft straight pipe f_1 (100/0.08742)	= 1,144 f_1
Exit loss	= 1.0
	$\Sigma K_1 = 2.0 + 1{,}144 f_1$

2. 2-in components

Entrance loss, inward projection	= 1.0
100 ft straight pipe f_2 (100/0.1723)	= 580.4 f_2
Exit loss	= 1.0
	$\Sigma K_2 = 2.0 + 580.4 f_2$

$h_f = \Sigma K_1 V_1^2/2g = \Sigma K_2 V_2^2/2g$

From the continuity equation, $Q = AV$

$$\Sigma K_1 \frac{Q_1^2}{2gA_1^2} = \Sigma K_2 \frac{Q_2^2}{2gA_2^2}$$

Solving for Q_1/Q_2,

$$\frac{Q_1}{Q_2} = \frac{A_1}{A_2}\sqrt{\frac{\Sigma K_2}{\Sigma K_1}} = \left(\frac{D_1}{D_2}\right)^2 \sqrt{\frac{\Sigma K_2}{\Sigma K_1}} = \left(\frac{D_1}{D_2}\right)^2 \sqrt{\frac{2.0 + 580.4 f_2}{2.0 + 1{,}144 f_1}}$$

For first trial assume flow is completely turbulent,

$$\frac{Q_1}{Q_2} = \left(\frac{0.08742}{0.1723}\right)^2 \sqrt{\frac{2.0 + 580.4 \times 0.01899}{2.0 + 1{,}144 \times 0.02249}}$$

$$\frac{Q_1}{Q_2} = 0.1764 \qquad Q = Q_1 + Q_2 = 0.1764\, Q_2 + Q_2$$

$0.5000 = 1.1764\, Q_2 \qquad Q_2 = 0.4250$

$Q_1 = 0.5000 - 0.4250 = 0.0750$

For second trial use first-trial values,

$$V_1 = Q_1/A_1 = 0.0750/(\pi/4)(0.08742)^2 = 12.50$$
$$V_2 = Q_2/A_2 = 0.4250/(\pi/4)(0.1723)^2 = 18.23$$
$$\mathbf{R}_1 = \rho_1 V_1 D_1/\mu_1 = (1.705)(12.50)(0.08742)/(13.62 \times 10^{-6})$$
$$\mathbf{R}_1 = 136{,}800 > 4{,}000 \therefore \text{ flow is turbulent}$$
$$\mathbf{R}_2 = \rho_2 V_2 D_2/\mu_2 = (1.705)(18.23)(0.1723)/(13.62 \times 10^{-6})$$
$$\mathbf{R}_2 = 393{,}200 > 4{,}000 \therefore \text{ flow is turbulent}$$

Using the Colebrook equation and Fig. 24,

$$\frac{1}{\sqrt{f_1}} = -2\log_{10}\left(\frac{1.716 \times 10^{-3}}{3.7} + \frac{2.51}{136{,}800\sqrt{0.024}}\right)$$
$$f_1 = 0.02389$$
$$\frac{1}{\sqrt{f_2}} = -2\log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7} + \frac{2.51}{393{,}200\sqrt{0.020}}\right)$$
$$f_2 = 0.01981$$
$$h_f = \Sigma K_1 \frac{V_1^2}{2g} = \Sigma K_2 \frac{V_2^2}{2g}$$
$$\Sigma K_1 V_1^2/2g = (2.0 + 1{,}144 \times 0.02389)(12.50)^2/(2 \times 32.17) = 71.23$$
$$\Sigma K_2 V_2^2/2g = (2.0 + 580.4 \times 0.01981)(18.23)^2/(2 \times 32.17) = 69.80$$

71.23 ≈ 69.80; further trials not justifiable because of accuracy of f, K, L/D. Use average or 70.52, so that $\Delta p = \rho g h_f = (1.705 \times 32.17 \times 70.52)/144 = 26.86$ lbf/in² $= p_1 - p_2 = 100 - p_2$, $p_2 = 100 - 26.86 = 73.40$ lbf/in² (5.061×10^5 N/m²).

Branch Flow Problems of a single line feeding several points may be solved as shown in the following example.

EXAMPLE. Ethyl alcohol at 68°F (20°C) flows from tank A, which is maintained at a constant pressure of 100 lb/in² through 200 ft of 2-in cast-iron schedule 40 pipe to a Y branch connection ($K = 0.5$) where 100 ft of 2-in pipe goes to tank B, which is maintained at 80 lbf/in² and 50 ft of 2-in pipe to tank C, which is also maintained at 80 lbf/in². All tank connections are flush and sharp-edged and are at the same elevation. Estimate the flow rate to each tank.

$$D = 2.067/12 = 0.1723 \text{ ft}$$
$$\epsilon/D = 850 \times 10^{-6}/0.1723 = 4.933 \times 10^{-3}$$

For turbulent flow, $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7}\right)$

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{4.933 \times 10^{-3}}{3.7}\right) \qquad f = 0.03025$$
$$h_{A/B} = (p_A - p_B)/\rho g = 144(100 - 80)/(1.532 \times 32.17) = 58.44$$
$$h_{A/C} = (p_A - p_C)/\rho g = h_{A/B} = 58.44$$

Let point X be just before the Y; then

	K
1. From tank A to Y	
Entrance loss, sharp-edged	= 0.5
200 ft straight pipe = f_{AX} (200/0.1723)	= 1,161 f_{AX}
	$\Sigma K_{AX} = 0.5 + 1{,}161 f_{AX}$
2. From Y to tank B	
Y branch	= 0.5
100 ft straight pipe = f_{XB} (100/0.1723)	= 580.4 f_{XB}
Exit loss	= 1.0
	$\Sigma K_{XB} = 1.5 + 580.4 f_{XB}$
3. From Y to tank C	
Y branch	= 0.5
50 ft straight pipe = f_{XC} (50/0.1723)	= 290.2 f_{XC}
Exit loss	= 1.0
	$\Sigma K_{XC} = 1.5 + 290.2 f_{XC}$

Balance of flows:

$$Q_{AX} = Q_{XB} + Q_{XC}$$

and from continuity, ($A_{AX} = A_{XB} = A_{XC}$), $V_{AX} = V_{XB} + V_{XC}$; then

$$h_{A/B} = \Sigma K_{AX}\frac{V_{AX}^2}{2g} + \Sigma K_{XB}\frac{V_{XB}^2}{2g}$$
$$h_{A/C} = \Sigma K_{AX}\frac{V_{AX}^2}{2g} + \Sigma K_{XC}\frac{V_{XC}^2}{2g}$$

For first trial assume completely turbulent flow

$$h_{A/B} = \frac{(0.5 + 1{,}161 f_{AX})V_{AX}^2}{2g} + \frac{(1.5 + 580.4 f_{XB})V_{XB}^2}{2g}$$
$$58.44 = \frac{(0.5 + 1{,}161 \times 0.03025)V_{AX}^2}{2 \times 32.17} + \frac{(1.5 + 580.4 \times 0.03025)V_{XB}^2}{2 \times 32.17}$$
$$58.44 = 0.5536\, V_{AX}^2 + 0.2962\, V_{XB}^2$$

and in a like manner

$$h_{A/C} = 58.44 = 0.5536\, V_{AX}^2 + 0.1598\, V_{XC}^2$$

Equating $h_{A/B} = h_{A/C}$,

$$0.5536\, V_{AX}^2 + 0.2962\, V_{XB}^2 = 0.5536\, V_{AX}^2 + 0.1598\, V_{XC}^2$$

or $V_{XC} = 1.3615\, V_{XB}$ and since $V_{AX} = V_{XB} + V_{XC}$

$V_{AX} = V_{XB} + 1.3615\, V_{XB} = 2.3615\, V_{XB}$

so that

$$h_{A/B} = 58.44 = 0.5536(2.3615\, V_{XB}^2) + 0.2962\, V_{XB}^2$$
$$V_{XB} = 4.156$$
$$V_{XC} = 1.3615(4.156) = 5.658$$
$$V_{AX} = 4.156 + 5.658 = 9.814$$

Second trial,

$$\mathbf{R}_{AX} = \frac{\rho V_{AX} D}{\mu} = \frac{1.532 \times 9.814 \times 0.1723}{25.06 \times 10^{-6}}$$
$$\mathbf{R}_{AX} = 103{,}400 > 4{,}000 \therefore \text{ flow is turbulent}$$

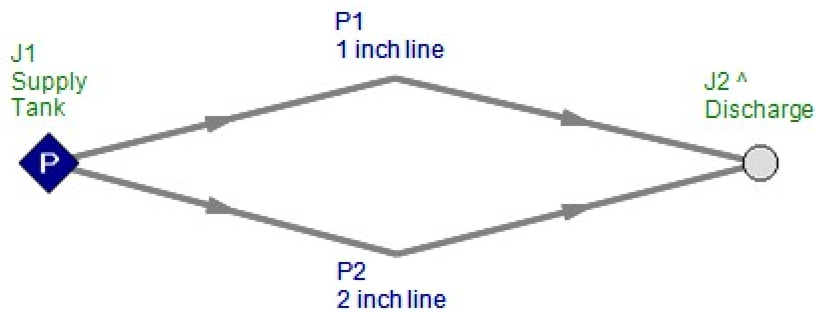
In a like manner,

$$\mathbf{R}_{XB} = 43{,}780 \qquad \mathbf{R}_{XC} = 59{,}600$$

Using the Colebrook equation and Fig. 24,

View Verification Case 43 Model

Verification Case 43



Verification Case 44

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify44.fth

REFERENCE: Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-59, 60

FLUID: Ethyl Alcohol (Ethanol) at 68 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Marks'	AFT Fathom
Flow rate to Tank B (ft3/sec)	0.09491	0.09474
Flow rate to Tank C (ft3/sec)	0.1304	0.13023

DISCUSSION:

All inputs were provided explicitly by the problem statement.

List of All Verification Models

Verification Case 44 Problem Statement

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Theodore Baumeister, Eugene Avallone, Theodore Baumeister III, Marks' Standard Handbook for Mechanical Engineers, 8th ed., 1978, McGraw-Hill, Page 3-59, 60

Marks' Handbook Title Page

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2. 2-in components

Entrance loss, inward projection	$= 1.0$
100 ft straight pipe f_2 (100/0.1723)	$= 580.4 f_2$
Exit loss	$= 1.0$
	$\Sigma K_2 = 2.0 + 580.4 f_2$

$h_f = \Sigma K_1 V_1^2/2g = \Sigma K_2 V_2^2/2g$

From the continuity equation, $Q = AV$

$$\Sigma K_1 \frac{Q_1^2}{2gA_1^2} = \Sigma K_2 \frac{Q_2^2}{2gA_2^2}$$

Solving for Q_1/Q_2,

$$\frac{Q_1}{Q_2} = \frac{A_1}{A_2}\sqrt{\frac{\Sigma K_2}{\Sigma K_1}} = \left(\frac{D_1}{D_2}\right)^2 \sqrt{\frac{\Sigma K_2}{\Sigma K_1}} = \left(\frac{D_1}{D_2}\right)^2 \sqrt{\frac{2.0 + 580.4 f_2}{2.0 + 1{,}144 f_1}}$$

For first trial assume flow is completely turbulent,

$$\frac{Q_1}{Q_2} = \left(\frac{0.08742}{0.1723}\right)^2 \sqrt{\frac{2.0 + 580.4 \times 0.01899}{2.0 + 1{,}144 \times 0.02249}}$$

$$\frac{Q_1}{Q_2} = 0.1764 \qquad Q = Q_1 + Q_2 = 0.1764\, Q_2 + Q_2$$

$0.5000 = 1.1764\, Q_2 \qquad Q_2 = 0.4250$

$Q_1 = 0.5000 - 0.4250 = 0.0750$

For second trial use first-trial values,

$V_1 = Q_1/A_1 = 0.0750/(\pi/4)(0.08742)^2 = 12.50$

$V_2 = Q_2/A_2 = 0.4250/(\pi/4)(0.1723)^2 = 18.23$

$\mathbf{R}_1 = \rho_1 V_1 D_1/\mu_1 = (1.705)(12.50)(0.08742)/(13.62 \times 10^{-6})$

$\mathbf{R}_1 = 136{,}800 > 4{,}000$ ∴ flow is turbulent

$\mathbf{R}_2 = \rho_2 V_2 D_2/\mu_2 = (1.705)(18.23)(0.1723)/(13.62 \times 10^{-6})$

$\mathbf{R}_2 = 393{,}200 > 4{,}000$ ∴ flow is turbulent

Using the Colebrook equation and Fig. 24,

$$\frac{1}{\sqrt{f_1}} = -2 \log_{10}\left(\frac{1.716 \times 10^{-3}}{3.7} + \frac{2.51}{136{,}800\sqrt{0.024}}\right)$$

$f_1 = 0.02389$

$$\frac{1}{\sqrt{f_2}} = -2 \log_{10}\left(\frac{8.706 \times 10^{-4}}{3.7} + \frac{2.51}{393{,}200\sqrt{0.020}}\right)$$

$f_2 = 0.01981$

$$h_f = \Sigma K_1 \frac{V_1^2}{2g} = \Sigma K_2 \frac{V_2^2}{2g}$$

$\Sigma K_1 V_1^2/2g$

$= (2.0 + 1{,}144 \times 0.02389)(12.50)^2/(2 \times 32.17) = 71.23$

$\Sigma K_2 V_2^2/2g$

$= (2.0 + 580.4 \times 0.01981)(18.23)^2/(2 \times 32.17) = 69.80$

71.23 = 69.80; further trials not justifiable because of accuracy of f, K, L/D. Use average or 70.52, so that $\Delta p = \rho g h_f = (1.705 \times 32.17 \times 70.52)/144 = 26.86$ lbf/in² $= p_1 - p_2 = 100 - p_2$, $p_2 = 100 - 26.86 = 73.40$ lbf/in² (5.061×10^5 N/m²).

Branch Flow Problems of a single line feeding several points may be solved as shown in the following example.

EXAMPLE. Ethyl alcohol at 68°F (20°C) flows from tank A, which is maintained at a constant pressure of 100 lb/in² through 200 ft of 2-in cast-iron schedule 40 pipe to a Y branch connection ($K = 0.5$) where 100 ft of 2-in pipe goes to tank B, which is maintained at 80 lbf/in² and 50 ft of 2-in pipe to tank C, which is also maintained at 80 lbf/in². All tank connections are flush and sharp-edged and are at the same elevation. Estimate the flow rate to each tank.

$D = 2.067/12 = 0.1723$ ft

$\epsilon/D = 850 \times 10^{-6}/0.1723 = 4.933 \times 10^{-3}$

For turbulent flow, $\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{\epsilon/D}{3.7}\right)$

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{4.933 \times 10^{-3}}{3.7}\right) \qquad f = 0.03025$$

$h_{Af_B} = (p_A - p_B)/\rho g = 144(100 - 80)/(1.532 \times 32.17) = 58.44$

$h_{Af_C} = (p_A - p_C)/\rho g = h_{Af_B} = 58.44$

Let point X be just before the Y; then

1. From tank A to Y

	K
Entrance loss, sharp-edged	$= 0.5$
200 ft straight pipe $= f_{AX}$ (200/0.1723)	$= 1{,}161 f_{AX}$
	$\Sigma K_{AX} = 0.5 + 1{,}161 f_{AX}$

2. From Y to tank B

Y branch	$= 0.5$
100 ft straight pipe $= f_{XB}$ (100/0.1723)	$= 580.4 f_{XB}$
Exit loss	$= 1.0$
	$\Sigma K_{XB} = 1.5 + 580.4 f_{XB}$

3. From Y to tank C

Y branch	$= 0.5$
50 ft straight pipe $= f_{XC}$ (50/0.1723)	$= 290.2 f_{XC}$
Exit loss	$= 1.0$
	$\Sigma K_{XC} = 1.5 + 290.2 f_{XC}$

Balance of flows:

$$Q_{AX} = Q_{XB} + Q_{XC}$$

and from continuity, ($A_{AX} = A_{XB} = A_{XC}$), $V_{AX} = V_{XB} + V_{XC}$; then

$$h_{Af_B} = \Sigma K_{AX}\frac{V_{AX}^2}{2g} + \Sigma K_{XB}\frac{V_{XB}^2}{2g}$$

$$h_{Af_C} = \Sigma K_{AX}\frac{V_{AX}^2}{2g} + \Sigma K_{XC}\frac{V_{XC}^2}{2g}$$

For first trial assume completely turbulent flow

$$h_{Af_B} = \frac{(0.5 + 1{,}161 f_{AX})V_{AX}^2}{2g} + \frac{(1.5 + 580.4 f_{XB})V_{XB}^2}{2g}$$

$$58.44 = \frac{(0.5 + 1{,}161 \times 0.03025)V_{AX}^2}{2 \times 32.17} + \frac{(1.5 + 580.4 \times 0.03025)V_{XB}^2}{2 \times 32.17}$$

$58.44 = 0.5536\, V_{AX}^2 + 0.2962\, V_{XB}^2$

and in a like manner

$$h_{Af_C} = 58.44 = 0.5536\, V_{AX}^2 + 0.1598\, V_{XC}^2$$

Equating $h_{Af_B} = h_{Af_C}$,

$$0.5536\, V_{AX}^2 + 0.2962\, V_{XB}^2 = 0.5536\, V_{AX}^2 + 0.1598\, V_{XC}^2$$

or $V_{XC} = 1.3615\, V_{XB}$ and since $V_{AX} = V_{XB} + V_{XC}$

$V_{AX} = V_{XB} + 1.3615\, V_{XB} = 2.3615\, V_{XB}$

so that

$h_{Af_B} = 58.44 = 0.5536(2.3615\, V_{XB}^2) + 0.2962\, V_{XB}^2$

$V_{XB} = 4.156$

$V_{XC} = 1.3615(4.156) = 5.658$

$V_{AX} = 4.156 + 5.658 = 9.814$

Second trial,

$$\mathbf{R}_{AX} = \frac{\rho V_{AX} D}{\mu} = \frac{1.532 \times 9.814 \times 0.1723}{25.06 \times 10^{-6}}$$

$\mathbf{R}_{AX} = 103{,}400 > 4{,}000$ ∴ flow is turbulent

In a like manner,

$$\mathbf{R}_{XB} = 43{,}780 \qquad \mathbf{R}_{XC} = 59{,}600$$

Using the Colebrook equation and Fig. 24,

$$\frac{1}{\sqrt{f_{AX}}} = -2 \log_{10}\left(\frac{4.933 \times 10^{-3}}{3.7} + \frac{2.51}{103{,}400\sqrt{0.031}}\right)$$
$$f_{AX} = 0.03116$$

In a like manner,

$$f_{XB} = 0.03231 \qquad f_{XC} = 0.03179$$

$$h_{Af_B} = \frac{(0.5 + 1{,}161 \times 0.03116)V_{AX}^2}{2 \times 32.17} + \frac{(1.5 + 580.4 \times 0.03231)V_{XB}^2}{2 \times 32.17}$$
$$h_{Af_B} = 0.5700\ V_{AX}^2 + 0.3148\ V_{XB}^2$$
$$h_{Af_C} = 0.5700\ V_{AX}^2 + \frac{(1.5 + 290.2 \times 0.03179)V_{XC}^2}{2 \times 32.17}$$
$$h_{Af_C} + 0.5700\ V_{AX}^2 + 0.1667\ V_{XC}^2$$
$$0.3148\ V_{XB}^2 = 0.1667\ V_{XC}^2$$
$$V_{XC} = 1.374\ V_{XB}$$
$$V_{AX} = V_{XB} + 1.374\ V_{XB} = 2.374\ V_{XB}$$

so that

$$h_{Af_B} = 58.44 = 0.5700\ (2.374\ V_{XB})^2 + 0.3148\ V_{XB}^2$$
$$V_{XB} = 4.070 \qquad V_{XC} = 5.592 \qquad V_{AX} = 9.663$$

Further trials are not justified.

$$A = \pi D^2/4 = (\pi/4)(0.1723)^2 = 0.02332\ \text{ft}^2$$
$$Q_{XB} = V_{XB}\,A = 4.070 \times 0.02332 = 0.09491\ \text{ft}^3/\text{s}\ (2.686 \times 10^{-3}\ \text{m}^3/\text{s})$$
$$Q_{XC} = V_{XC}\,A = 5.592 \times 0.02332 = 0.1304\ \text{ft}^3/\text{s}\ (3.693 \times 10^{-3}\ \text{m}^3/\text{s})$$

Siphons are arrangements of hose or pipe which cause liquids to flow from one level A in Fig. 25 to a lower level C over an intermediate summit B. Performance of siphons may be evaluated from the equation of motion between points A and B:

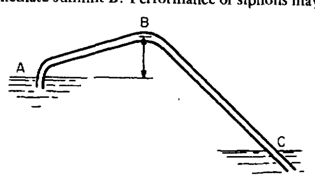


Fig. 25 Siphon.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{Af_B}$$

Noting that on the surface $V_A = 0$ and the minimum pressure that can exist at point B is the vapor pressure p_v, the maximum elevation of point B is

$$z_B - z_A = \frac{p_A}{\gamma} - \left(\frac{p_v}{\gamma} + \frac{V_B^2}{2g} + h_{Af_B}\right)$$

The friction loss $h_f = \Sigma K_{AB} V_B^2/2g$, and let $V_B = V$; then

$$z_B - z_A = \frac{p_A - p_v}{\rho g} - (1 + \Sigma K_{AB})\frac{V^2}{2g}$$

Flow under this maximum condition will be uncertain. The air pump or ejector used for priming the pipe (flow will not take place unless the siphon is full of water) might have to be operated occasionally to remove accumulated air and vapor. Values of $z_B - z_A$ less than those calculated by the above equation should be used.

Example. The siphon shown in Fig. 25 is composed of 2,000 ft of 6-in schedule 40 cast-iron pipe. Reservoir A is at elevation 800 ft and C at 600 ft. Estimate the maximum height for $z_B - z_A$ if the water temperature may reach 104°F (40°C), and the amount of straight pipe from A to B is 100 ft. For the first bend $L/D = 25$ and the second (at B) $L/D = 50$. Atmospheric pressure is 14.70 lbf/in². For 6-in schedule 40 pipe $D = 6.065/12 = 0.5054$ ft, $\epsilon/D = 850 \times 10^{-6}/0.5054 = 1.682 \times 10^{-3}$. Turbulent friction factor $1/\sqrt{f} = -2 \log_{10}\left(\frac{\epsilon/D}{3.7}\right) = -2 \log_{10}(1.682 \times 10^{-3}/3.7) = 0.02238$.

	K
1. Components from A to B (Note loss in second bend takes place in downstream piping)	
Entrance (inward projection)	= 1.0
100 ft straight pipe f (100/0.5054)	= 197.9 f
First bend	= 25 f
ΣK_{AB}	= 1.0 + 227.9 f
2. Components from A to C	
ΣK_{AB}	= 1.0 + 2,229 f
1,900 ft of straight pipe f(1,900/0.5054)	= 3,759.4 f
Second bend	= 50 f
Exit loss	= 1
ΣK_{AC}	= 2.0 + 4,032 f

First trial assume complete turbulence. Writing the equation of motion between A and C,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + \Sigma K_{AC}\frac{V^2}{2g}$$

Noting $V_A = V_C = 0$, and $p_A = p_C = 14.7$ lbf/in²,

$$V = \sqrt{\frac{2g(z_A - z_C)}{\Sigma K_{AC}}} = \sqrt{\frac{2g(z_A - z_C)}{2.0 + 4{,}032 f}} = \sqrt{\frac{2 \times 32.17\ (800 - 600)}{2.0 + 4{,}032 f}}$$
$$= \frac{113.44}{\sqrt{2.0 + 4{,}032 \times 0.02238}} = 11.81$$

Second trial, use first trial values,

$$\mathbf{R} = \frac{\rho V D}{\mu} = (1.925)(11.81)(0.5054)/(13.61 \times 10^{-6})$$
$$\mathbf{R} = 846{,}200 > 4{,}000\ \therefore \text{flow is turbulent}$$

From Fig. 24 and the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{1.682 \times 10^{-3}}{3.7} + \frac{2.51}{844{,}200\sqrt{0.023}}\right)$$
$$f = 0.02263$$
$$V = \frac{113.44}{\sqrt{2.0 + 4{,}032 \times 0.02263}} = 11.75 \qquad \text{(close check)}$$

From Sec. 4 steam tables at 104°F, $p_v = 1.070$ lbf/in², the maximum height

$$z_B - z_A = \frac{p_A - p_v}{\rho g} - (1 + \Sigma K_{AB})\frac{V^2}{2g}$$
$$z_B - z_A = \frac{144(14.70 - 1.070)}{1.925 \times 32.17} - (1 + 1 + 227.9 \times 0.02262)\frac{(11.75)^2}{2 \times 32.17} = 16.58\ \text{ft}\ (5.053\ \text{m})$$

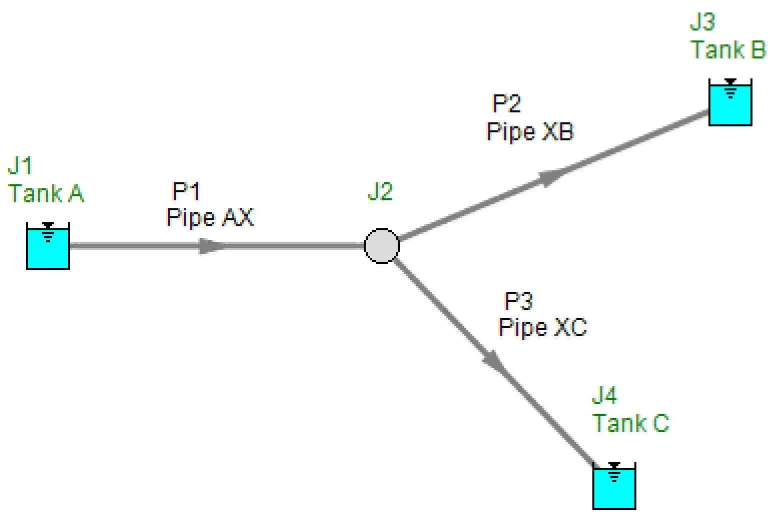
Note that if a ±10 percent error exists in calculation of pressure loss, maximum height should be limited to ≈ 15 ft (5 m).

ASME PIPELINE FLOWMETERS

Parameters Dimensional analysis of the flow of an incompressible fluid flowing in a pipe of diameter D, surface rough-

View Verification Case 44 Model

Verification Case 44



Verification Case 45

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify45.fth

REFERENCE: James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 168-169, Example 6.3

FLUID: Water at 25 deg. C

ASSUMPTIONS: Flow rate is 0.01 m3/sec (omitted in problem statement)

RESULTS:

Parameter	John & Haberman	AFT Fathom
Pressure drop in cast iron pipe (kPa)	21.81	21.90
Prssure drop in pipe 2 (kPa)	13.46	13.63

DISCUSSION:

Results disagree slightly because John & Haberman read friction factors from a chart, which is less accurate than AFT Fathom's correlation based method.

List of All Verification Models

Verification Case 45 Problem Statement

Verification Case 45

James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 168-169, Example 6.3

John and Haberman Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Note that for large values of relative roughness, the dividing line between he two regions occurs at a lower Reynolds number than for small values of ·elative roughness. This is because the thickness of the laminar sublayer lecreases as the Reynolds number, and hence the turbulence of the flow, ncreases. So, at low Reynolds numbers, with a relatively thick laminar ublayer, only very rough pipes will have protuberances that will project hrough the sublayer.

In order to determine a value of the friction factor f from the Moody liagram, a knowledge of relative roughness is necessary. Typical values of roughness for various types of pipe are shown in Table 6.1.

Table 6.1.

Type	ϵ (mm)
Glass	Smooth
Asphalted cast iron	0.12
Galvanized iron	0.15
Cast iron	0.26
Wood stave	0.18–0.90
Concrete	0.30–3.0
Riveted steel	1.0–10
Drawn tubing	0.0015

It should be noted that after pipes have been in service for a time, deposits build up on the pipe walls which may substantially increase the preceding values of ϵ.

For noncircular pipes, good correlation for fully developed turbulent pipe flow is obtained by using Figure 6.18, with Re_{D_h} substituted for Re_D and ϵ/D_h for ϵ/D. The values of critical Reynolds number from Section 6.1 can also be used, with Re_{D_h} in place of Re_D.

For laminar flow in noncircular pipes, however, an appreciable error can result from using the value of f for circular pipes, as was shown for flow through the rectangular channel worked out previously in this section. The magnitude of the error is dependent on the shape of the cross section.

The following example illustrates the use of the Moody diagram.

EXAMPLE 6.3. Ten liters per second of water at 25°C is to flow through a horizontal pipe 100 m long. Compare the pressure drop in a cast iron pipe of circular cross section of 10 cm diameter with the pressure drop in a wood pipe ($\epsilon = 0.30$ mm) of square cross section 10 cm on a side.

Solution: The water velocity in the circular pipe, $\bar{V}$, is given by

$$\bar{V} = \frac{10 \times 10^{-3}\ \text{m}^3/\text{s}}{(\pi/4)(0.10)^2\ \text{m}^2} = 1.273\ \text{m/s}$$

At 25°C, $\nu = 0.898 \times 10^{-6}\ \text{m}^2/\text{s}$, so that

$$\text{Re}_D = \frac{(1.273\ \text{m/s})(0.10)\ \text{m}}{0.898 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.418 \times 10^5$$

This is well over the critical value of 2200 given in Section 6.1, so we are in the region of fully developed turbulent pipe flow. In order to find f from Figure 6.18, we need relative roughness:

$$\frac{\epsilon}{D} = \frac{0.30 \times 10^{-3}\ \text{m}}{0.10\ \text{m}} = 0.0030 \quad \text{for the circular pipe}$$

With relative roughness and Reynolds number, we now go to the Moody diagram to find $f = 0.027$. From (6.5),

$$p_2 - p_1 = -\frac{1}{2}\rho\bar{V}^2\frac{fL}{D}$$

$$= -\frac{1}{2}(997\ \text{kg/m}^3)(1.273)^2\ \text{m}^2/\text{s}^2\left(\frac{0.027 \times 100\ \text{m}}{0.10\ \text{m}}\right)$$

$$= \underline{-21.81\ \text{kPa}}$$

For the square wood pipe,

$$\bar{V} = \frac{10 \times 10^{-3}\ \text{m}^3/\text{s}}{(0.10)^2\ \text{m}^2} = 1.0\ \text{m/s}$$

$$D_h = 10\ \text{cm} \qquad \text{(see Section 6.2)}$$

$$\text{Re}_{D_h} = \frac{(1.0\ \text{m/s})(0.10\ \text{m})}{0.898 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.114 \times 10^5$$

$$\frac{\epsilon}{D_h} = \frac{0.30 \times 10^{-3}\ \text{m}}{0.10\ \text{m}} = 0.003$$

so that, from Figure 6.18,

$$f = 0.027$$

$$p_2 - p_1 = -\frac{1}{2}\rho\bar{V}^2\frac{fL}{D_h}$$

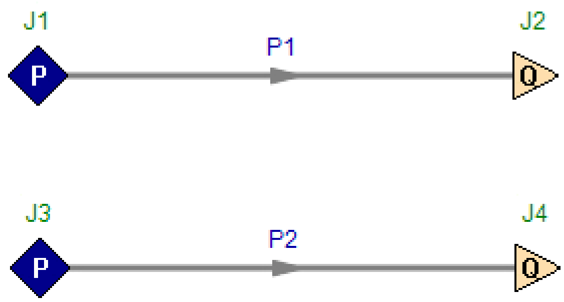
$$= -\frac{1}{2}(997\ \text{kg/m}^3)(1.0)^2\ \text{m}^2/\text{s}^2\ \frac{0.027 \times 100\ \text{m}}{0.10\ \text{m}}$$

$$= \underline{-13.46\ \text{kPa}}$$

The discussion of the friction factor has dealt solely with fully developed flow. At a pipe inlet, however, the flow is not fully developed. As can be seen in Figure 6.9, the velocity gradient at the wall and resultant shear stress are actually greater near the inlet. The problem of trying to predict an equivalent friction coefficient for flow near an inlet and its variation with axial distance is extremely complex. Fortunately, it has been shown experimentally that the local friction factor attains its fully developed value roughly 10 diameters from the inlet for turbulent flow; a somewhat greater length is required for laminar flow. It can be seen, then, that any inlet effect can be neglected

View Verification Case 45 Model

Verification Case 45



Verification Case 46

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify46.fth

REFERENCE: James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 176-178, Example 6.4

FLUID: Water at 20 deg. C

ASSUMPTIONS: Use the K factors provided

RESULTS:

Parameter	John & Haberman	AFT Fathom
Pressure at discharge (kPa gage)	116	115.0

DISCUSSION:

The K factors used by John & Haberman differ from Crane's and those in the AFT Fathom library. The model therefore uses the same K factors as used in the problem solution. In the problem solution, the K factor for contraction area change is applied to the 7.5 cm pipe, while AFT Fathom applies it to the 15 cm pipe. It therefore needed to be multiplied by the area ratio squared, which is 16.

The pipe lengths between fittings were not specified, and do not affect the results. The AFT Fathom assumed pipe lengths, with the total pipe length adding up to 130 m.

Results disagree slightly because John & Haberman read friction factors from a chart, which is less accurate than AFT Fathom's correlation based method. The friction factor for the 7.5 cm pipe was determined to be 0.027 by John & Haberman, and AFT Fathom calculated it as 0.028 based on the Colebrook-White method.

The resulting 115.0 kPa (g) pressure is obtained in AFT Fathom by looking at the exit pressure from the Assigned Flow junction at J8.

List of All Verification Models

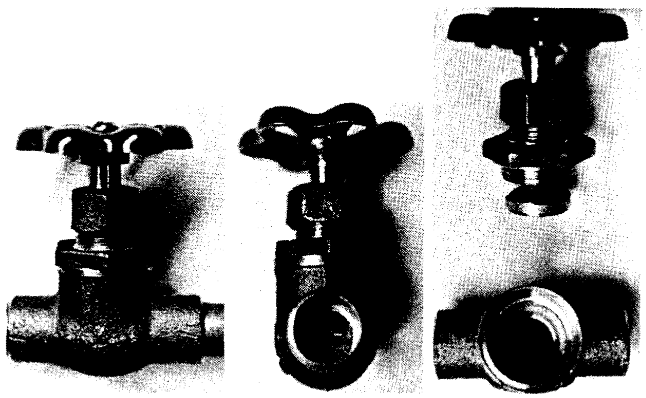
Verification Case 46 Problem Statement

Verification Case 46

James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 176-178, Example 6.4

John and Haberman Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.



(a) (b) (c)

Figure 6.30. Globe valve.

6.5 PIPING SYSTEMS

We are now in a position to work through several examples of complete piping systems. Three basic types of problems are encountered. First, for a given piping system and flow rate, it might be required to find the pressure drop. Second, for a given system and pressure drop, the flow rate might be required. Third, for a given flow and pressure drop, it might be required to design the system, that is, to determine the necessary pipe diameter. In the following material, an example of each type of problem is presented.

EXAMPLE 6.4. For the piping system shown in Figure 6.31 determine the pressure p_2. There are 100 m of 15-cm-I.D. cast iron pipe and 30 m of 7.5-cm-I.D. cast iron pipe. Water flow rate is 0.01 m³/s, the water temperature is 20°C, and the gage pressure at 1 is 250 kPa.

Solution: First write the modified Bernoulli equation (6.7) between 1 and 2, including minor losses:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \left(\frac{fL}{D}\frac{V^2}{2}\right)_{\substack{15\text{-cm}\\ \text{pipe}}} + \left(\sum K\frac{V^2}{2}\right)_{\substack{15\text{-cm}\\ \text{pipe}}} + \left(\frac{fL}{D}\frac{V^2}{2}\right)_{\substack{7.5\text{-cm}\\ \text{pipe}}}$$

$$+ \left(\sum K\frac{V^2}{2}\right)_{\substack{7.5\text{-cm}\\ \text{pipe}}} + \frac{V_2^2}{2} + gz_2 + \frac{p_2}{\rho}$$

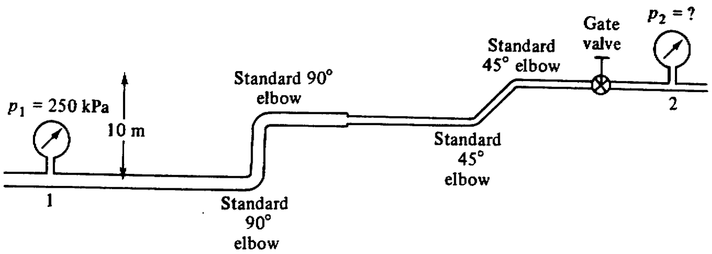


Figure 6.31

The velocity in the 15-cm pipe is

$$V_{15} = \frac{Q}{A} = \frac{0.01\ \text{m}^3/\text{s}}{(\pi/4)(0.15)^2\ \text{m}^2} = 0.5659\ \text{m/s}$$

and in the 7.5-cm pipe it is

$$V_{7.5} = \frac{Q}{A}\frac{0.01}{(\pi/4)(0.075)^2\ \text{m}^2} = 2.264\ \text{m/s}$$

At 20°C, for water, $\nu = 1.00 \times 10^{-6}\ \text{m}^2/\text{s}$, so that

$$\text{Re}_{15} = \frac{(0.5659\ \text{m/s})(0.15\ \text{m})}{1.00 \times 10^{-6}\ \text{m}^2/\text{s}} = 8.489 \times 10^4$$

$$\text{Re}_{7.5} = \frac{(2.264\ \text{m/s})(0.075\ \text{m})}{1.00 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.698 \times 10^5$$

Also,

$$\left(\frac{\epsilon}{D}\right)_{15} = \frac{0.26 \times 10^{-3}\ \text{m}}{0.15\ \text{m}} = 0.00173$$

$$\left(\frac{\epsilon}{D}\right)_{7.5} = \frac{0.26 \times 10^{-3}\ \text{m}}{0.075\ \text{m}} = 0.00346$$

From the Moody diagram, we obtain

$$f_{15} = 0.025$$
$$f_{7.5} = 0.027$$

The minor losses in the 15-cm pipe are given by

$$(\Sigma K)_{15} = \underset{\text{standard elbow}}{0.75} + \underset{\text{standard elbow}}{0.75} = 1.50$$

For the 7.5-cm pipe,

$$(\Sigma K)_{7.5} = \underset{\text{contraction}}{0.33} + \underset{\text{elbow}}{0.35} + \underset{\text{elbow}}{0.35} + \underset{\text{gate valve}}{0.20} = 1.23$$

Substituting into the Bernoulli equation, we have

$$\frac{p_2 - p_1}{\rho} = g(z_1 - z_2) + \frac{V_{15}^2}{2}\left(1 - \frac{fL}{D} - -\Sigma K\right)_{15} - \frac{V_{7.5}^2}{2}\left(1 + \frac{fL}{D} + \Sigma K\right)_{7.5}$$

$$= 9.81 \text{ m/s}^2(-10 \text{ m}) + \left(\frac{0.5659^2}{2} \text{ m}^2/\text{s}^2\right)\left(1 - \frac{0.025 \times 100}{0.15} - 1.50\right)$$

$$- \left(\frac{2.264^2}{2} \text{ m}^2/\text{s}^2\right)\left(1 + \frac{0.027 \times 30}{0.075} + 1.23\right)$$

$$= -98.1 \text{ m}^2/\text{s}^2 + (0.1601 \text{ m}^2/\text{s}^2)(-17.17) - (2.563 \text{ m}^2/\text{s}^2)(13.03)$$

$$= -98.1 \text{ m}^2/\text{s}^2 - 2.749 \text{ m}^2/\text{s}^2 - 33.40 \text{ m}^2/\text{s}^2$$

$$= -134.2 \text{ m}^2/\text{s}^2$$

or

$$p_2 - p_1 = -(998.3 \text{ kg/m}^3)(134.2 \text{ m}^2/\text{s}^2)$$
$$= -134 \text{ kPa}$$

or

$$p_2 = 250 - 134$$
$$= \underline{116 \text{ kPa}} \text{ (gage)}$$

EXAMPLE 6.5. For the piping system shown in Figure 6.32, determine the water flow rate. The water temperature is 25°C, with all pipe asphalted cast iron. There are 250 m of 10-cm-I.D. pipe.

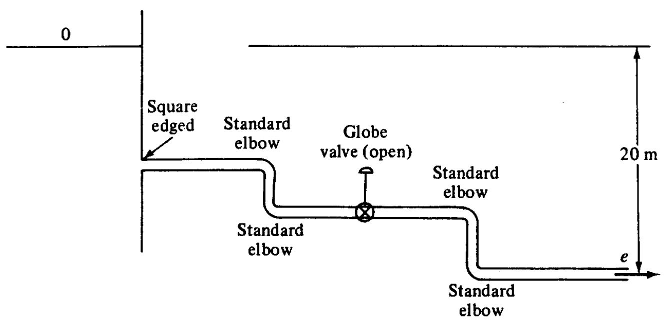


Figure 6.32

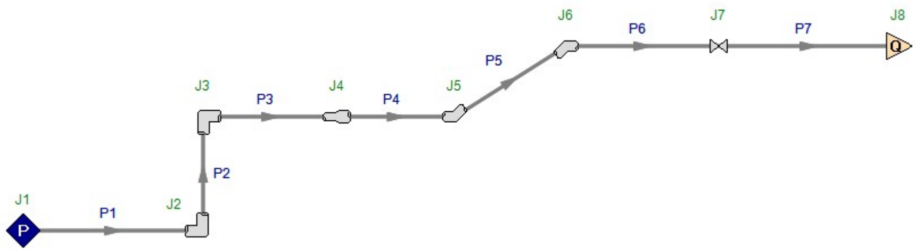
Solution: Write the modified Bernoulli equation between the reservoir surface and pipe outlet:

$$\frac{p_0}{\rho} + gz_0 + \frac{V_0^2}{2} = \frac{fL}{D}\frac{V^2}{2} + \sum K\frac{V^2}{2} + \frac{p_e}{\rho} + gz_e + \frac{V_e^2}{2}$$

where $p_0 = p_e$ = atmospheric pressure; $z_0 - z_e = 20$ m; and $V_0 = 0$. For this case, with flow velocity the unknown, Reynolds numbers cannot be directly

View Verification Case 46 Model

Verification Case 46



Verification Case 47

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify47.fth

REFERENCE: James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 178-179, Example 6.5

FLUID: Water at 20 deg. C (incorrectly stated as 25 deg. C)

ASSUMPTIONS: Use the K factors provided

RESULTS:

Parameter	John & Haberman	AFT Fathom
Flow rate through pipe line (liter/sec)	13.55*	19.33
Flow rate through pipe line (liter/sec)	19.17**	19.33

(*) Calculation error in text explained below

(**) Corrected flow rate calculation as explained below

DISCUSSION:

The K factors used by John & Haberman differ from Crane's and those in the AFT Fathom library. The model therefore uses the same K factors as used in the problem solution. The pipe lengths between fittings were not specified, and do not affect the results. The AFT Fathom assumed pipe lengths, with the total pipe length adding up to 250 m.

The problem statement is for water at 25 deg. C, but when the Reynolds number is calculated on page 179 the kinematic viscosity is that at 20 deg. C. Therefore the AFT Fathom model uses 20 deg. C water.

The results disagree significantly. A review of John & Haberman page 179 shows a calculation error. The text says the following:

$$\frac{V_e^2}{2} = \frac{9.81(20)}{55 + 9.9 + 1}$$

Or

$$V_e = 1.725\ m/s$$

which is incorrect. Solving the above equation by a hand calculator shows that

$$V_e = 2.44\ m/s$$

If the incorrect velocity is used the final flow rate in John & Haberman is significantly different that Fathom.

$$Q = AV$$

$$= \left[\frac{\pi}{4}(0.10)^2\ m^2\right] 1.725\ \mathrm{m/s}$$

$$= 0.01355\ m^3/s$$

$$= 13.55\ l/s$$

When using the corrected velocity the final flow rate in John & Haberman is much closer to the Fathom result.

$$Q = AV$$

$$= \left[\frac{\pi}{4}(0.10)^2\ m^2\right] 2.44\ m/s$$

$$= 0.01917\ m^3/s$$

$$= 19.17\ l/s$$

Using the correct velocity results in the correct flow rate, which agrees closely with AFT Fathom.

List of All Verification Models

Verification Case 47 Problem Statement

Verification Case 47

James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 178-179, Example 6.5

John and Haberman Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

178 *Pipe Flow*

Substituting into the Bernoulli equation, we have

$$\frac{p_2 - p_1}{\rho} = g(z_1 - z_2) + \frac{V_{15}^2}{2}\left(1 - \frac{fL}{D} - - \Sigma K\right)_{15} - \frac{V_{7.5}^2}{2}\left(1 + \frac{fL}{D} + \Sigma K\right)_{7.5}$$

$$= 9.81 \text{ m/s}^2(-10 \text{ m}) + \left(\frac{0.5659^2}{2} \text{ m}^2/\text{s}^2\right)\left(1 - \frac{0.025 \times 100}{0.15} - 1.50\right)$$

$$- \left(\frac{2.264^2}{2} \text{ m}^2/\text{s}^2\right)\left(1 + \frac{0.027 \times 30}{0.075} + 1.23\right)$$

$$= -98.1 \text{ m}^2/\text{s}^2 + (0.1601 \text{ m}^2/\text{s}^2)(-17.17) - (2.563 \text{ m}^2/\text{s}^2)(13.03)$$

$$= -98.1 \text{ m}^2/\text{s}^2 - 2.749 \text{ m}^2/\text{s}^2 - 33.40 \text{ m}^2/\text{s}^2$$

$$= -134.2 \text{ m}^2/\text{s}^2$$

or

$$p_2 - p_1 = -(998.3 \text{ kg/m}^3)(134.2 \text{ m}^2/\text{s}^2) = -134 \text{ kPa}$$

or

$$p_2 = 250 - 134 = \underline{116 \text{ kPa}} \text{ (gage)}$$

EXAMPLE 6.5. For the piping system shown in Figure 6.32, determine the water flow rate. The water temperature is 25°C, with all pipe asphalted cast iron. There are 250 m of 10-cm-I.D. pipe.

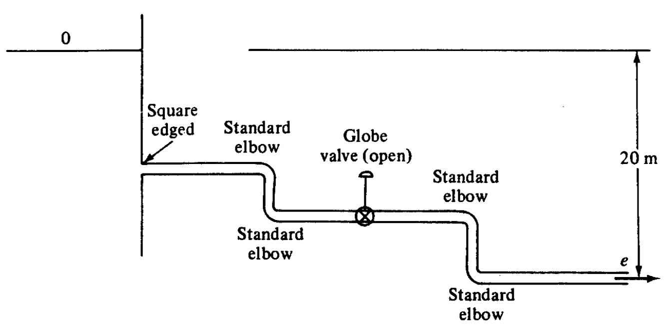


Figure 6.32

Solution: Write the modified Bernoulli equation between the reservoir surface and pipe outlet:

$$\frac{p_0}{\rho} + gz_0 + \frac{V_0^2}{2} = \frac{fL}{D}\frac{V^2}{2} + \sum K\frac{V^2}{2} + \frac{p_e}{\rho} + gz_e + \frac{V_e^2}{2}$$

where $p_0 = p_e$ = atmospheric pressure; $z_0 - z_e = 20$ m; and $V_0 = 0$. For this case, with flow velocity the unknown, Reynolds numbers cannot be directly determined; a trial-and-error solution is called for. As a first trial, assume $f = 0.02$.

$$(9.81\ \text{m/s}^2)(20\ \text{m}) = \left[\frac{0.02 \times 250}{0.10} + (\underset{\text{square-edged inlet}}{0.5} + \underset{\text{4 elbows}}{3.0} + \underset{\text{globe valve}}{6.4}) + 1\right]\frac{V_e^2}{2}$$

$$\frac{V_e^2}{2} = \frac{9.81(20)}{50 + 9.9 + 1}\ \text{m}^2/\text{s}^2$$

or

$$V_e = 1.795\ \text{m/s}$$

For this velocity,

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.795\ \text{m/s})(0.10\ \text{m})}{1.00 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.795 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{0.12 \times 10^{-3}\ \text{m}}{0.10\ \text{m}} = 1.2 \times 10^{-3}$$

From the Moody diagram, we obtain $f = 0.022$. Since this value does not agree with our initial trial, we shall now make a second trial, starting with $f = 0.022$.

$$\frac{fL}{D} = \frac{0.022 \times 250}{0.10} = 55$$

$$\frac{V_e^2}{2} = \frac{9.81(20)}{55 + 9.9 + 1}$$

or

$$V_e = 1.725\ \text{m/s}$$

For this velocity,

$$\text{Re} = \frac{VD}{\nu} = \frac{1.725 \times 0.10}{10^{-6}} = 1.725 \times 10^5$$

From the Moody diagram, we find $f = 0.022$. It can be seen that generally we are able to converge on the correct answer quite rapidly; no more than two trials are required, as a rule. The water flow rate is

$$\begin{aligned} Q &= AV \\ &= \left[\frac{\pi}{4}(0.10)^2\ \text{m}^2\right] 1.725\ \text{m/s} \\ &= 0.01355\ \text{m}^3/\text{s} \\ &= \underline{13.55\ \text{l/s}} \end{aligned}$$

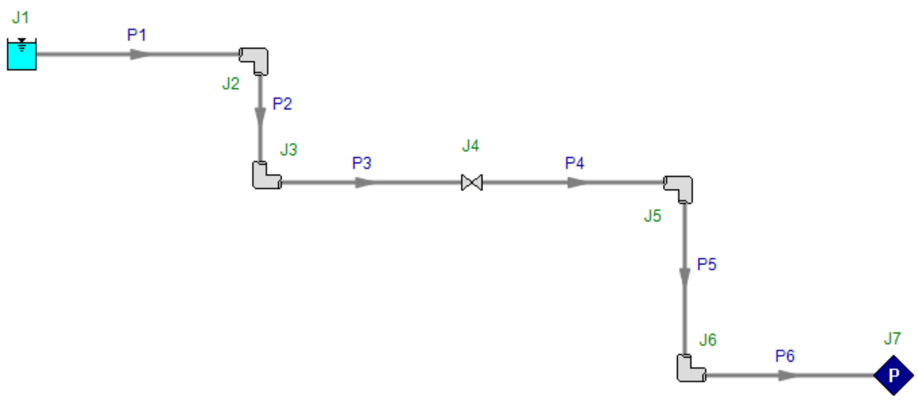
EXAMPLE 6.6. A pump is to be used to supply 5 liters per second of water from a reservoir to a point 400 m from the reservoir at the same level as the reservoir surface. Determine the minimum-size cast iron pipe required. The water temperature is 15°C; assume that minor losses can be neglected. (See Figure 6.33.) The pump supplies 50 kW of power to the water flow.

Solution: From (6.8) we can express the pump work in terms of reservoir surface and outlet conditions:

$$-\frac{1}{\dot{m}}\frac{dW_p}{dt} = \frac{p_e - p_0}{\rho} + g(z_e - z_0) + \frac{fL}{D}\frac{V^2}{2} + \frac{V_e^2}{2}$$

View Verification Case 47 Model

Verification Case 47



Verification Case 48

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify48.fth

REFERENCE: James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 184-186, Example 6.8

FLUID: Water at 20 deg. C

ASSUMPTIONS: All pipes at same elevation. Common exit pressure is 1 atm.

RESULTS:

Parameter	John & Haberman	AFT Fathom
Flow rate through pipe 1 (m3/sec)	0.095	0.0943
Flow rate through pipe 2 (m3/sec)	0.105	0.1057

DISCUSSION:

The problem statement does not have any pressure information, because it is not required if the only purpose is to calculate flow rates. However, AFT Fathom requires at least one pressure boundary because AFT Fathom offers pressure information in the output for all systems. Therefore, a pressure of 1 atm was assumed for the common junction at the exit of the system.

List of All Verification Models

Verification Case 48 Problem Statement

Verification Case 48

James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Page 184-186, Example 6.8

John and Haberman Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

184 *Pipe Flow*

or, with $f = 0.02$,

$$\frac{\bar{V}^2}{D_h} = \frac{5.23(2)}{1.06(0.02)} = 493.7 \text{ m/s}^2$$

Now $Q = \bar{V}A = \bar{V}w(0.08)$.

$$\bar{V} = \frac{5}{60(0.08)w} = \frac{1.042}{w}$$

and

$$D_h = \frac{2dw}{(d + w)} = 2\frac{0.08w}{0.08 + w}$$

Hence

$$\frac{\bar{V}^2}{D_h} = \frac{(1.042)^2(w + 0.08)}{w^3 0.16} = 6.782\frac{w + 0.08}{w^3}$$

Thus

$$493.7 = 6.782\frac{w + 0.08}{w^3} \quad \text{or} \quad 72.75w^3 = w + 0.08$$

Using a trial-and-error solution for w,

w	$72.75w^3$	$w + 0.08$
0.1	0.073	0.18
0.15	0.246	0.23
0.14	0.200	0.22
0.145	0.222	0.225
0.146	0.226	0.226

we obtain w = <u>146 mm.</u>

EXAMPLE 6.8. Twelve cubic meters per minute of water flow into a horizontal pipe network consisting of two cast iron pipe branches (Figure 6.37). The first branch is 100 m long with a 200-mm pipe diameter, and the second branch is 200 m long with a 250-mm pipe diameter and a half-open gate valve. Determine the flow through each pipe and the pressure drop across the branches. Water temperature is 20°C.

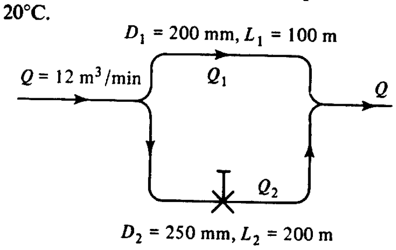


Figure 6.37

Solution: From (6.7) and Section 6.5, the pressure drop in each branch is given by:

$$\Delta p = f\frac{L}{D}\frac{V^2}{2} + \sum K\frac{V^2}{2}$$

The velocity in each branch can be expressed as $V = Q/(\pi/4)D^2$, so that, for each branch:

$$\Delta p = \frac{16}{\pi^2}\left(\frac{fL}{D} + \sum K\right)\frac{Q^2}{D^4}$$

But $\Delta p_1 = \Delta p_2$ and $\sum K_1 = 0$; therefore, the ratio of volume flow rates is

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^2\sqrt{\frac{f_2L_2/D_2 + \sum K_2}{f_1L_1/D_1}}$$

As a first trial, assume $f_1 = f_2 = 0.020$. From Table 6.3, $K_2 = 4.5$ for a half-open gate valve; hence

$$\frac{f_2L_2}{D_2} + \sum K_2 = 0.020\frac{200}{0.250} + 4.5 = 20.5$$

and

$$\frac{f_1L_1}{D_1} = 0.020\frac{100}{0.2} = 10$$

Therefore,

$$\frac{Q_1}{Q_2} = \left(\frac{0.200}{0.250}\right)^2\sqrt{\frac{20.5}{10}} = 0.916$$

$$\text{Total flow } Q = Q_1 + Q_2, \quad \text{or} \quad \frac{Q}{Q_2} = \frac{Q_1}{Q_2} + 1 = 1.916$$

and

$$Q_2 = \frac{\frac{12}{60}}{1.916} = 0.1044 \text{ m}^3/\text{s}$$

with $Q_1 = 0.2 - 0.1044 = 0.0956 \text{ m}^3/\text{s}$

Then

$$V_2 = \frac{0.1044}{(\pi/4)((0.250)^2} = 2.13 \text{ m/s}$$

From Appendix A, $\nu = 1.00 \times 10^{-6}$ m²/s at 20°C for water, so

$$\text{Re}_2 = \frac{V_2D_2}{\nu} = \frac{(2.13 \text{ m/s})(0.250 \text{ m})}{1.00 \times 10^{-6} \text{ m}^2/\text{s}} = 5.32 \times 10^5$$

From Table 6.1, $\epsilon = 0.26$ mm for cast iron pipe; hence,

$$\frac{\epsilon}{D_2} = \frac{0.00026}{0.250} = 0.001$$

Hence, from Figure 6.18, we find $f_2 = 0.02$, which was our initial assumption.

Now,

$$\Delta p_2 = \frac{\rho V_2^2}{2}\left(\frac{f_2L_2}{D} + K_2\right) = \frac{(998.3 \text{ kg/m}^3)(2.13^2 \text{ m}^2/\text{s}^2)}{2}20.5$$

$$= 46.5 \text{ kPa}$$

where $\rho = 998.3$ kg/m³ at 20°C. Further,

$$V_1 = \frac{0.0956 \text{ m}^3/\text{s}}{(\pi/4)(0.200)^2 \text{ m}^2} = 3.04 \text{ m/s}$$

and

$$\text{Re}_1 = \frac{(3.04 \text{ m/s})(0.2 \text{ m})}{1 \times 10^{-6} \text{ m}^2/\text{s}} = 6.09 \times 10^5$$

$$\frac{\epsilon}{D_1} = \frac{0.00026 \text{ m}}{0.200 \text{ m}} = = 0.0013$$

From Figure 6.17, $f_1 = 0.0208$, and hence

$$\Delta p_1 = \frac{\rho V_1^2}{2}\frac{fL_1}{D_1} = \frac{(998.3 \text{ kg/m}^3)(3.04^2 \text{ m}^2/\text{s}^2)}{2}\frac{0.0208 \text{ (100 m)}}{0.2 \text{ m}}$$

$$= 48.0 \text{ kPa}$$

as compared to $\Delta p_2 = 46.5$ kPa.

For our second trial, we increase Q_2 and decrease Q_1 to balance the pressure drops. Let $Q_2 = 0.105$ m³/s and $Q_1 = 0.095$ m³/s. Then

$$V_2 = \frac{0.105 \text{ m}^3/\text{s}}{(\pi/4)(0.25)^2} = 2.14 \text{ m/s}$$

$$\text{Re}_2 = \frac{(2.14 \text{ m/s})(0.25 \text{ m})}{1 \times 10^{-6} \text{ m}^2/\text{s}} = 5.34 \times 10^5$$

Therefore,

$$f_2 = 0.02 \quad \text{and} \quad \Delta p_2 = 47.0 \text{ kPa}$$

$$V_1 = \frac{0.095 \text{ m}^3/\text{s}}{(\pi/4)(0.200)^2 \text{ m}^2} = 3.02 \text{ m/s}$$

$$\text{Re}_1 = 6.05 \times 10^5$$

$$f_1 = 0.0208, \qquad \Delta p_1 = 47.3 \text{ kPa}$$

Thus the solution is

$$Q_1 = 0.095 \text{ m}^3/\text{s}, \qquad Q_2 = 0.105 \text{ m}^3/\text{s}$$

with a pressure drop Δp of 47 kPa. Note that, although branch 2 has a larger diameter than branch 1, it carries less flow due to the presence of a valve and due to its greater length.

PROBLEMS

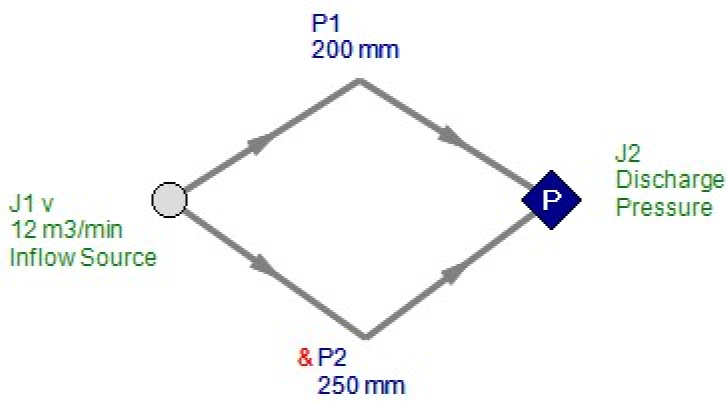
6.1. Water at 10°C flows steadily through a 5-cm-diameter pipe. The pressure drop in a 3-m length of pipe is found to be 750 Pa. Compute the wall shear stress.

6.2. Air at 20°C flows steadily through a rectangular duct, 5 cm by 10 cm with a velocity of 2 m/s. Is the flow laminar or turbulent?

6.3. For Problem 6.2, compute the pressure drop per meter of duct, with $f = 0.02$, $\epsilon = 0.10$ mm.

View Verification Case 48 Model

Verification Case 48



Verification Case 49

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify49.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 153-154, Example 5.2

FLUID: Water

ASSUMPTIONS: Water assumed to be 20 deg. C

RESULTS:

Parameter	Janna	AFT Fathom
Pressure drop in pipe (kPa)	347.1	347.1

DISCUSSION:

The friction factor for this problem was defined explicitly based on the problem statement rather than using the iterative Colebrook-White equation.

List of All Verification Models

Verification Case 49 Problem Statement

Verification Case 49

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 153-154, Example 5.2

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

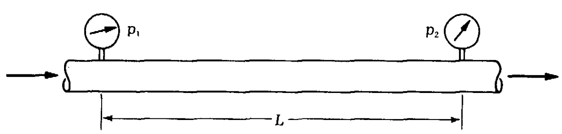


Figure 5.10. Flow in a constant-diameter pipe.

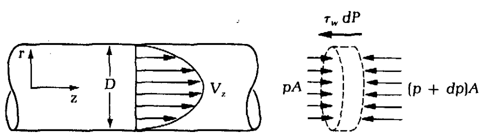


Figure 5.11. Control volume of a system: flow in a duct.

Integrating this expression from points 1 to 2 a distance L apart in the conduit yields

$$p_2 - p_1 = -\frac{\rho V^2}{2g_c}\frac{fL}{D} \tag{5.7}$$

Equation 5.7 gives the pressure drop in the duct due to friction. With this equation, the Bernoulli equation can now be rewritten as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D}\frac{V^2}{2g} \tag{5.8}$$

which now takes wall friction into account. The summation indicates that if several pipes of different diameter are connected in series, the total friction drop is due to the combined effect of them all. The frictional effect is manifested in either a heat loss from the fluid or a gain in internal energy of the fluid.

EXAMPLE 5.2

A 2-nominal pipe is inclined at an angle of 30° with the horizontal and conveys 0.001 m³/s of water uphill. Determine the pressure drop in the pipe if it is 70 m long (see Figure 5.12). Take the friction factor f to be 0.03.

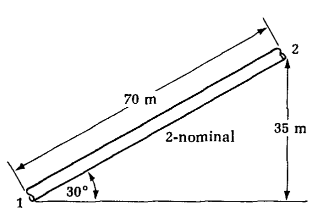


Figure 5.12. Sketch for Example 5.2.

SOLUTION

The Bernoulli equation with the friction term applies:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D}\frac{V^2}{2g}$$

or

$$p_1 - p_2 = \frac{\rho}{2g_c}(V_2^2 - V_1^2) + \frac{\rho g}{g_c}(z_2 - z_1) + \frac{\rho V^2}{2g_c}\frac{fL}{D}$$

From continuity, $A_1V_1 = A_2V_2$ or $V_1 = V_2$. If z_1 is our reference point, then $z_1 = 0$ and $z_2 = 35$ m. From Appendix Table C-1, the inside diameter of 2-nominal pipe is 5.252 cm for schedule 40. (Because a schedule was not specified in the problem statement, assume the standard.) The average velocity in the pipe is

$$V = \frac{Q}{A} = \frac{0.001}{0.002\,2} = 0.455 \text{ m/s}$$

where the area A for schedule 40, 2-nominal pipe is obtained also from Appendix Table C-1. By substitution into the Bernoulli equation, we obtain

$$p_1 - p_2 = 0 + 1\,000(9.8)(35 - 0) + \frac{1\,000(0.455)^2}{2}\frac{(0.03)(70)}{0.052\,52}$$

$$= 343\,000 + 4\,139 = 347\,139 \text{ Pa}$$

$$\underline{p_1 - p_2 = 347.1 \text{ kPa}}$$

5.5 / The Friction Factor and Pipe Roughness

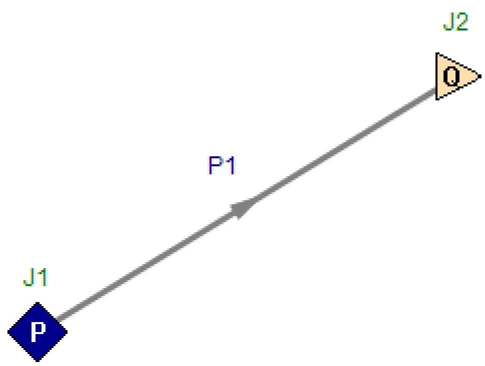
The preceding section introduced the concept of friction and a friction factor. In this section we will discuss this concept further for two flow regimes—laminar and turbulent flow. Because the friction factor is influenced by the fluid velocity, we begin with a velocity distribution for circular pipe flow and later extend the results to other cross sections.

Consider laminar flow in a circular pipe as shown in Figure 5.13. Cylindrical coordinates are chosen with z as the axial direction. A control volume is selected as shown, and in applying the momentum equation we obtain

$$\Sigma F_z = \iint_{CS} V_z \frac{\rho}{g_c} V_n \, dA$$

View Verification Case 49 Model

Verification Case 49



Verification Case 50

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify50.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 166-167, Example 5.3

FLUID: Water at 22 deg. C

ASSUMPTIONS: N/A

RESULTS:

Parameter	John & Haberman	AFT Fathom
Head loss in pipe (meters)	0.744	0.759
Pressure drop in pipe (kPa)	7.279	7.434

DISCUSSION:

Results disagree slightly because Janna reads friction factors from a chart, which is less accurate than AFT Fathom's correlation based method. The friction factor was determined to be 0.022 by Janna, and AFT Fathom calculated it as 0.02244 based on the Colebrook-White method, which is 2% higher.

List of All Verification Models

Verification Case 50 Problem Statement

Verification Case 50

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 166-167, Example 5.3

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

In simple piping systems there are six variables that enter the problem:

L = pipe length

D = pipe diameter or hydraulic diameter of conduit

$\nu = \dfrac{\mu g_c}{\rho}$ = kinematic viscosity of fluid

ϵ = wall roughness

Q = volume flow rate

$h_f = \dfrac{V^2}{2g}\dfrac{fL}{D}$ = head loss due to friction

Of these variables, L, ν, and ϵ are generally known. Thus three types of problems are commonly encountered:

1. Given L, ν, ϵ, Q, and D, find h_f.
2. Given L, ν, ϵ, D, and h_f, find Q.
3. Given L, ν, ϵ, h_f, and Q, find D.

The solution technique for each of these problems is illustrated by example.

EXAMPLE 5.3

Water at 0.02 m³/s flows through 350 m of 8-nominal cast iron pipe. Determine the head loss if the water temperature is 22°C.

SOLUTION

We know that $Q = 0.02$ m³/s and $L = 350$ m. From Appendix Table C-1, $D = 20.27$ cm for schedule 40 pipe and $A = 322.70$ cm². From Appendix Table A-4, $\nu = 0.954\,8 \times 10^{-3}/997.8 = 9.569 \times 10^{-7}$ m²/s. By definition,

$$V = \frac{Q}{A} = \frac{0.02}{0.032\,270} = 0.62 \text{ m/s}$$

so

$$\text{Re} = \frac{VD}{\nu} = \frac{0.62(0.202\,7)}{9.569 \times 10^{-7}} = 1.312 \times 10^5$$

Also, from Table 5.2,

$$\epsilon = 0.025 \text{ cm}$$

Thus

$$\frac{\epsilon}{D} = \frac{0.025}{20.27} = 0.001\,2$$

On the $\epsilon/D = 0.001$ line of Figure 5.15*a*, follow to the left from the turbulence zone until the Re $= 1.312 \times 10^5$ point is reached. At this intersection, read

$$f = 0.022$$

By substitution into the equation for head loss, we obtain

$$h_f = \frac{fL}{D}\frac{V^2}{2g} = \frac{0.022(350)}{0.2027}\frac{(0.62)^2}{2(9.81)}$$

$$\underline{h_f = 0.744 \text{ m of water}}$$

Further,

$$\Delta p = \frac{\rho g}{g_c} h_f = 997(9.81)(0.744) = 7\,279 \text{ N/m}^2$$

or

$$\underline{\Delta p = 7.279 \text{ kPa}}$$

The method and calculations in the preceding example are quite straightforward. In the following example, Q is the unknown and therefore V and f are unknowns. A trial and error solution is required, but the technique is simple.

EXAMPLE 5.4

Benzene flows through a schedule 80 12-nominal wrought iron pipe. The pressure drop measured at points 1200 ft apart is 15 psi. Determine the volume flow rate through the pipe.

SOLUTION

We know that $L = 1200$ ft and $\Delta p = 15$ psi. From Appendix Table A-5, $\rho = 0.876(62.4) = 54.7$ lbm/ft^3 and $\mu g_c = 4.04 \times 10^{-4}$ lbm/ft-s for benzene. From Appendix Table C-1, $D = 0.9478$ ft for schedule 80 12-nominal pipe. From Table 5.2, $\epsilon = 0.00015$ ft. The head loss is

$$h_f = \frac{\Delta p}{\rho}\frac{g_c}{g} = \frac{15(144)}{54.7}\frac{32.2}{32.2} = 39.49 \text{ ft of benzene}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{0.00015}{0.9478} = 0.00016$$

View Verification Case 50 Model

Verification Case 50

Verification Case 51

View Model Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify51.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 167-168, Example 5.4

FLUID: Benzene at 77 deg. F

ASSUMPTIONS: N/A

RESULTS:

Parameter	Janna	AFT Fathom
Flow rate (ft3/sec)	8.51	8.49

DISCUSSION:

Although Janna only performed one iteration for friction factor while AFT Fathom performed further iterations, the results were still very similar due to the small difference in calculated friction factors.

List of All Verification Models

Verification Case 51 Problem Statement

Verification Case 51

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 167-168, Example 5.4

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

On the $\epsilon/D = 0.001$ line of Figure 5.15*a*, follow to the left from the turbulence zone until the Re $= 1.312 \times 10^5$ point is reached. At this intersection, read

$$f = 0.022$$

By substitution into the equation for head loss, we obtain

$$h_f = \frac{fL}{D}\frac{V^2}{2g} = \frac{0.022(350)}{0.2027}\frac{(0.62)^2}{2(9.81)}$$

$$h_f = 0.744 \text{ m of water}$$

Further,

$$\Delta p = \frac{\rho g}{g_c} h_f = 997(9.81)(0.744) = 7\,279 \text{ N/m}^2$$

or

$$\Delta p = 7.279 \text{ kPa}$$

The method and calculations in the preceding example are quite straightforward. In the following example, Q is the unknown and therefore V and f are unknowns. A trial and error solution is required, but the technique is simple.

EXAMPLE 5.4

Benzene flows through a schedule 80 12-nominal wrought iron pipe. The pressure drop measured at points 1200 ft apart is 15 psi. Determine the volume flow rate through the pipe.

SOLUTION

We know that $L = 1200$ ft and $\Delta p = 15$ psi. From Appendix Table A-5, $\rho = 0.876(62.4) = 54.7$ lbm/ft^3 and $\mu g_c = 4.04 \times 10^{-4}$ lbm/ft-s for benzene. From Appendix Table C-1, $D = 0.9478$ ft for schedule 80 12-nominal pipe. From Table 5.2, $\epsilon = 0.00015$ ft. The head loss is

$$h_f = \frac{\Delta p}{\rho}\frac{g_c}{g} = \frac{15(144)}{54.7}\frac{32.2}{32.2} = 39.49 \text{ ft of benzene}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{0.00015}{0.9478} = 0.00016$$

Using Figure 5.15*a* (or *b*), as a first trial estimate the friction factor for this flow to be that for completely turbulent flow; thus $f = 0.0138$. By definition,

$$h_f = \frac{fL}{D}\frac{V^2}{2g}$$

or

$$V = \sqrt{\frac{2gh_fD}{fL}} = \sqrt{\frac{2(32.2)(39.49)(0.9478)}{0.0138(1200)}} = 12.06 \text{ ft/s}$$

Also

$$\text{Re} = \frac{\rho VD}{\mu g_c} = \frac{54.7(12.06)(0.9478)}{4.04 \times 10^{-4}} = 1.55 \times 10^6$$

At Re = 1.55×10^6 and $\epsilon/D = 0.00016$, we find (from Figure 5.15*a*),

$$f = 0.0139$$

which is close enough to our assumed value. So

$$V = 12.06 \text{ ft/s}$$

and

$$Q = AV = \frac{\pi}{4}(0.9478)^2(12.06)$$

$$\underline{Q = 8.51 \text{ ft}^3/\text{s}}$$

If the value of f determined did not check with the assumed value, then for our second trial we would begin with $f = 0.0139$. The method converges very rapidly. Seldom are more than two trials necessary.

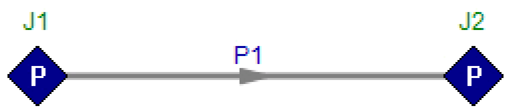
In the next example diameter is unknown. Therefore f and V are also unknown and again a trial and error solution is required. It is convenient to develop equations to streamline the calculations before proceeding, however. By definition,

$$h_f = \frac{fL}{D}\frac{V^2}{2g}$$

Combining with

View Verification Case 51 Model

Verification Case 51



Verification Case 52

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify52.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 176-177, Example 5.6

FLUID: Turpentine

ASSUMPTIONS: Use Janna K factors

RESULTS:

Parameter	Janna	AFT Fathom
Static Pressure difference (kPa)	3.012	2.938

DISCUSSION:

The various pipe lengths and intermediate junction elevations are not specified by Janna, so values were assigned to the AFT Fathom model such that the total lengths of 60 m and 22 m resulted, and the end-point elevations were correct.

Results disagree slightly because Janna reads friction factors from a chart, which is less accurate than AFT Fathom's correlation based method.

The pressure drop in AFT Fathom can be obtained by subtracting the exit pressure from the inlet in the Output window junction results table, or in the Junction Deltas table at the top of the Output window. The junction delta was created in the Output Control window.

List of All Verification Models

Verification Case 52 Problem Statement

Verification Case 52

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 176-177, Example 5.6

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

EXAMPLE 5.6

In a processing plant, turpentine is piped from tanks to cans that are to be sealed and sold to retail outlets. A portion of the pipeline is sketched in Figure 5.22. There are 60 m of 12-nominal pipe and 22 m of 8-nominal pipe. All elbows are standard and flanged, and the line is made of schedule 80 wrought iron pipe. Determine the pressure drop $p_1 - p_2$ if the volume rate of flow is 0.05 m^3/s.

SOLUTION

First we write the modified Bernoulli equation from section 1 to 2:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \left(\frac{fL}{D} + \Sigma K\right)\frac{V^2}{2g}\bigg|_{\text{12-pipe}} + \left(\frac{fL}{D} + \Sigma K\right)\frac{V^2}{2g}\bigg|_{\text{8-pipe}}$$

From Appendix Table C-1 (for 12-nominal and 8-nominal pipe):

$$D_{12} = 28.89 \text{ cm} = 0.289 \text{ m} \qquad A_{12} = 0.065\,550 \text{ m}^2$$
$$D_8 = 0.194 \text{ m} \qquad A_8 = 0.029\,470 \text{ m}^2$$

Therefore

$$V_{12} = \frac{Q}{A_{12}} = \frac{0.05}{0.065\,550} = 0.762 \text{ m/s}$$
$$V_8 = \frac{Q}{A_8} = \frac{0.05}{0.029\,470} = 1.697 \text{ m/s}$$

From Appendix Table A-5 for turpentine, $\rho = 870$ kg/m^3 and $\mu = 1.375 \times 10^{-3}$ N-s/m^2. Thus

$$Re_{12} = \frac{\rho V D}{\mu g_c} = \frac{870(0.762)(0.289)}{1.375 \times 10^{-3}} = 1.39 \times 10^5$$
$$Re_8 = \frac{870(1.697)(0.194)}{1.375 \times 10^{-3}} = 2.08 \times 10^5$$

From Table 5.2 for wrought iron, $\epsilon = 0.004\,6$ cm; we find

$$\frac{\epsilon}{D_{12}} = \frac{0.004\,6}{28.89} = 0.000\,16$$
$$\frac{\epsilon}{D_8} = \frac{0.004\,6}{19.37} = 0.000\,24$$

From the Moody diagram,

$$f_{12} = 0.018$$
$$f_8 = 0.017\,5$$

The minor losses in the 12-pipe are from one gate valve and four standard elbows:

$$\Sigma K|_{12} = 0.15 + 4(0.31) = 1.39$$

The minor losses in the 8-pipe area are from the contraction ($D_8^2/D_{12}^2 = 0.45$) and two 45° elbows:

$$\Sigma K|_8 = 0.29 + 2(0.17) = 0.63$$

After rearranging the Bernoulli equation and substituting, we obtain

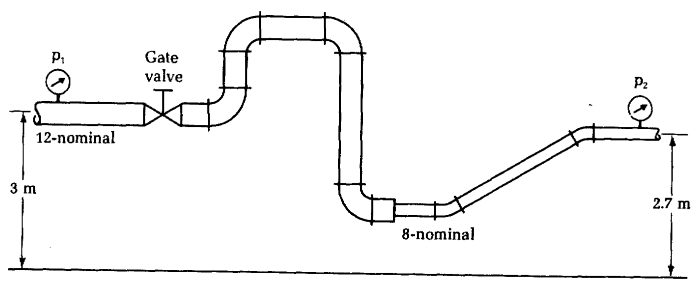
$$\begin{aligned}\frac{p_1 - p_2}{\rho}\frac{g_c}{g} &= \frac{(1.697)^2}{2(9.81)} - \frac{(0.762)^2}{2(9.81)} + (2.7 - 3)\\ &+\left[\frac{0.018(60)}{0.289} + 1.39\right]\frac{(0.762)^2}{2(9.81)}\\ &+\left[\frac{0.017\,5(22)}{0.194} + 0.63\right]\frac{(1.697)^2}{2(9.81)}\\ &= 0.146\,8 - 0.029\,6 + (-0.3) + 0.152 + 0.384\\ &= 0.353 \text{ m of turpentine}\end{aligned}$$

or

$$p_1 - p_2 = 0.353(870)(9.81) = 3\,012 \text{ N/m}^2$$

$$p_1 - p_2 = 3.012 \text{ kPa}$$

Figure 5.22. A pipeline for Example 5.6.

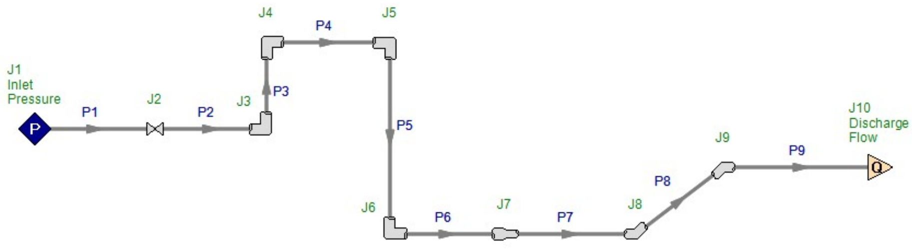


EXAMPLE 5.7

A water tank is fitted with a drain and outlet pipe as sketched in Figure 5.23. The system has 82 ft of cast iron pipe of 1½-nominal diameter. Determine the flow rate through the pipe. All fittings are threaded and regular.

View Verification Case 52 Model

Verification Case 52



Verification Case 53

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify53.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 177-179, Example 5.7

FLUID: Water

ASSUMPTIONS: Water at 77F

RESULTS:

Parameter	Janna	AFT Fathom
Flow rate (ft3/sec)	0.0918	0.0905

DISCUSSION:

The various pipe lengths and intermediate junction elevations are not specified by Janna, so values were assigned to the AFT Fathom model such that the total length of 82 feet resulted, and the endpoint elevations were correct.

Results disagree slightly because Janna reads friction factors from a chart, which is less accurate than AFT Fathom's correlation based method.

List of All Verification Models

Verification Case 53 Problem Statement

Verification Case 53

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 177-179, Example 5.7

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

The minor losses in the 12-pipe are from one gate valve and four standard elbows:

$$\Sigma K|_{12} = 0.15 + 4(0.31) = 1.39$$

The minor losses in the 8-pipe area are from the contraction ($D_8^2/D_{12}^2 = 0.45$) and two 45° elbows:

$$\Sigma K|_8 = 0.29 + 2(0.17) = 0.63$$

After rearranging the Bernoulli equation and substituting, we obtain

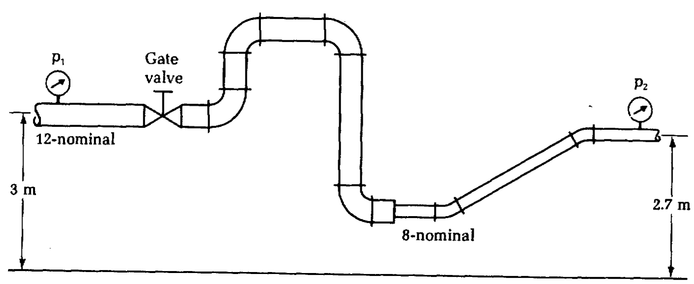
$$\begin{aligned}\frac{p_1 - p_2}{\rho}\frac{g_c}{g} &= \frac{(1.697)^2}{2(9.81)} - \frac{(0.762)^2}{2(9.81)} + (2.7 - 3) \\ &+ \left[\frac{0.018(60)}{0.289} + 1.39\right]\frac{(0.762)^2}{2(9.81)} \\ &+ \left[\frac{0.017\,5(22)}{0.194} + 0.63\right]\frac{(1.697)^2}{2(9.81)} \\ &= 0.146\,8 - 0.029\,6 + (-0.3) + 0.152 + 0.384 \\ &= 0.353 \text{ m of turpentine}\end{aligned}$$

or

$$p_1 - p_2 = 0.353(870)(9.81) = 3\,012 \text{ N/m}^2$$

$$\underline{p_1 - p_2 = 3.012 \text{ kPa}}$$

Figure 5.22. A pipeline for Example 5.6.



EXAMPLE 5.7

A water tank is fitted with a drain and outlet pipe as sketched in Figure 5.23. The system has 82 ft of cast iron pipe of 1½-nominal diameter. Determine the flow rate through the pipe. All fittings are threaded and regular.

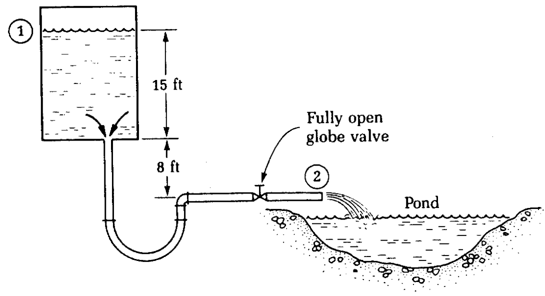


Figure 5.23. Piping system for Example 5.7.

SOLUTION

First write the Bernoulli equation from 1 to 2:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D}\frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

The figure shows that $p_1 = p_2$ and that if $z_2 = 0$ then $z_1 = 23$ ft. The reservoir surface velocity V_1 is negligible compared to V_2. Because all the piping is of the same diameter, the equation becomes

$$23 = \frac{V_2^2}{2g}\left(1 + \frac{fL}{D} + \Sigma K\right).$$

The minor losses include the entrance, the return bend, the elbow and the globe valve. If we include also the loss due to the exit fitting, then the exit kinetic energy term in the above equation $V_2^2/2g$ should be zero. It is redundant to include the exit loss and a nonzero exit kinetic energy term. Thus we have

$\Sigma K = 0.5 + 1.4 + 10.0 = 13.4$ (Table 5.4)

$D = 0.1342$ ft and $A = 0.01414$ ft^2 (Appendix Table C-1)

$\rho = 62.4$ lbm/ft^3 and $\mu g_c = 6 \times 10^{-4}$ lbm/ft-s (Appendix Table A-5)

$\epsilon = 0.00085$ ft (Table 5-2)

By substitution, the Bernoulli equation becomes

$$23(2)(32.2) = V_2^2\left[1 + \frac{f(82)}{0.1342} + 13.4\right]$$

or

$$V_2 = \left(\frac{1481.2}{14.4 + 611.0f}\right)^{1/2}$$

Because V_2 and f are unknown, a trial and error solution is required. As a first trial assume $f = 0.03$; then

$$V_2 = 6.72 \text{ ft/s}$$

$$\left.\begin{array}{l} \text{Re} = \dfrac{\rho VD}{\mu g_c} = \dfrac{62.4(6.72)(0.1342)}{6 \times 10^{-4}} = 9.38 \times 10^4 \\ \dfrac{\epsilon}{D} = \dfrac{0.00085}{0.1342} = 0.0063 \end{array}\right\} \begin{array}{l}\text{Moody diagram:} \\ f = 0.0335\end{array}$$

As a second trial assume $f = 0.0335$; then

$$V_2 = 6.51 \text{ ft/s}$$

$$\left.\begin{array}{l} \text{Re} = 9.1 \times 10^4 \\ \dfrac{\epsilon}{D} = 0.0063 \end{array}\right\} \begin{array}{l}\text{Moody diagram} \\ f = 0.034 \quad \text{(close enough)}\end{array}$$

Using $f = 0.034$, $V_2 = 6.49$ ft/s, and the continuity equation, we find

$$Q = A_2V_2 = 0.01414(6.49)$$

$$\underline{Q = 0.0918 \text{ ft}^3/\text{s} = 5.51 \text{ ft}^3/\text{min}}$$

EXAMPLE 5.8

Methyl alcohol is used in a processing plant where a flow rate of 0.8 m^3/min of the liquid must be supplied. The available liquid pump can supply this flow rate only if the pressure drop in the supply line is less than 15 m head of water. The pipeline is made up of 50 m of soldered drawn copper tubing and follows the path shown in Figure 5.24. Determine the minimum size of tubing required.

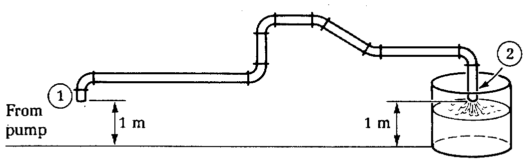


Figure 5.24. Sketch for Example 5.8.

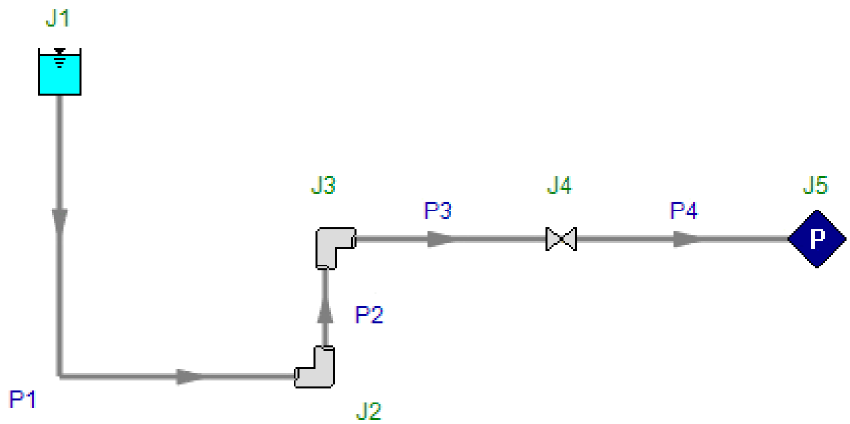
SOLUTION

Write the Bernoulli equation from 1 to 2 as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D}\frac{V^2}{2g} + \Sigma K \frac{V^2}{2g}$$

View Verification Case 53 Model

Verification Case 53



Verification Case 54

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify54.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 183-186, Example 5.9

FLUID: Water

ASSUMPTIONS: Water at 77 deg. F

RESULTS:

Parameter	Janna	AFT Fathom
Flow rate pipe 2 (ft^3/sec)	0.19	0.1947
Flow rate pipe 3 (ft^3/sec)	0.11	0.1053

DISCUSSION:

The problem statement does not have any pressure information, because it is not required if the only purpose is to calculate flow rates. However, AFT Fathom requires at least one pressure boundary because AFT Fathom offers pressure information in the output for all systems. Therefore, a pressure of 1 atm was assumed for the common junction at the exit of the system.

An "out of balance" warning may appear in the AFT Fathom Output. The amount that is out of balance is very small and therefore the warning can be neglected.

List of All Verification Models

Verification Case 54 Problem Statement

Verification Case 54

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 183-186, Example 5.9

Janna Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

(A_2 or A_3) is the same. Usually the diameters of all the pipes are known; the problem involves finding Q_2 and Q_3. The solution technique requires use of the Moody diagram and trial and error. The technique is best illustrated by example.

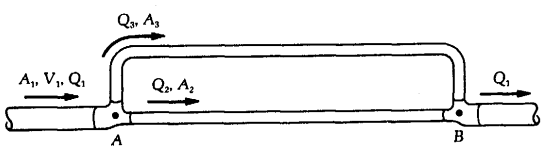


Figure 5.26. Pipes in parallel.

EXAMPLE 5.9

For the piping system of Figure 5.26, the distance from A to B is 4500 ft and the main line is made of schedule 40, 10-nominal wrought iron pipe. The attached loop is schedule 40, 8-nominal wrought iron pipe. The flow Q_1 is 0.3 ft³/s of water. Determine the flow rate in both branches.

SOLUTION

Minor losses are usually neglected in problems of this type. From the continuity equation,

$$Q_1 = Q_2 + Q_3 = 0.3 \text{ ft}^3/\text{s}$$

Also $p_A - p_B$ along Q_2 must be equal to $p_A - p_B$ along Q_3. Thus

$$(p_A - p_B)|_2 = (p_A - p_B)|_3$$

or

$$\frac{f_2 L_2}{D_2}\frac{V_2^2}{2g} = \frac{f_3 L_3}{D_3}\frac{V_3^2}{2g}$$

With $L_2 = L_3$, the equation reduces to

$$\frac{f_2 V_2^2}{D_2} = \frac{f_3 V_3^2}{D_3}$$

In terms of volume flow rate,

$$\frac{f_2}{D_2}\frac{16Q_2^2}{\pi^2 D_2^4} = \frac{f_3}{D_3}\frac{16Q_3^2}{\pi^2 D_3^4}$$

or

$$\frac{f_2 Q_2^2}{D_2^5} = \frac{f_3 Q_3^2}{D_3^5}$$

From Appendix Table C-1,

$$D_2 = 0.8350 \text{ ft} \quad \text{and} \quad A_2 = 0.5476 \text{ ft}^2$$
$$D_3 = 0.6651 \text{ ft} \quad \text{and} \quad A_3 = 0.3474 \text{ ft}^2$$

By substitution, the equation now becomes

$$2.46 f_2 Q_2^2 = 7.68 f_3 Q_3^2$$

or

$$Q_2 = Q_3 \sqrt{\frac{3.119 f_3}{f_2}}$$

From Table 5.2, $\epsilon = 0.00015$; hence

$$\frac{\epsilon}{D_2} = 0.000180$$

$$\frac{\epsilon}{D_3} = 0.000226$$

From Appendix Table A-5, $\rho = 62.4$ lbm/ft^3 and $\mu g_c = 6 \times 10^{-4}$ lbm/ft-s. The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{\rho D}{\mu g_c}\frac{4Q}{\pi D^2} = \frac{4\rho Q}{\pi D \mu g_c}$$

For each line,

$$\text{Re}_2 = \frac{4(62.4)Q_2}{\pi(0.8350)(6 \times 10^{-4})} = 1.586 \times 10^5 Q_2$$
$$\text{Re}_3 = 1.991 \times 10^5 Q_3$$

From the calculations above, our working equations are

$$Q_2 + Q_3 = Q = 0.3 \text{ ft}^3/\text{s}$$
$$\text{Re}_2 = 1.586 \times 10^5 Q_2 \qquad \epsilon/D_2 = 0.000180$$
$$\text{Re}_3 = 1.991 \times 10^5 Q_3 \qquad \epsilon/D_3 = 0.000226$$
$$Q_2 = Q_3 \sqrt{\frac{3.119 f_3}{f_2}}$$

Because $D_2 > D_3$ we expect that $Q_2 > Q_3$.

As a first trial assume $Q_2 = \frac{1}{2}Q = 0.15$ ft³/s; then

$$Q_3 = 0.30 - 0.15 = 0.15$$
$$Re_2 = 2.37 \times 10^4$$
$$Re_3 = 2.99 \times 10^4$$

From the Moody diagram,

$$f_2 = 0.0255$$
$$f_3 = 0.0241$$

and by substitution into the working equation we get

$$Q_2 = 0.258 \text{ ft}^3/\text{s}$$

If this value is used for the second trial, the next iteration gives $Q_2 = 0.0898$. The third iteration gives $Q_2 = 0.33$, which shows that successive trials lead to divergence. A more successful method involves taking as a second assumed value the average of the initially assumed Q_2 and the calculated Q_2. For the second iteration, therefore, assume

$$Q_2 = \frac{0.15 + 0.26}{2} = 0.20$$

then

$$Q_3 = 0.1$$
$$Re_2 = 3.17 \times 10^4$$
$$Re_3 = 1.99 \times 10^4$$
$$f_2 = 0.024$$
$$f_3 = 0.0265$$

and, by equation,

$$Q_2 = 0.18$$

For the next trial assume

$$Q_2 = \frac{0.18 + 0.20}{2} = 0.19$$

then

$$Q_3 = 0.11$$
$$Re_2 = 3.013 \times 10^4$$
$$Re_3 = 2.19 \times 10^4$$
$$f_2 = 0.024$$
$$f_3 = 0.026$$

and, by equation,

$$Q_2 = 0.20$$

which is close enough to the assumed value. Thus the solution is

$$Q_2 = 0.19 \text{ ft}^3/\text{s} \quad \text{and} \quad Q_3 = 0.11 \text{ ft}^3/\text{s}$$

All pipe friction problems in this chapter have been solved by using the Moody diagram. Although reference to a figure is highly impractical if one is using a computer for the calculations, the computer is well suited for these problems, especially the trial and error type. Thus an equation fit of the Moody diagram is helpful. Among the many equations that have been written are the following:

Hagen–Poiseuille equation:

$$f = \frac{64}{\text{Re}} \quad \text{(laminar flow, circular pipe)} \tag{5.35}$$

Prandtl equation:

$$\frac{1}{\sqrt{f}} = 2.0 \log(\text{Re}\sqrt{f}) - 0.8 \quad [3\,000 < \text{Re} < 3.4 \times 10^6] \tag{5.36}$$

Von Karman equation:

$$\frac{1}{\sqrt{f}} = 2.0 \log\left(\frac{D}{\epsilon}\right) + 1.74 \quad \frac{D}{\epsilon}\frac{1}{\text{Re}\sqrt{f}} > 0.01 \tag{5.37}$$

Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon}{3.7065D} + \frac{2.5226}{\text{Re}\sqrt{f}}\right) \quad \frac{D}{\epsilon}\frac{1}{\text{Re}\sqrt{f}} < 0.005 \tag{5.38}$$

Except for the laminar-flow expression, all these equations have $\sqrt{f}$ appearing on both sides or in the condition. This difficulty is overcome by the Chen equation, which is good for any ϵ/D and Re:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left\{\frac{\epsilon}{3.7065D} - \frac{5.0452}{\text{Re}} \log\left[\frac{1}{2.8257}\left(\frac{\epsilon}{D}\right)^{1.1098} + \frac{5.8506}{\text{Re}^{0.8981}}\right]\right\} \tag{5.39}$$

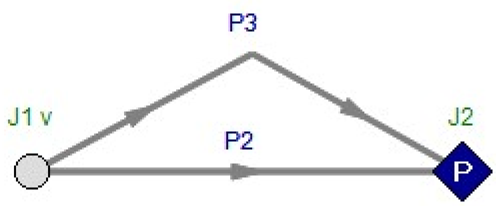
If ϵ/D and Re are known then f can be determined, just as with the Moody diagram.

5.10 / Pumps and Piping Systems

To make fluid flow from one point to another in a closed conduit, a driving force is required. Where elevation differences exist this force may be grav-

View Verification Case 54 Model

Verification Case 54



Verification Case 55

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify55.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 189-190, Example 5.10

FLUID: Water

ASSUMPTIONS: Water at 75 deg. F

RESULTS:

Parameter	Janna	AFT Fathom
Pump ideal power (hp)	0.737	0.734

DISCUSSION:

The losses due to elbows are included as a fitting and loss value in the pipes. The ideal power usage is given the Output window Pump Summary.

List of All Verification Models

Verification Case 55 Problem Statement

Verification Case 55

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Page 189-190, Example 5.10

Janna Title Page

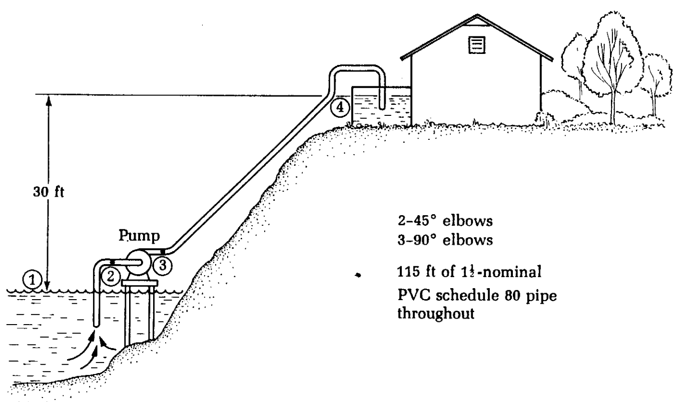
Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

which is converted to pressure head after the liquid leaves the impeller. Pumping power requirements are determined with the energy equation as illustrated by the following example.

EXAMPLE 5.10

A house is located near a freshwater lake. The homeowner decides to install a pump near the lake to deliver 50 gpm of water to a tank adjacent to the house. The water can then be used for lavatory facilities or sprinkling the lawn. For the system sketched in Figure 5.30, determine the pump power required.

Figure 5.30. Pumping system for Example 5.10.



SOLUTION

The following equations can be written for the system:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right)$$

$$\frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_3 g_c}{\rho g} + \frac{V_3^2}{2g} + z_3 + \frac{1}{\dot{m}}\frac{dW}{dt}\frac{g_c}{g}$$

$$\frac{p_3 g_c}{\rho g} + \frac{V_3^2}{2g} + z_3 = \frac{p_4 g_c}{\rho g} + \frac{V_4^2}{2g} + z_4 + \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right)$$

These equations can be added to obtain a single equation for the entire system:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_4 g_c}{\rho g} + \frac{V_4^2}{2g} + z_4 + \sum \frac{fL}{D}\frac{V_4^2}{2g} + \sum K \frac{V_4^2}{2g} + \frac{1}{\dot{m}}\frac{dW}{dt}\frac{g_c}{g}$$

Thus only inlet and outlet conditions must be known. At sections 1 and 4 (both are reservoir surfaces), velocity is negligible. With section 1 as the reference,

$$-\frac{1}{\dot{m}}\frac{dW}{dt} = \frac{z_4 g}{g_c} + \frac{V_4^2}{2g_c}\left(1 + \frac{fL}{D} + \Sigma K\right)$$

The volume flow required is

$$Q = 50 \text{ gpm} \times 0.002228 = 0.111 \text{ ft}^3/\text{s}$$

For water, $\rho = 62.4$ lbm/ft³ and, from Appendix Table A-4, $\mu g_c = 6.122 \times 10^{-4}$; so

$$\dot{m} = \rho Q = 6.926 \text{ lbm/s}$$

From Appendix Table C-1 for $1\frac{1}{2}$-nominal schedule 80 pipe, we find

$$D = 0.125 \text{ ft} \quad \text{and} \quad A = 0.01227 \text{ ft}^2$$

The velocity is

$$V = \frac{Q}{A} = \frac{0.111}{0.01227} = 9.05 \text{ ft/s}$$

with which we obtain

$$\text{Re} = \frac{62.4(9.05)(0.125)}{6.122 \times 10^{-4}} = 1.15 \times 10^5$$

It is permissible to use the "smooth pipe" curve on the Moody diagram for PVC pipe. At $\text{Re} = 1.15 \times 10^5$, therefore, $f = 0.018$. From Table 5.4, $K = 1.4$ for 90° elbows, $K = 0.35$ for 45° elbows and 1.0 for an exit fitting. By substitution into the equation, we obtain

$$-\frac{dW}{dt} = 6.926\left\{30 + \frac{(9.05)^2}{2(32.2)}\left[1 + \frac{0.018(115)}{0.125} + 2(0.35) + 3(1.4)\right]\right\}$$

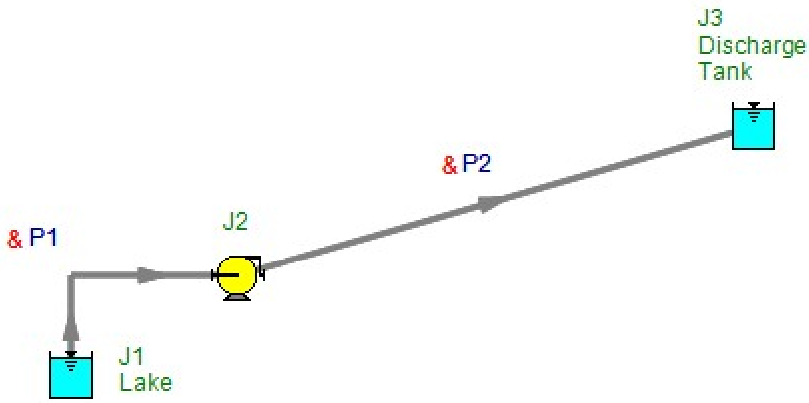
$$= 6.926(58.56) = 405.6 \text{ ft-lbf/s}$$

By definition, 1 hp = 550 ft-lbf/s; therefore

$$\frac{dW}{dt} = -0.737 \text{ hp}$$

View Verification Case 55 Model

Verification Case 55



Verification Case 56

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify56.fth

REFERENCE: Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-4, Example 6-3

FLUID: Fuel Oil at 300 deg. F

ASSUMPTIONS: No elevation changes. Inlet pressure is 100 psia.

RESULTS:

Parameter	Chopey	AFT Fathom
Pressure drop (psid)	1.17	1.172

DISCUSSION:

No absolute pressure information was provided. It was assumed the inlet pressure was 100 psia. This does not affect the answer.

The flow in the pipe is laminar.

List of All Verification Models

Verification Case 56 Problem Statement

Verification Case 56

Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-4, Example 6-3

Chopey Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

6-3 Pressure Loss in Piping with Laminar Flow

Fuel oil at 300°F (422 K) and having a specific gravity of 0.850 is pumped through a 30,000-ft-long 24-in pipe at the rate of 500 gal/min (0.032 m^3/s). What is the pressure loss if the viscosity of the oil is 75 cP?

Calculation Procedure:

1. Determine the type of flow that exists.

Flow is laminar (also termed viscous) if the Reynolds number Re for the liquid in the pipe is less than about 2000. Turbulent flow exists if the Reynolds number is greater than about 4000. Between these values is a zone in which either condition may exist, depending on the roughness of the pipe wall, entrance conditions, and other factors. Avoid sizing a pipe for flow in this critical zone because excessive pressure drops result without a corresponding increase in the pipe discharge.

Compute the Reynolds number from $\text{Re} = 3.162G/kd$, where G = flow rate, gal/min; k = kinematic viscosity of liquid, Cst = viscosity z cP/specific gravity of the liquid S; d = inside diameter of pipe, in. From a table of pipe properties, $d = 22.626$ in. Also, $k = z/S = 75/0.85 = 88.2$ Cst. Then, $\text{Re} = 3162(500)/[88.2(22.626)] = 792$. Since $\text{Re} < 2000$, laminar flow exists in this pipe.

2. Compute the pressure loss using the Poiseuille formula.

The Poiseuille formula gives the pressure drop ρ_d lb/in² $= 2.73(10^{-4})luG/d^4$, where l = total length of pipe, including equivalent length of fitting, ft; u = absolute viscosity of liquid, cP; G = flow rate, gal/min; d = inside diameter of pipe, in. For this pipe, $\rho_d = 2.73\ (10^{-4})(30{,}000)(75)(500)/262{,}078 = 1.17$ lb/in² (8.07 kPa).

Related Calculations: Use this procedure for any pipe in which there is laminar flow of liquid. Table 6-1 gives a quick summary of various ways in which the Reynolds num-

TABLE 6-1 Reynolds Number

Reynolds number Re	Numerator: Coefficient	Numerator: First symbol	Numerator: Second symbol	Numerator: Third symbol	Denominator: Fourth symbol	Denominator: Fifth symbol
$Dv\rho/\mu$	—	ft	ft/s	lb/ft³	lb mass/ft·s	—
$124dv\rho/z$	124	in	ft/s	lb/ft³	cP	—
$50.7G\rho/dz$	50.7	gal/min	lb/ft³	—	in	cP
$6.32W/dz$	6.32	lb/h	—	—	in	cP
$35.5B\rho/dz$	35.5	bbl/h	lb/ft³	—	in	cP
$7{,}742dv/k$	7,742	in	ft/s	—	—	cP
$3{,}162G/dk$	3,162	gal/min	—	—	in	cP
$2{,}214B/dk$	2,214	bbl/h	—	—	in	cP
$22{,}735q\rho/dz$	22,735	ft³/s	lb/ft³	—	in	cP
$378.9Q\rho/dz$	378.9	ft³/min	lb/ft³	—	in	cP

View Verification Case 56 Model

Verification Case 56

Verification Case 57

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify57.fth

REFERENCE: Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-5, 11, Example 6-4

FLUID: Kerosene at 65 deg. F

ASSUMPTIONS: No elevation changes. Inlet pressure is 100 psia.

RESULTS:

Parameter	Chopey	AFT Fathom
Pressure drop (psid)	17.3	17.24

DISCUSSION:

The kinematic viscosity of 2.95 cSt was converted to dynamic viscosity, obtaining 0.00161 lbm/ft-s.

No absolute pressure information was provided. It was assumed the inlet pressure was 100 psia. This does not affect the answer.

List of All Verification Models

Verification Case 57 Problem Statement

Verification Case 57

Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-5, 11, Example 6-4

Chopey Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

ber can be expressed. The symbols in Table 6-1, in the order of their appearance, are D = inside diameter of pipe, ft; v = liquid velocity, ft/s; ρ = liquid density, lb/ft^3; μ = absolute viscosity of liquid, lb mass/ft·s; d = inside diameter of pipe, in. From a table of pipe properties, d = 22.626 in. Also, $k = z/S$ liquid flow rate, lb/h; B = liquid flow rate, bbl/h; k = kinematic viscosity of the liquid, Cst; q = liquid flow rate, ft^3/s; Q = liquid flow rate, ft^3/min. Use Table 6-1 to find the Reynolds number for any liquid flowing through a pipe.

6-4 Determining the Pressure Loss in Pipes

What is the pressure drop in a 5000-ft-long 6-in oil pipe conveying 500 bbl/h (0.022 m^3/s) kerosene having a specific gravity of 0.813 at 65°F, which is the temperature of the liquid in the pipe? The pipe is schedule 40 steel.

Calculation Procedure:

1. Determine the kinematic viscosity of the oil.

Use Fig. 6-1 and Table 6-2 or the Hydraulic Institute—*Pipe Friction Manual* kinematic viscosity and Reynolds number chart to determine the kinematic viscosity of the liquid. Enter Table 6-2 at kerosene and find the coordinates as X = 10.2, Y = 16.9. Using these coordinates, enter Fig. 6-1 and find the absolute viscosity of kerosene at 65°F as 2.4 cP. Using the method of Example 6-2, the kinematic viscosity, in cSt, equals absolute viscosity, cP/specific gravity of the liquid = 2.4/0.813 = 2.95 cSt. This value agrees closely with that given in the *Pipe Friction Manual.*

2. Determine the Reynolds number of the liquid.

The Reynolds number can be found from the *Pipe Friction Manual* chart mentioned in step 1 or computed from Re = 2214 B/dk = 2214(500)/[(6.065)(2.95)] = 61,900.

To use the *Pipe Friction Manual* chart, compute the velocity of the liquid in the pipe by converting the flow rate to cubic feet per second. Since there are 42 gal/bbl and 1 gal = 0.13368 ft^3, 1 bbl = (42)(0.13368) = 5.6 ft^3. With a flow rate of 500 bbl/h, the equivalent flow in ft^3 = (500)(5.6) = 2800 ft^3/h, or 2800/3600 s/h = 0.778 ft^3/s. Since 6-in schedule 40 pipe has a cross-sectional area of 0.2006 ft^2 internally, the liquid velocity, in ft/s, equals 0.778/0.2006 = 3.88 ft/s. Then, the product (velocity, ft/s)(internal diameter, in) = (3.88)(6.065) = 23.75. In the *Pipe Friction Manual,* project horizontally from the kerosene specific-gravity curve to the vd product of 23.75 and read the Reynolds number as 61,900, as before. In general, the Reynolds number can be found faster by computing it using the appropriate relation given in Table 6-1, unless the flow velocity is already known.

3. Determine the friction factor of this pipe.

Enter Fig. 6-2 at the Reynolds number value of 61,900 and project to the curve 4 as indicated by Table 6-3. Read the friction factor as 0.0212 at the left. Alternatively, the *Pipe Friction Manual* friction-factor chart could be used, if desired.

VISCOSITIES

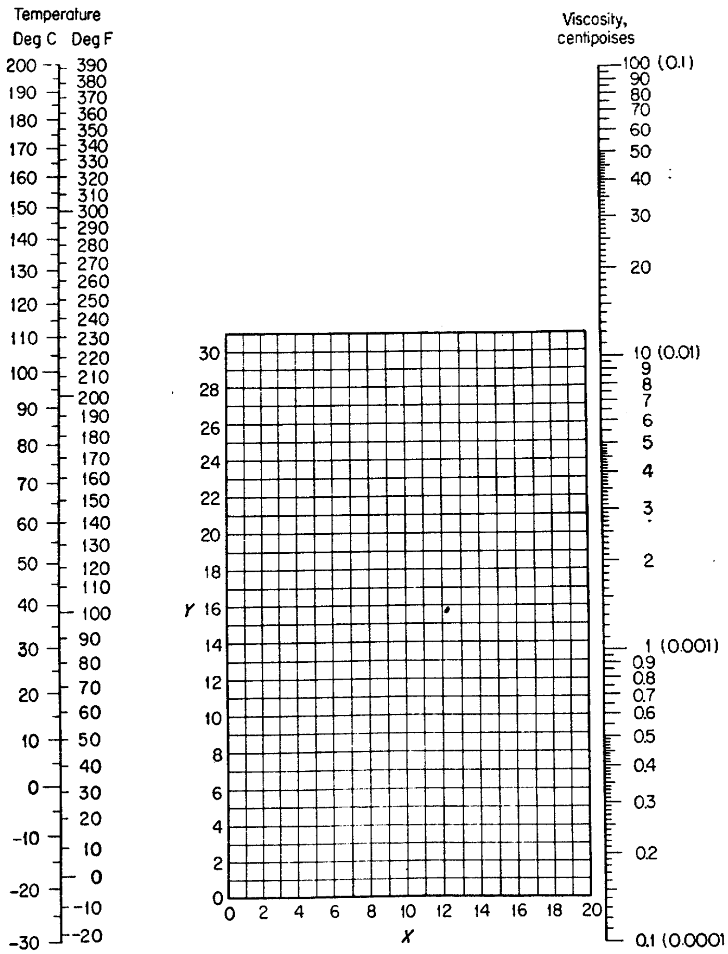


FIG. 6-1 Viscosities of liquids at 1 atm (101.3 kPa). For coordinates, see Table 6-2.

6-6

TABLE 6-2 Viscosities of Liquids (coordinates for use with Fig. 6-1)

No.	Liquid	X	Y
1	Acetaldehyde	15.2	4.8
	Acetic acid:		
2	100%	12.1	14.2
3	70%	9.5	17.0
4	Acetic anhydride	12.7	12.8
	Acetone:		
5	100%	14.5	7.2
6	35%	7.9	15.0
7	Allyl alcohol	10.2	14.3
	Ammonia:		
8	100%	12.6	2.0
9	26%	10.1	13.9
10	Amyl acetate	11.8	12.5
11	Amyl alcohol	7.5	18.4
12	Aniline	8.1	18.7
13	Anisole	12.3	13.5
14	Arsenic trichloride	13.9	14.5
15	Benzene	12.5	10.9
	Brine:		
16	$CaCl_2$, 25%	6.6	15.9
17	NaCl, 25%	10.2	16.6
18	Bromine	14.2	13.2
19	Bromotoluene	20.0	15.9
20	Butyl acetate	12.3	11.0
21	Butyl alcohol	8.6	17.2
22	Butyric acid	12.1	15.3
23	Carbon dioxide	11.6	0.3
24	Carbon disulfide	16.1	7.5
25	Carbon tetrachloride	12.7	13.1
26	Chlorobenzene	12.3	12.4
27	Chloroform	14.4	10.2
28	Chlorosulfonic acid	11.2	18.1
	Chlorotoluene:		
29	Ortho	13.0	13.3
30	Meta	13.3	12.5
31	Para	13.3	12.5
32	Cresol, meta	2.5	20.8
33	Cyclohexanol	2.9	24.3
34	Dibromoethane	12.7	15.8
35	Dichloroethane	13.2	12.2
36	Dichloromethane	14.6	8.9
37	Diethyl oxalate	11.0	16.4
38	Dimethyl oxalate	12.3	15.8
39	Diphenyl	12.0	18.3
40	Dipropyl oxalate	10.3	17.7
41	Ethyl acetate	13.7	9.1
	Ethyl alcohol:		
42	100%	10.5	13.8
43	95%	9.8	14.3
44	40%	6.5	16.6
45	Ethyl benzene	13.2	11.5
46	Ethyl bromide	14.5	8.1
47	Ethyl chloride	14.8	6.0
48	Ethyl ether	14.5	5.3
49	Ethyl formate		
50	Ethyl iodide	14.7	10.3
51	Ethylene glycol	6.0	23.6
52	Formic acid	10.7	15.8
53	Freon-11	14.4	9.0
54	Freon-12	16.8	5.6
55	Freon-21	15.7	7.5
56	Freon-22	17.2	4.7
57	Freon-13	12.5	11.4
	Glycerol:		
58	100%	2.0	30.0
59	50%	6.9	19.6
60	Heptene	14.1	8.4
61	Hexane	14.7	7.0
62	Hydrochloric acid, 31.5%	13.0	16.6
63	Isobutyl alcohol	7.1	18.0
64	Isobutyric acid	12.2	14.4
65	Isopropyl alcohol	8.2	16.0
66	Kerosene	10.2	16.9
67	Linseed oil, raw	7.5	27.2
68	Mercury	18.4	16.4
	Methanol		
69	100%	12.4	10.5
70	90%	12.3	11.8
71	40%	7.8	15.5
72	Methyl acetate	14.2	8.2
73	Methyl chloride	15.0	3.8
74	Methyl ethyl ketone	13.9	8.6
75	Naphthalene	7.9	18.1
	Nitric acid:		
76	95%	12.8	13.8
77	60%	10.8	17.0
78	Nitrobenzene	10.6	16.2
79	Nitrotoluene	11.0	17.0
80	Octane	13.7	10.0
81	Octyl alcohol	6.6	21.1
82	Pentachloroethane	10.9	17.3
83	Pentane	14.9	5.2
84	Phenol	6.9	20.8
85	Phosphorus tribromide	13.8	16.7
86	Phosphorus trichloride	16.2	10.9
87	Propionic acid	12.8	13.8
88	Propyl alcohol	9.1	16.5
89	Propyl bromide	14.5	9.6
90	Propyl chloride	14.4	7.5
91	Propyl iodide	14.1	11.6
92	Sodium	16.4	13.9
93	Sodium hydroxide, 50%	3.3	25.8
94	Stannic chloride	13.5	12.8
95	Sulfur dioxide	15.2	7.1
	Sulfuric acid:		
96	110%	7.2	27.4
97	98%	7.0	24.8
98	60%	10.2	21.3
99	Sulfuryl chloride	15.2	12.4
100	Tetrachloroethane	11.9	15.7
101	Tetrachloroethylene	14.2	12.7
102	Titanium tetrachloride	14.4	12.3
103	Toluene	13.7	10.4
104	Trichloroethylene	14.8	10.5
105	Turpentine	11.5	14.9
106	Vinyl acetate		
107	Water	10.2	13.0
	Xylene:		
108	Ortho	13.5	12.1
109	Meta	13.9	10.6
110	Para	13.9	10.9

6-7

6-8

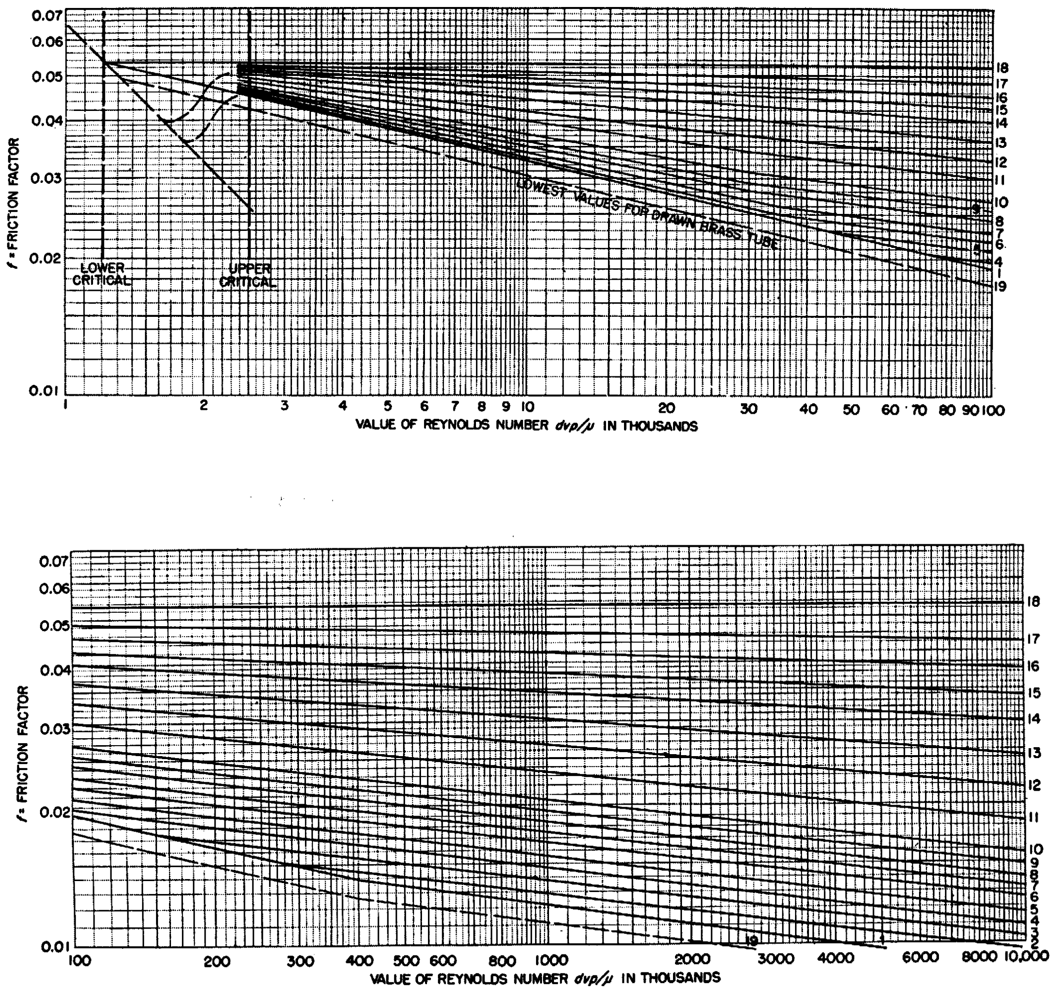


FIG. 6-2 Friction-factor curves. *(Mechanical Engineering.)*

6-9

TABLE 6-3 Data for Fig. 6-2

Percent roughness	For value of *f* see curve	Diameter (actual of drawn tubing, nominal of standard-weight pipe)											
		Drawn tubing, brass, tin, lead, glass		Clean steel, wrought iron		Clean, galvanized		Best cast iron		Average cast iron		Heavy riveted, spiral riveted	
		in	mm	in	mm	in	mm	in	mm	in	mm	in	mm
0.2	1	0.35 up	8.89 up	72	1829	—	—	—	—	—	—	—	—
1.35	4	—	—	6–12	152–305	10–24	254–610	20–48	508–1219	42–96	1067–2438	84–204	2134–5182
2.1	5	—	—	4–5	102–127	6–8	152–203	12–16	305–406	24–36	610–914	48–72	1219–1829
3.0	6	—	—	2–3	51–76	305	76–127	5–10	127–254	10–20	254–508	20–42	508–1067
3.8	7	—	—	1½	38	2½	64	3–4	76–102	6–8	152–203	16–18	406–457
4.8	8	—	—	1–1¼	25–32	1½–2	38–51	2–2½	51–64	4–5	102–127	10–14	254–356
6.0	9	—	—	¾	19	1¼	32	1½	38	3	76	8	203
7.2	10	—	—	½	13	1	25	1¼	32	—	—	5	127
10.5	11	—	—	⅜	9.5	¾	19	1	35	—	—	4	102
14.5	12	—	—	¼	6.4	½	13	—	—	—	—	3	76
24.0	14	0.125	3.18	—	—	⅜	9.5	—	—	—	—	—	—
31.5	16	—	—	—	—	¼	6.4	—	—	—	—	—	—
37.5	18	0.0625	1.588	—	—	⅛	3.2	—	—	—	—	—	—

4. Compute the pressure loss in the pipe.

Use the Fanning formula $p_d = 1.06(10^{-4})f\rho lB^2/d^5$. In this formula, ρ = density of the liquid, lb/ft^3. For kerosene, p = (density of water, lb/ft^3)(specific gravity of the kerosene) = (62.4)(0.813) = 50.6 lb/ft^3. Then, $p_d = 1.06(10^{-4})(0.0212)(50.6)(5000)(500)^2/8206 = 17.3$ lb/in^2 (119 kPa).

Related Calculations: The Fanning formula is popular with oil-pipe designers and can be stated in various ways: (1) with velocity v, in ft/s, $p_d = 1.29(10^{-3})f\rho v^2 l/d$; (2) with velocity V, in ft/min, $p_d = 3.6(10^{-7})f\rho V^2 l/d$; (3) with flow rate G, in gal/min, $p_d = 2.15(10^{-4})f\rho lG^2/d^2$; (4) with the flow rate W, in lb/h, $p_d = 3.36(10^{-6})flW^2/d^5\rho$.

Use this procedure for any fluid—crude oil, kerosene, benzene, gasoline, naptha, fuel oil, Bunker C, diesel oil toluene, etc. The tables and charts presented here and in the *Pipe Friction Manual* save computation time.

6-5 Equivalent Length of a Complex-Series Pipeline

Figure 6-3 shows a complex-series pipeline made up of four lengths of different size pipe. Determine the equivalent length of this pipe if each size of pipe has the same friction factor.

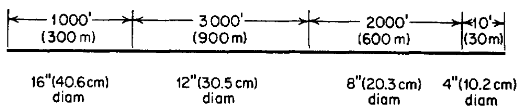


FIG. 6-3 Complex-series pipeline.

Calculation Procedure:

1. Select the pipe size for expressing the equivalent length.

The usual procedure when analyzing complex pipelines is to express the equivalent length in terms of the smallest, or next-to-smallest, diameter pipe. Choose the 8-in size as being suitable for expressing the equivalent length.

2. Find the equivalent length of each pipe.

For any complex-series pipeline having equal friction factors in all the pipes, L_e = equivalent length, ft, of a section of constant diameter = (actual length of section, ft)(inside diameter, in, of pipe used to express the equivalent length/inside diameter, in, of section under consideration)^5.

For the 16-in pipe, $L_e = (1000)(7.981/15.000)^5 = 42.6$ ft. The 12-in pipe is next; for it, $L_e = (3000)(7.981/12.00)^5 = 390$ ft. For the 8-in pipe, the equivalent length = actual length = 2000 ft. For the 4-in pipe, $L_e = (10)(7.981/4.026)^5 = 306$ ft. Then, the total equivalent length of 8-in pipe = sum of the equivalent lengths = 42.6 + 390 + 2000 + 306 = 2738.6 ft, or rounding off, 2740 ft of 8-in pipe (835 m of 0.2-m pipe) will have a frictional resistance equal to the complex-series pipeline shown in Fig. 6-3. To compute the actual frictional resistance, use the methods given in previous Calculation Procedures.

View Verification Case 57 Model

Verification Case 57

Verification Case 58

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify58.fth

REFERENCE: Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-41, 46, Example 6-21

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Parameter	Chopey	AFT Fathom
Head rise case A (feet)	125	124.4
Head rise case B (feet)	125	124.4
Head rise case C (feet)	125	124.4
Power usage case A (hp)	90.2	89.8
Power usage case B (hp)	90.2	89.8
Power usage case C (hp)	90.2	89.8

DISCUSSION:

The hydraulic loss data used by Chopey is based on equivalent length. All equivalent lengths were converted to K values and included as a fitting and loss value in the pipe or as custom entrance/exit loss factors at the tanks.

List of All Verification Models

Verification Case 58 Problem Statement

Verification Case 58

Nicholas P. Chopey, Handbook of Chemical Engineering Calculations, 1994, McGraw-Hill, Page 6-41, 46, Example 6-21

Chopey Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

TABLE 6-21 Suction Specific-Speed Ratings

Single-suction pump	Double-suction pump	Rating
Above 11,000	Above 14,000	Excellent
9,000–11,000	11,000–14,000	Good
7,000–9,000	9,000–11,000	Average
5,000–7,000	7,000–9,000	Poor
Below 5,000	Below 7,000	Very poor

Source: Peerless Pump Division, FMC Corporation.

Related Calculations: Use this procedure for any type of centrifugal pump handling water for plant services, cooling, process, fire protection, and similar requirements. This procedure is the work of R. P. Horwitz, Hydrodynamics Division, Peerless Pump, FMC Corporation, as reported in *Power* magazine.

6-21 Total Head on a Pump Handling Vapor-Free Liquid

Sketch three typical pump piping arrangements with static suction lift and submerged, free, and varying discharge head. Prepare similar sketches for the same pump with static suction head. Label the various heads. Compute the total head on each pump if the elevations are as shown in Fig. 6-13 and the pump discharges a maximum of 2000 gal/min (0.126 m^3/s) of water through 8-in schedule 40 pipe. What horsepower is required to drive the pump? A swing check valve is used on the pump suction line and a gate valve on the discharge line.

Calculation Procedure:

1. Sketch the possible piping arrangements.

Figure 6-13 shows the six possible piping arrangements for the stated conditions of the installation. Label the total static head—i.e., the *vertical* distance from the surface of the source of the liquid supply to the free surface of the liquid in the discharge receiver, or to the point of free discharge from the discharge pipe. When both the suction and discharge surfaces are open to the atmosphere, the total static head equals the vertical difference in elevation. Use the free-surface elevations that cause the maximum suction lift and discharge head—i.e., the *lowest* possible level in the supply tank and the *highest* possible level in the discharge tank or pipe. When the supply source is *below* the pump centerline, the vertical distance is called the "static suction lift." With the supply *above* the pump centerline, the vertical distance is called "static suction head." With variable static suction head, use the lowest liquid level in the supply tank when computing total static head. Label the diagrams as shown in Fig. 6-13.

2. Compute the total static head on the pump.

The total static head, in feet, is H_{ts} = static suction lift, in feet, h_{sl} + static discharge head, in feet, h_{sd}, where the pump has a suction lift, *s* in Fig. 6-13*a, b,* and *c*. In these

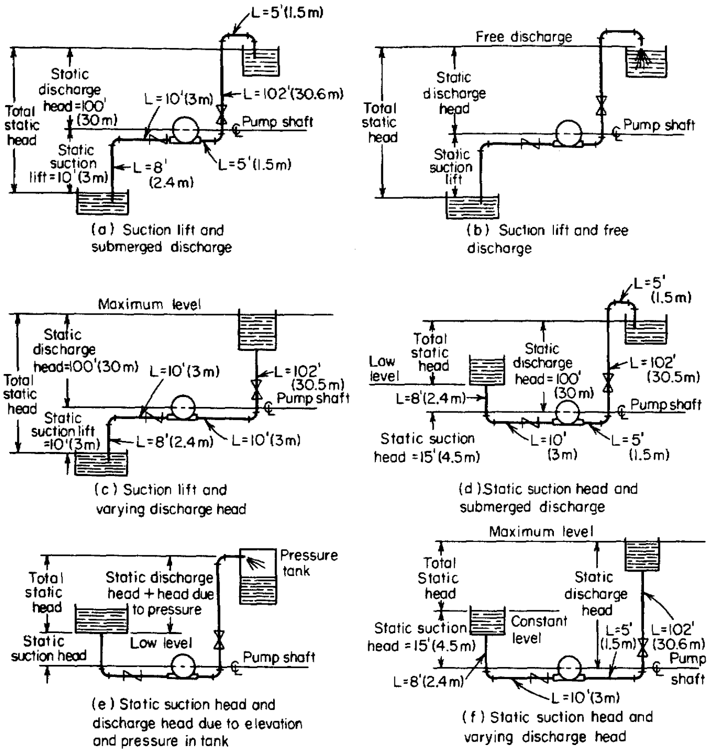


FIG. 6-13 Typical pump suction and discharge piping arrangements.

installations, $H_{ts} = 10 + 100 = 110$ ft. Note that the static discharge head is computed between the pump centerline and the water level with an underwater discharge (Fig. 6-13*a*), to the pipe outlet with a free discharge (Fig. 6-13*b*), and to the maximum water level in the discharge tank (Fig. 6-13*c*). When a pump is discharging into a closed compression tank, the total discharge head equals the static discharge head plus the head equivalent, in feet of liquid, of the internal pressure in the tank, or 2.31 × tank pressure, in lb/in^2.

Where the pump has a static suction head, as in Fig. 6-13*d*, *e*, and *f*, the total static head, in feet, is $H_{ts} = h_{sd}$ − static suction head, in feet, h_{sh}. In these installations, $H_t = 100 - 15 = 85$ ft.

The total static head, as computed above, refers to the head on the pump without liquid flow. To determine the total head on the pump, the friction losses in the piping system during liquid flow must also be determined.

3. Compute the piping friction losses.

Mark the length of each piece of straight pipe on the piping drawing. Thus, in Fig. 6-13*a*, the total length of straight pipe L_t, in feet, is 8 + 10 + 5 + 102 + 5 = 130 ft, starting at the suction tank and adding each length until the discharge tank is reached. To the total length of straight pipe must be added the *equivalent* length of the pipe fittings. In Fig. 6-13*a* there are four long-radius elbows, one swing check valve, and one globe valve. In addition, there is a minor head loss at the pipe inlet and at the pipe outlet.

The equivalent length of one 8-in-long-radius elbow is 14 ft of pipe, from Table 6-22. Since the pipe contains four elbows, the total equivalent length is 4(14) = 56 ft of straight pipe. The open gate valve has an equivalent resistance of 4.5 ft, and the open swing check valve has an equivalent resistance of 53 ft.

The entrance loss h_e, in feet, assuming a basket-type strainer is used at the suction-pipe inlet, is $Kv^2/(2g)$, where K = a constant from Fig. 6-14; v = liquid velocity, in ft/s; and g = 32.2 ft/s^2. The exit loss occurs when the liquid passes through a sudden enlargement, as from a pipe to a tank. Where the area of the tank is large, causing a final velocity that is zero, $h_{ex} = v^2/2g$.

The velocity v, in feet per second, in a pipe is (gal/min)/(2.448d^2). For this pipe, v = 2000/[2.448(7.98)^2] = 12.82 ft/s. Then, h_e = 0.74(12.82)^2/[2(32.2)] = 1.89 ft, and h_{ex} = (12.82)^2/[2(32.2)] = 2.56 ft (0.78 m). Hence the total length of the piping system in Fig. 6-13*a* is 130 + 56 + 4.5 + 53 + 1.89 + 2.56 = 247.95 ft (75.6 m), say 248 ft (75.6 m).

Use a suitable head-loss equation, or Table 6-23, to compute the head loss for the pipe and fittings. Enter Table 6-23 at an 8-in (203.2-mm) pipe size and project horizontally across to 2000 gal/min (126.2 L/s) and read the head loss as 5.86 ft of water per 100 ft (1.8 m/30.5 m) of pipe.

The total length of pipe and fittings computed above is 288 ft (87.8 m). Then, total friction-head loss with a 2000-gal/min (126.2 L/s) flow is H_f = (5.86)(248/100) = 14.53 ft (4.5 m).

4. Compute the total head on the pump.

The total head on the pump $H_t = H_{ts} + H_f$. For the pump in Fig. 6-13*a*, H_t = 110 + 14.53 = 124.53 ft (38.0 m), say 125 ft (38.0 m). The total head on the pump in Fig. 6-13*b* and *c* would be the same. Some engineers term the total head on a pump the "total dyamic head" to distinguish between static head (no-flow vertical head) and operating head (rated flow through the pump).

The total head on the pumps in Fig. 6-13*d, c,* and *f* is computed in the same way as described above, except that the total static head is less because the pump has a static suction head—that is, the elevation of the liquid on the suction side reduces the total distance through which the pump must discharge liquid; thus the total static head is less. The static suction head is *subtracted* from the static discharge head to determine the total static head on the pump.

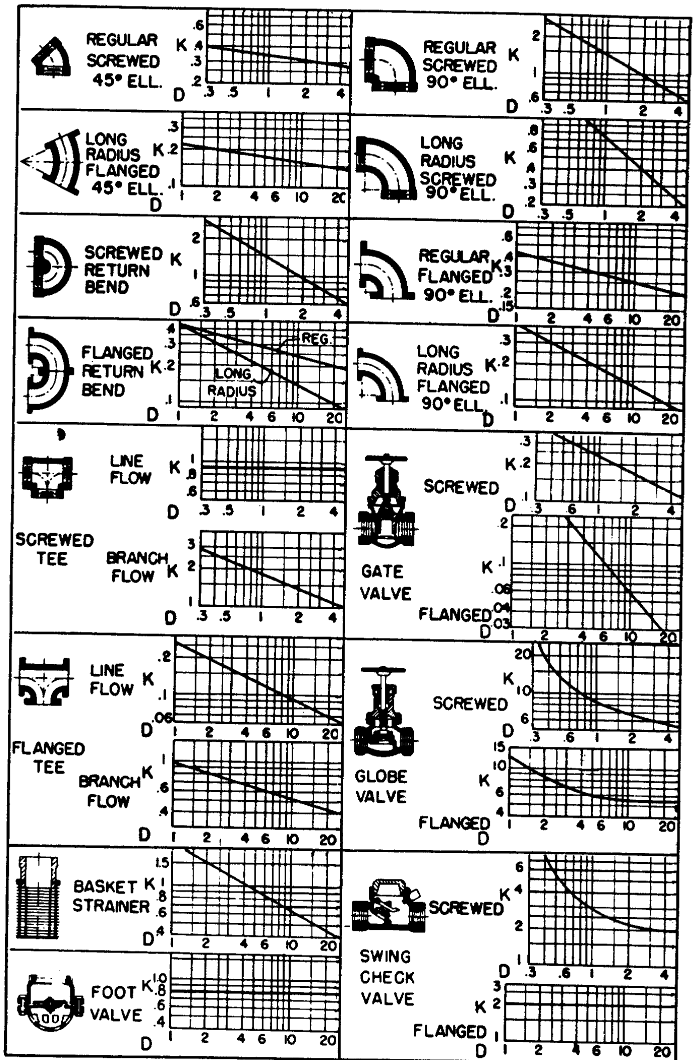
5. Compute the horsepower required to drive the pump.

The brake horsepower input to a pump equals (gal/min)(H_t)(s)/3960e, where s = specific gravity of the liquid handled, and e = hydraulic efficiency of the pump, expressed as a decimal. The usual hydraulic efficiency of a centrifugal pump is 60 to 80 percent;

6-44

TABLE 6-22 Resistance of Fittings and Valves (length of straight pipe giving equivalent resistance)

Pipe size		Standard ell		Medium-radius ell		Long-radius ell		45° Ell		Tee		Gate valve, open		Globe valve, open		Swing check, open	
in	mm	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m	ft	m
6	152.4	16	4.9	14	4.3	11	3.4	7.7	2.3	33	10.1	3.5	1.1	160	48.8	40	12.2
8	203.2	21	6.4	18	5.5	14	4.3	10	3.0	43	13.1	4.5	1.4	220	67.0	53	16.2
10	254.0	26	7.9	22	6.7	17	5.2	13	3.9	56	17.1	5.7	1.7	290	88.4	67	20.4
12	304.8	32	9.8	26	7.9	20	6.1	15	4.6	66	20.1	6.7	2.0	340	103.6	80	24.4



$$h = k\frac{v^2}{2g} \text{ feet of fluid.}$$

FIG. 6-14 Resistance coefficients of pipe fittings. *(Hydraulic Institute.)*

6-45

TABLE 6-23 Pipe Friction Loss for Water (wrought-iron or steel schedule 40 pipe in good condition)

Diameter		Flow		Velocity		Velocity head		Friction loss/100 ft (30.5 m) pipe	
in	mm	gal/min	L/s	ft/s	m/s	ft water	m water	ft water	m water
6	152.4	1000	63.1	11.1	3.4	1.92	0.59	6.17	1.88
6	152.4	2000	126.2	22.2	6.8	7.67	2.3	23.8	7.25
6	152.4	4000	252.4	44.4	13.5	30.7	9.4	93.1	28.4
8	203.2	1000	63.1	6.41	1.9	0.639	0.195	1.56	0.475
8	203.2	2000	126.2	12.8	3.9	2.56	0.78	5.86	1.786
8	203.2	4000	252.4	25.7	7.8	10.2	3.1	22.6	6.888
10	254.0	1000	63.1	3.93	1.2	0.240	0.07	0.497	0.151
10	254.0	3000	189.3	11.8	3.6	2.16	0.658	4.00	1.219
10	254.0	5000	315.5	19.6	5.9	5.99	1.82	10.8	3.292

reciprocating pumps, 55 to 90 percent; rotary pumps, 50 to 90 percent. For each class of pump, the hydraulic efficiency decreases as the liquid viscosity increases.

Assume that the hydraulic efficiency of the pump in this system is 70 percent and the specific gravity of the liquid handled is 1.0. Then, input brake horsepower equals (2000)(125)(1.0)/[3960(0.70)] = 90.2 hp (67.4 kW).

The theoretical or *hydraulic horsepower* equals (gal/min)$(H_t)(s)$/3960 = (2000)(125)(1.0)/3900 = 64.1 hp (47.8 kW).

Related Calculations: Use this procedure for any liquid–water, oil, chemical, sludge, etc.—whose specific gravity is known. When liquids other than water are being pumped, the specific gravity and viscosity of the liquid must be taken into consideration. The procedure given here can be used for any class of pump—centrifugal, rotary, or reciprocating.

Note that Fig. 6-14 can be used to determine the equivalent length of a variety of pipe fittings. To use Fig. 6-14, simply substitute the appropriate K value in the relation $h = Kv^2/2g$, where h = equivalent length of straight pipe; other symbols as before.

6-22 Pump Selection for Any Pumping System

Give a step-by-step procedure for choosing the class, type, capacity, drive, and materials for a pump that will be used in an industrial pumping system.

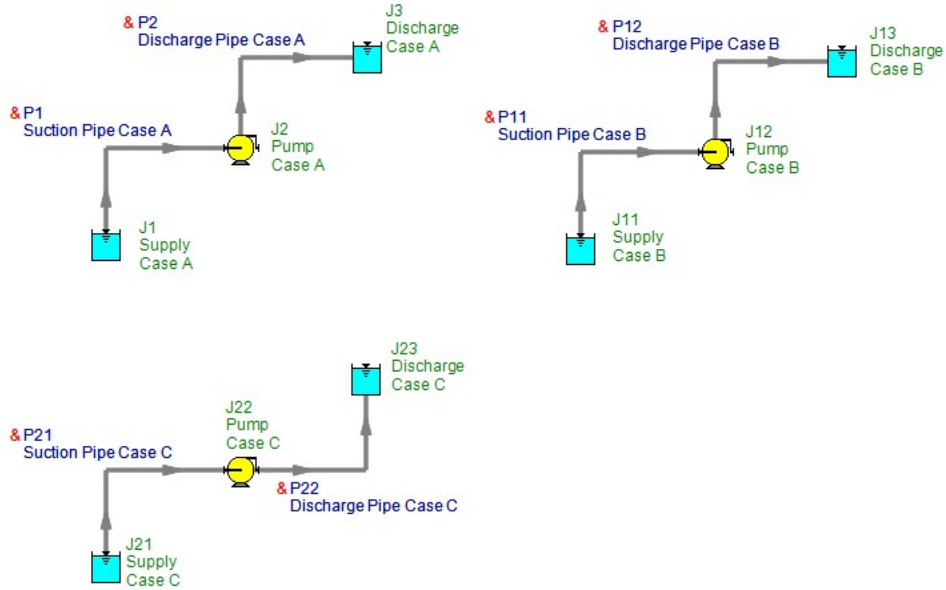
Calculation Procedure:

1. Sketch the proposed piping layout.

Use a single-line diagram (Fig. 6-15) of the piping system. Base the sketch on the actual job conditions. Show all the piping, fittings, valves, equipment, and other units in the system. Mark the *actual* and *equivalent* pipe length (see the previous example) on the sketch. Be certain to include all vertical lifts, sharp bends, sudden enlargements, storage tanks, and similar equipment in the proposed system.

View Verification Case 58 Model

Verification Case 58



Verification Case 59

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify59.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 84-85

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7
Jeppson	0.841	0.666	1.334	0.573	0.761	0.175	0.398
AFT Fathom	0.8411	0.6664	1.3336	0.5726	0.761	0.1748	0.3979

Pipe Head Loss (feet)	1	2	3	4	5	6	7
Jeppson	3.15	17.63	15.46	1.5	11.42	0.674	3.24
AFT Fathom	*3.126	17.535	15.387	1.484	*11.374	0.664	*3.210

Node EGL (feet)	1	2	3	4
Jeppson	114.2	96.6	112	113.5
AFT Fathom	114.23	96.7	112.08	113.57

Pump Head (feet)	1	2	3
Jeppson	17.36	18.46	21.78
AFT Fathom	17.36	18.46	21.78

* AFT Fathom results combine two pipes, as discussed below

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there are three pumps in the example, there are three additional pipes in the AFT Fathom model. AFT Fathom pipes 1 and 8 together represent Jeppson pipe 1. Similarly, AFT Fathom pipes 5 and 9 represent Jeppson pipe 5, and AFT Fathom pipes 7 and 10 represent Jeppson pipe 7.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

The printed Jeppson results also give pressures at junctions/nodes 1-4. However, pressures can only be determined if elevation data is given for these locations, of which there is no information in the problem statement. Therefore no comparison is made to pressure.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 59 Problem Statement

Verification Case 59

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 84-85

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

84 PIPE NETWORK ANALYSES

work. The number of these pseudo loops must equal the difference between the number of unknown flow rates, i.e. N and (J + L).

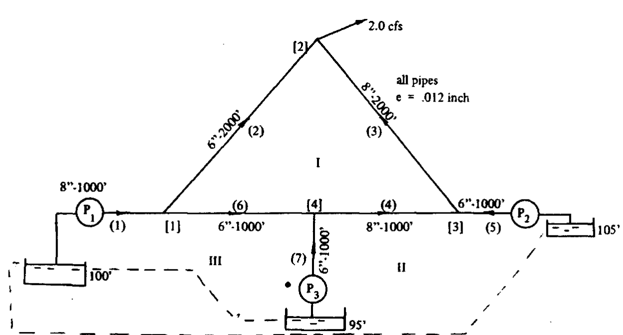
4. As many additional linear equations of the form $G - Q = B/2A$ (Eq. 5-5) are written as pumps exist.
5. The nonlinear energy equations are linearized by defining coefficients K' of the Q's which are obtained by $K' = KQ(m)^{n-1}$ and coefficients K'_G for the G unknowns are obtained by $K'_G = AG$.
6. The resulting system is solved iteratively, adjusting the coefficients as described earlier to reflect the average of the flow from the past two solutions until convergence occurs.

Should any of the details involved in these steps be vague, following them through for a simple example will be helpful. Consider the seven-pipe, one loop network supplied by three identical pumps shown below. Each pump supply head according to the equation

$$h_p = -10.328\,Q_p^2 + 2.823\,Q_p + 22.289$$

Since there are seven pipes in this network there will be seven unknown flow rates, plus three additional unknowns, i.e. the G's of Eq. 5-5 for the three pumps which supply flow. Consequently a total of 10 simultaneous equations are needed. Four of these equations are the junction continuity equations; three are from Eq. 5-5 relating the three G's to Q_1, Q_5, and Q_7; and consequently three energy equations are needed, one from the real loop and two from pseudo loops connecting pump reservoirs with no flow pipes. With the K's in the exponential formula approximately computed by the Hazen-Williams equation, these equations are:

$$\text{Continuity Equations}\quad \begin{cases} -Q_1 + Q_2 + Q_6 = 0 \\ -Q_2 - Q_3 = -2.0 \\ Q_3 - Q_4 - Q_5 = 0 \\ Q_4 - Q_6 - Q_7 = 0 \end{cases}$$



Real Loop
$$\begin{cases} 55.7\,Q_2^{1.85} - 13.7\,Q_3^{1.85} - 6.86\,Q_4^{1.85} \\ -27.8\,Q_6^{1.85} = 0 \end{cases}$$

Pseudo Loop
$$\begin{cases} 6.86\,Q_1^{1.85} + 6.86\,Q_4^{1.85} - 27.8\,Q_5^{1.85} \\ + 27.8\,Q_6^{1.85} + 10.33\,G_1 - 10.33\,G_2 = -5 \\ 6.86\,Q_1^{1.85} + 27.8\,Q_6^{1.85} - 27.8\,Q_7^{1.85} \\ + 10.33\,G_1 - 10.33\,G_3 = 5 \end{cases}$$

Transformation Eq. 5-5
$$\begin{cases} -Q_1 + G_1 = -0.137 \\ -Q_5 + G_2 = -0.137 \\ -Q_7 + G_3 = -0.137 \end{cases}$$

In applying the linear theory method, the three nonlinear energy equations are linearized as described previously, and the resulting linear system solved. After three such iterative solutions of the linearized system the following solution results:

Pipe No.	1	2	3	4	5	6	7
Flow rate (cfs)	0.841	0.666	1.334	0.573	0.761	0.175	0.398
Head loss	3.15	17.63	15.46	1.50	11.42	0.674	3.24

Junction No.	1	2	3	4
Head (ft)	114.2	96.6	112.0	113.5
Pressure (psi)	49.5	41.9	48.5	49.2

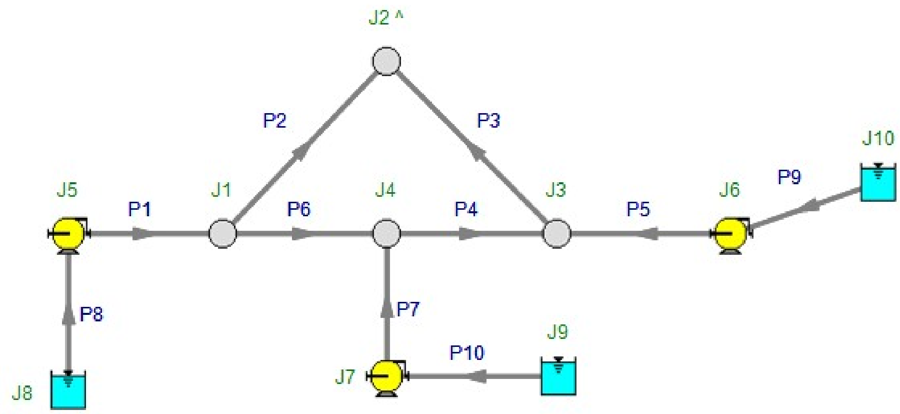
Pump	Head (ft)
1	17.36
2	18.46
3	21.78

Including Pressure Reduction Valves Into Linear Theory Method

A pressure reducing valve (denoted PRV) is designed to maintain a constant pressure downstream from it regardless of how large the upstream pressure is. The exceptions to this occurrence are: (1) If the upstream pressure becomes less than the valve setting, and (2) if the downstream pressure exceeds the pressure setting of the valve so that if the PRV were not present the flow would be in the opposite direction to the downstream flow direction of the valve. If the first condition occurs, the valve has no effect on flow conditions. The PRV acts as a check valve, preventing reverse flow if the second condition occurs. By preventing

View Verification Case 59 Model

Verification Case 59



Verification Case 60

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify60.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 86-87

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7
Jeppson	1.03	0.98	0.017	0.97	0.96	0.041	0.003
AFT Fathom	1.0268	0.984	0.016	0.973	0.960	0.039	0.003

Pipe Head Loss (feet)	1	2	3	4	5	6	7
Jeppson	23.35	21.43	0.009	4.21	40.92	0.05	23.68
AFT Fathom	*23.324	21.449	0.0065	4.194	40.829	*0.046	21.443

Node EGL (feet)	1	2	3	4
Jeppson	117.21	95.79	95.79	57.98
AFT Fathom	117.25	95.8	95.81	54.98

* AFT Fathom results combine two pipes, as discussed below

DISCUSSION:

Jeppson's method of applying pump and PRV data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps and PRVs at boundaries between pipes. Pumps and PRVs are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump or PRV is split into two equivalent pipes in AFT Fathom. In the case of the pump, where the split is made will have no impact on the results. If the PRV control pressure is specified in terms of head, the elevation of the PRV becomes important. In such cases, Jeppson specifies the elevation and AFT Fathom incorporates this.

Because there is one pump and one PRV in the example, there are two additional pipes in the AFT Fathom model. AFT Fathom pipes 1 and 9 together represent Jeppson pipe 1. Similarly, AFT Fathom pipes 6 and 8 represent Jeppson pipe 6.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 60 Problem Statement

Verification Case 60

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 86-87

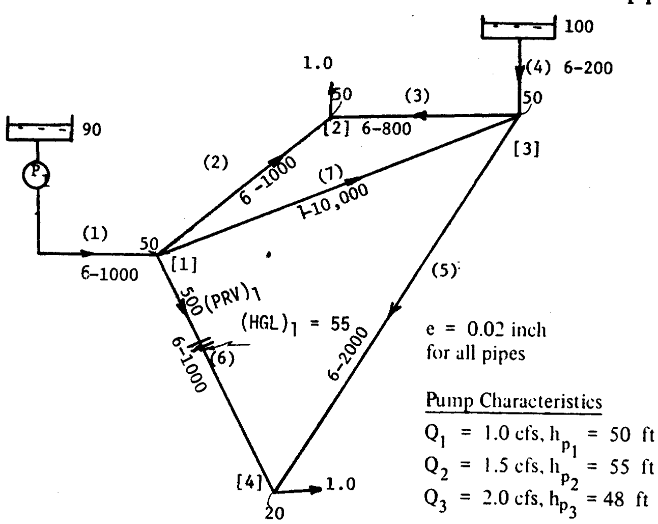
Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

reverse flow, the PRV allows the pressure immediately downstream from the valve to exceed its pressure setting. Thus, PRV's are used to reduce pressures in portions of a pipe distribution system if the pressures would otherwise be excessive, and they may also be used to control from which sources of supply the flow comes under various demand levels. In the latter applications the PRV acts as a check valve until the pressure is reduced to critical levels by large demands at which time additional sources of supply are drawn upon.

The analysis of a pipe network containing one or more PRV's must be capable of determining which of these conditions exist. Methods for accomplishing this which are consistent with the linear theory method are discussed in this section. Problems 11 through 15 in the next section provide additional examples, beyond those given in this section, of small networks containing PRV's.

Example No. 1 Methods for including PRV's in network analysis by the linear theory method will be illustrated by several examples. First consider the 7-pipe network shown below in which a PRV exists in pipe



6, 500 feet downstream from the beginning of this pipe. The system of Q-equations for this network consists of four junction continuity equations and three energy equations obtained from loops. The junction continuity equations are identical to those that would be written if the PRV were not present. In forming the loops for the energy equations, however, the pipe containing the PRV (pipe 6) is disconnected from its upstream junction

and the PRV is replaced by a reservoir. (This assumes the PRV reduces the pressure to a constant value. The PRV acting as a check valve is discussed later.) Of the three loop equations, one is obtained by summing the head losses around the real loop formed by pipes 2, 3, and 7. The second equation comes from the pseudo loop which connects the upper reservoir to the reservoir from which the pump obtains water (i.e., the source pump) by the pipes 4, 7, and 1. The third loop equation connects the artificial reservoir created by the PRV to another reservoir by a series of pipes such as 6, 5, and 4. Note that with pipe 6 disconnected from junction 1, only one real loop is availabe whereas two independent real loops existed before this. The real loop which is lost through the disconnection is compensated for by the additional pseudo loop from the artificial reservoir created by the PRV. Therefore, the number of equations which are available always equals the number of unknown flow rates Q. Using this scheme, the eight equations (eight are used instead of seven because one equation is added by the pump transformation as described in the previous section) needed for a solution by the linear theory method are:

$$\left.\begin{aligned} -Q_1 + Q_2 + Q_6 + Q_7 &= 0.0 \\ -Q_2 - Q_3 &= -1.0 \\ Q_3 - Q_4 + Q_5 - Q_7 &= 0.0 \\ -Q_5 - Q_6 &= -1.0 \end{aligned}\right\} \text{Junction Continuity Equations}$$

$$-K_2 Q_2^{n_2} + K_3 Q_3^{n_3} + K_7 Q_7^{n_7} = 0.0 \qquad \text{Real Loop}$$

$$K_4 Q_4^{n_4} - K_7 Q_7^{n_7} - K_1 Q_1^{n_1} + A_1 G_1^2 = 100 - 90 - h_{01} \qquad \text{Pseudo Loop}$$

$$G_1 - Q_1 = (B/2A)_1 \qquad \text{Pump Transformation}$$

$$K_6 Q_6^{n_6} - K_5 Q_5^{n_5} - K_4 Q_4^{n_4} = 55 - 100 \qquad \text{Pseudo Loop from PRV}$$

in which K_6 is determined only for the portion of pipe 6 downstream from the PRV.

Upon solving this system of equations by the linear theory method, using the procedure described previously, the following solution results:

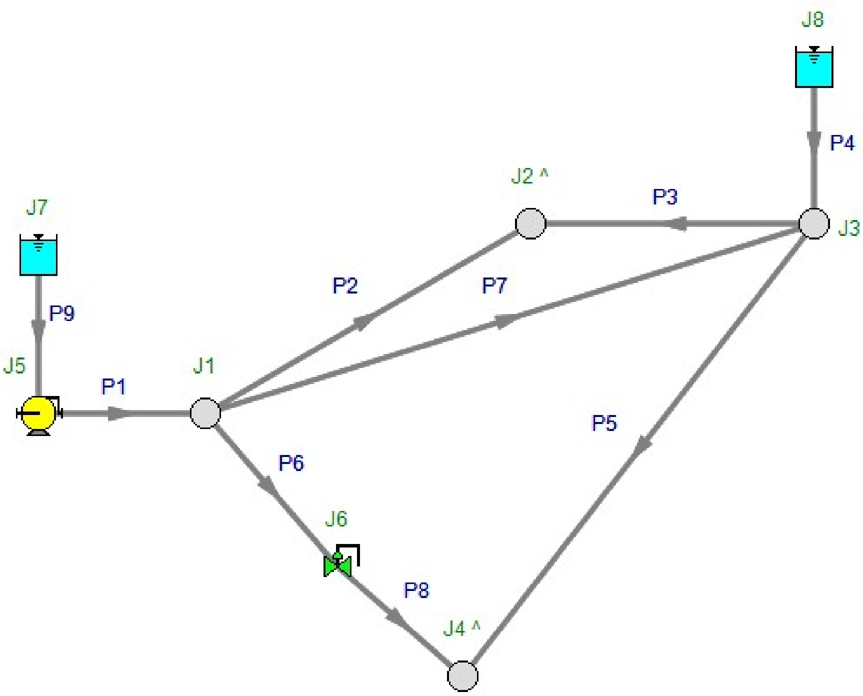
Pipe No.	1	2	3	4	5	6	7
Flow rate (cfs)	1.03	0.98	0.017	0.97	0.96	0.041	0.003
Head loss (ft)	23.35	21.43	0.009	4.21	40.92	0.05	23.68

The hydraulic grade line elevations at the junctions are:

Junction No.	1	2	3	4
Elevation of HGL (ft)	117.21	95.79	95.79	57.98

View Verification Case 60 Model

Verification Case 60



Verification Case 61

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify61.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 135-137

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	4.975	1.653	0.114	2.777	1.277	2.045	-1.039	-0.960
AFT Fathom	4.986	1.656	0.114	2.781	1.281	2.048	-1.042	-0.965

Pipe Flow Rate (ft3/sec)	9	10	11	12	13	14	15	16
Jeppson	1.415	0.889	0.609	0.109	0.413	1.587	2.391	0.025
AFT Fathom	1.413	0.891	0.608	0.108	0.411	1.589	2.395	0.014

Pipe Head Loss (feet)	1	2	3	4	5	6	7	8
Jeppson	11.32	10.60	0.03	8.41	2.22	13.25	4.48	1.83
AFT Fathom	*11.284	10.56	0.03	8.37	2.22	13.20	-4.48	-1.83

Pipe Head Loss (feet)	9	10	11	12	13	14	15	16
Jeppson	2.21	4.04	2.17	0.04	0.27	4.31	2.62	0.00
AFT Fathom	2.18	4.02	2.15	0.04	0.27	4.29	2.61	0.00

Node EGL (feet)	1	2	3	4	5	6	7	8
Jeppson	165.1	154.5	154.5	162.9	150.0	151.8	147.8	147.8
AFT Fathom	165.0	154.5	154.4	162.8	150.0	151.8	147.9	147.8

Node EGL (feet)	9
Jeppson	147.5
AFT Fathom	147.5

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there is one pump in the example, there is one additional pipe in the AFT Fathom model. AFT Fathom pipes 1 and 17 together represent Jeppson pipe 1.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the pipe head loss formula used by Jeppson differs from AFT Fathom. Jeppson's Hazen-Williams formula is given in his book and does not agree exactly with the accepted formula as used in AFT Fathom. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

Results for AFT Fathom also vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom help file.

List of All Verification Models

Verification Case 61 Problem Statement

Verification Case 61

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 135-137

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

$$F_1 = 2.074\,(5+\Delta Q_1)^{1.85} + 3.78\,(2+\Delta Q_1-\Delta Q_2)^{1.85} - 7.56\,(2-\Delta Q_1+\Delta Q_3)^{1.85} = 0$$

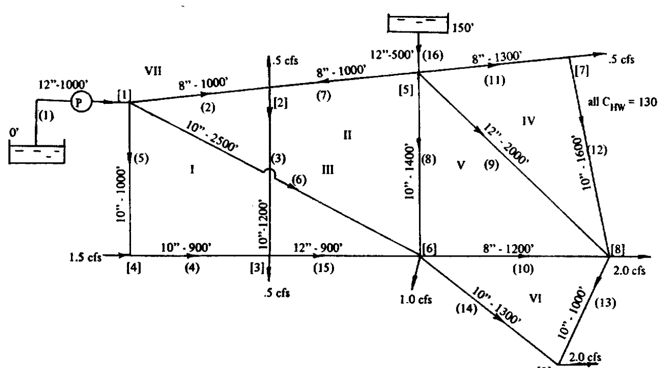
$$F_2 = 14.94\,(3+\Delta Q_2)^{1.85} + 30.32\,(.5+\Delta Q_2)^{1.85} - 5.04\,(4.5-\Delta Q_2)^{1.85} - 3.78\,(2-\Delta Q_2+\Delta Q_1)^{1.85} = 0$$

$$F_3 = 7.56\,(2.0+\Delta Q_3-\Delta Q_1)^{1.85} - 60.65\,(2.5-\Delta Q_3)^{1.85} - 11.21\,(5-\Delta Q_3)^{1.85} = 0$$

The solution to this system produces: $\Delta Q_1 = 0.712$, $\Delta Q_2 = -0.115$, $\Delta Q_3 = 2.24$, all in cfs. Therefore the flow rates and head losses in each pipe are:

Pipe No.	1	2	3	4	5	6	7	8
Q (cfs)	5.712	2.827	0.257	2.757	2.885	0.385	4.615	3.531
h_f (ft)	52.29	25.90	4.90	73.30	106.32	5.18	85.59	78.19

2. Solve for the flow rate in each pipe and the head at each junction of the network shown below. The pump characteristic curve is given by $h_p = -2.505\,Q_p^2 + 16.707\,Q_p + 155.29$ (h_p is in feet and Q_p in cfs)



Solution:

Because of the pump and reservoir one pseudo loop connecting the reservoir by a no-flow pipe is needed. To solve for the seven corrective flow rates, seven energy equations are written as follows around the seven loops:

$$F_1 = K_2 (Q_{02} + \Delta Q_1 + \Delta Q_3)^{1.85} + K_3 (Q_{03} + \Delta Q_1 - \Delta Q_L)^{1.85} - K_4 (Q_{04} - \Delta Q_1)^{1.85} + K_5 (Q_{05} - \Delta Q_1)^{1.85} = 0$$

$$F_2 = - K_7 (Q_{07} - \Delta Q_2 - \Delta Q_3)^{1.85} + K_8 (Q_{08} + \Delta Q_2 - \Delta Q_5)^{1.85} - K_{15} (Q_{015} - \Delta Q_2)^{1.85} - K_3 (Q_{03} - \Delta Q_2 + \Delta Q_1)^{1.85} = 0$$

$$F_3 = K_2 (Q_{02} + \Delta Q_3 + \Delta Q_1)^{1.85} - K_7 (Q_{07} - \Delta Q_3 - \Delta Q_2)^{1.85} + K_8 (Q_{08} + \Delta Q_3 + \Delta Q_2)^{1.85} - K_6 (Q_{06} - \Delta Q_3)^{1.85} = 0$$

$$F_4 = K_{11} (Q_{011} + \Delta Q_4)^{1.85} + K_{12} (Q_{012} + \Delta Q_4)^{1.85} - K_9 (Q_{09} - \Delta Q_4 + \Delta Q_5)^{1.85} = 0$$

$$F_5 = K_9 (Q_{09} + \Delta Q_5 - \Delta Q_4)^{1.85} - K_{10} (Q_{010} - \Delta Q_5 + \Delta Q_6)^{1.85} - K_8 (Q_{08} - \Delta Q_5 + \Delta Q_2)^{1.85} = 0$$

$$F_6 = K_{10} (Q_{010} + \Delta Q_6 - \Delta Q_5)^{1.85} + K_{13} (Q_{013} + \Delta Q_6)^{1.85} - K_{14} (Q_{014} - \Delta Q_6)^{1.85} = 0$$

$$F_7 = K_{16} (Q_{016} + \Delta Q_7)^{1.85} + K_7 (Q_{07} + \Delta Q_7 - \Delta Q_2 - \Delta Q_3)^{1.85} - K_2 (Q_{02} - \Delta Q_7 + \Delta Q_2 + \Delta Q_3)^{1.85} - K_1(Q_{01} - \Delta Q_7)^{1.85} + h_p - 150 = 0$$

In the last of these equations h_p is defined by the pump characteristic curve equation with $Q_{01} - \Delta Q_7$ replacing Q_p. After supplying an initial flow for each pipe, and solving by the Newton-Raphson method the following solution results:

Pipe No.	1	2	3	4	5	6	7
Q (cfs)	4.975	1.653	0.114	2.777	1.277	2.045	-1.039
h_f (ft)	11.32	10.60	0.03	8.41	2.22	13.25	4.48

Pipe No.	8	9	10	11	12	13	14
Q (cfs) h_f (ft)	-0.960 1.83	1.415 2.21	0.889 4.04	0.609 2.17	0.109 0.04	0.413 0.27	1.587 4.31

Pipe No.	15	16
Q (cfs) h_f (ft)	2.391 2.62	0.025 0.00

Junction No.	1	2	3	4	5	6	7	8	9
Head (ft)	165.1	154.5	154.5	162.9	150.0	151.8	147.8	147.8	147.5

3. Solve the network shown below. The wall roughness for all pipes is e = 0.012 inch.

Solution:

Pipe No.	1	2	3	4	5	6	7
Q (cfs) h_f (ft)	19.80 10.59	10.37 2.97	4.89 0.69	4.00 24.57	2.66 0.22	4.13 17.43	4.44 10.04

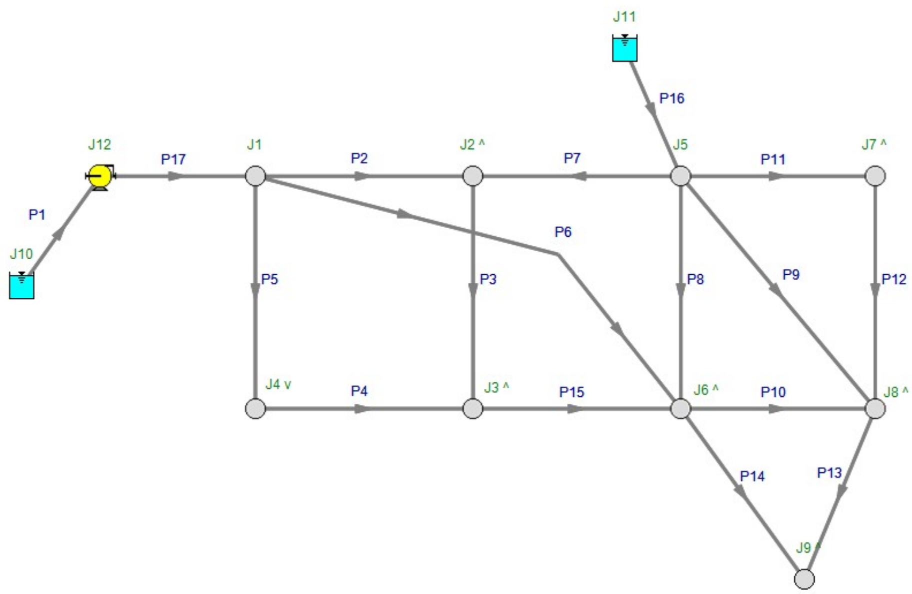
Pipe No.	8	9	10	11	12	13	14
Q (cfs) h_f (ft)	4.60 16.17	13.67 11.30	2.40 3.01	4.07 16.90	6.07 2.29	1.64 2.86	1.11 1.01

Pipe No.	15	16	17	18	19	20	21
Q (cfs) h_f (ft)	5.47 0.86	1.60 4.10	0.40 0.10	1.33 1.45	2.83 0.52	4.83 1.02	4.07 12.69

Pipe No.	22	23	24	25	26	27	28
Q (cfs) h_f (ft)	2.59 10.46	1.41 1.07	3.07 7.29	3.11 15.00	3.07 7.28	2.57 5.16	6.98 61.39

View Verification Case 61 Model

Verification Case 61



Verification Case 62

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify62.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 137-139

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	19.8	10.37	4.89	4	2.66	4.13	4.44	4.6
AFT Fathom	19.798	10.375	4.891	4.003	2.656	4.131	4.439	4.601

Pipe Flow Rate (ft3/sec)	9	10	11	12	13	14	15	16
Jeppson	13.67	2.4	4.07	6.07	1.64	1.11	5.47	1.6
AFT Fathom	13.667	2.402	4.066	6.066	1.638	1.108	5.468	1.598

Pipe Flow Rate (ft3/sec)	17	18	19	20	21	22	23	24
Jeppson	0.4	1.33	2.83	4.83	4.07	2.59	1.41	3.07
AFT Fathom	0.402	1.336	2.836	4.836	4.068	2.591	1.409	3.066

Pipe Flow Rate (ft3/sec)	25	26	27	28	29	30	31	32
Jeppson	3.11	3.07	2.57	6.98	2.48	8.02	10.26	12.26
AFT Fathom	3.112	3.067	2.573	6.975	2.483	8.019	10.262	12.262

Pipe Flow Rate (ft3/sec)	33	34	35	36	37	38	39	40
Jeppson	2.94	1.06	17.21	0.63	8.06	11.69	4.5	12.03
AFT Fathom	2.944	1.056	17.206	0.635	8.064	11.699	4.494	12.033

Pipe Flow Rate (ft3/sec)	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
Jeppson	12.55	8.16	29.73	26.58	19.58	2.48	4.96	9.74
AFT Fathom	12.548	8.160	29.731	26.581	19.571	2.480	4.964	9.745

Pipe Head Loss (feet)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Jeppson	10.59	2.97	0.69	24.57	0.22	17.43	10.04	16.17
AFT Fathom	10.562	2.961	0.686	24.513	0.214	17.385	10.019	16.129

Pipe Head Loss (feet)	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Jeppson	11.3	3.01	16.9	2.29	2.86	1.01	0.86	4.1
AFT Fathom	11.275	2.997	16.852	2.274	2.845	1.003	0.851	4.066

Pipe Head Loss (feet)	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Jeppson	0.1	1.45	0.52	1.02	12.69	10.46	1.07	7.29
AFT Fathom	0.098	1.438	0.518	1.007	12.650	10.428	1.062	7.251

Pipe Head Loss (feet)	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
Jeppson	15	7.28	5.16	61.39	0.81	7.89	1.46	5.16
AFT Fathom	14.939	7.259	5.143	61.221	0.802	7.869	1.449	5.137

Pipe Head Loss (feet)	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Jeppson	6.73	0.92	8.03	0.05	5.99	2.82	1.53	13.17
AFT Fathom	6.698	0.914	8.007	0.046	5.967	2.811	1.519	13.145

Pipe Head Loss (feet)	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
Jeppson	9.54	4.9	17.75	9.48	7.77	6.41	25.04	2.63
AFT Fathom	9.521	4.886	17.720	9.461	7.743	6.383	24.986	2.619

DISCUSSION:

The problem statement does not include any reservoirs or pressure junctions, and hence no EGL or pressure results can be obtained. Since AFT Fathom always displays EGL and pressure results, there must be a pressure. Thus junction 1 was chosen as a pressure junction and assigned a surface elevation of 200 feet. The particular value chosen does not affect the results.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson. The head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 62 Problem Statement

Verification Case 62

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 137-139

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pipe No.	8	9	10	11	12	13	14
Q (cfs)	-0.960	1.415	0.889	0.609	0.109	0.413	1.587
h_f (ft)	1.83	2.21	4.04	2.17	0.04	0.27	4.31

Pipe No.	15	16
Q (cfs)	2.391	0.025
h_f (ft)	2.62	0.00

Junction No.	1	2	3	4	5	6	7	8	9
Head (ft)	165.1	154.5	154.5	162.9	150.0	151.8	147.8	147.8	147.5

3. **Solve the network shown below. The wall roughness for all pipes is e = 0.012 inch.**

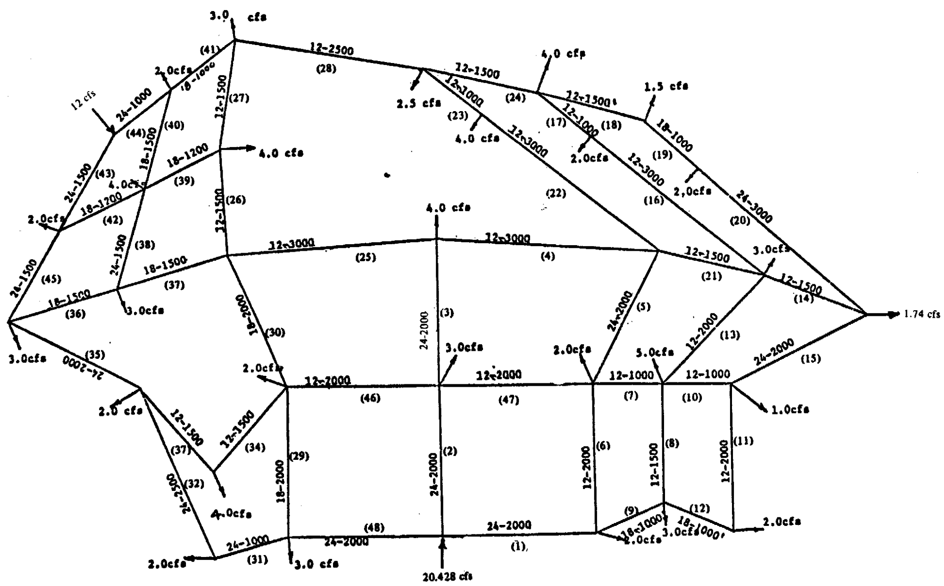
Solution:

Pipe No.	1	2	3	4	5	6	7
Q (cfs)	19.80	10.37	4.89	4.00	2.66	4.13	4.44
h_f (ft)	10.59	2.97	0.69	24.57	0.22	17.43	10.04

Pipe No.	8	9	10	11	12	13	14
Q (cfs)	4.60	13.67	2.40	4.07	6.07	1.64	1.11
h_f (ft)	16.17	11.30	3.01	16.90	2.29	2.86	1.01

Pipe No.	15	16	17	18	19	20	21
Q (cfs)	5.47	1.60	0.40	1.33	2.83	4.83	4.07
h_f (ft)	0.86	4.10	0.10	1.45	0.52	1.02	12.69

Pipe No.	22	23	24	25	26	27	28
Q (cfs)	2.59	1.41	3.07	3.11	3.07	2.57	6.98
h_f (ft)	10.46	1.07	7.29	15.00	7.28	5.16	61.39


3.0 cfs
12-2500
4.0 cfs
1.5 cfs
12 cfs
2.5 cfs
4.0 cfs
4.0 cfs
2.0cfs
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3.0cfs
1.74 cfs
3.0cfs
3.0cfs
2.0cfs
5.0cfs
1.0cfs
2.0cfs
2.0cfs
3.0 cfs
20.428 cfs
24-2000

Pipe No.	29	30	31	32	33	34	35
Q (cfs)	2.48	8.02	10.26	12.26	2.94	1.06	17.21
h_f (ft)	0.81	7.89	1.46	5.16	6.73	0.92	8.03

Pipe No.	36	37	38	39	40	41	42
Q (cfs)	0.63	8.06	11.69	4.50	12.03	12.55	8.16
h_f (ft)	0.05	5.99	2.82	1.53	13.17	9.54	4.90

Pipe No.	43	44	45	46	47	48
Q (cfs)	29.73	26.58	19.58	2.48	4.96	9.74
h_f (ft)	17.75	9.48	7.77	6.41	25.04	2.63

4. Solve the 48-pipe network of example problem 3 using the Hazen-Williams formula assuming the coefficient C_{HW} = 120 for all pipes.

Solution:

Pipe No.	1	2	3	4	5	6	7
Q (cfs)	19.65	10.25	4.79	3.93	2.60	4.06	4.42
h_f (ft)	11.44	3.42	0.84	25.51	0.27	18.06	10.53

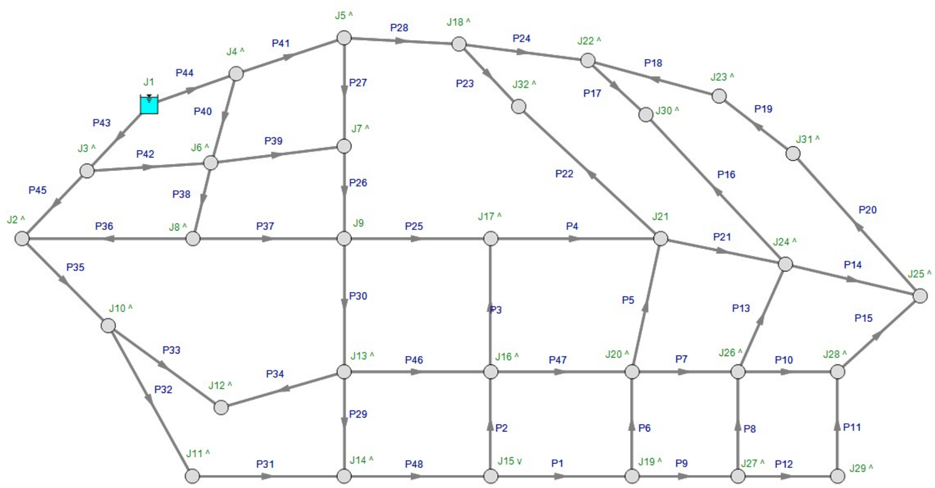
Pipe No.	8	9	10	11	12	13	14
Q (cfs)	4.58	13.59	2.39	4.01	6.01	1.61	1.09
h_f (ft)	16.87	11.72	3.37	17.64	2.59	3.23	1.78

Pipe No.	15	16	17	18	19	20	21
Q (cfs)	5.40	1.57	0.43	1.25	2.75	4.75	4.06
h_f (ft)	1.05	4.67	0.14	1.52	0.61	1.23	13.49

Pipe No.	22	23	24	25	26	27	28
Q (cfs)	2.48	1.52	3.18	3.14	3.04	2.47	7.20
h_f (ft)	10.88	1.46	8.60	16.83	7.93	5.39	65.07

View Verification Case 62 Model

Verification Case 62



Verification Case 63

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify63.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 94

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7
Jeppson	0.533	0.662	1.338	0.699	0.639	-0.129	0.828
AFT Fathom	0.5352	0.6636	1.3417	0.7008	0.6409	-0.1284	0.8291

Pipe Head Loss (feet)	1	2	3	4	5	6	7
Jeppson	1.306	17.384	15.563	2.201	8.127	0.38	13.474
AFT Fathom	1.303	17.390	15.569	2.192	8.124	-0.372	*13.460

Node EGL (feet)	1	2	3	4
Jeppson	98.7	81.3	96.9	99.1
AFT Fathom	98.7	81.31	96.88	99.07

Node pressure (psig)	1	2	3	4
Jeppson	8.1	0.56	7.32	8.28
AFT Fathom	8.095	0.566	7.307	8.256

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there is one pump in the example, there is one additional pipe in the AFT Fathom model. AFT Fathom pipes 7 and 8 together represent Jeppson pipe 7.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 63 Problem Statement

Verification Case 63

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 94

Jeppson Title Page

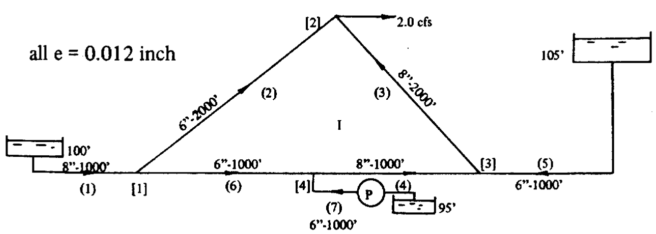
Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pipe No.	6	7	8	9	10
Flow rate (cfs) Head loss (ft)	1.432 38.15	0.326 3.86	0.174 0.85	1.758 12.83	2.242 20.70

PRV No.	1	2	3
Upstream elevation of HGL (ft) Downstream elevation of HGL (ft)	448.21 150.00	446.23 150.00	485.67 150.00

Example Problems Which Include Pumps and Reservoirs

1. Water supply comes from one pump and two reservoirs as shown in the sketch. How many pseudo loops need to be established? Write the system of equations whose solution provides the flow rate in each pipe using symbols K and n in the energy equations. The pump characteristic curve is given by: $h_p = -10.33\,Q_p^2 + 2.823\,Q_p + 22.29$.



Solution:

Two pseudo loops are required. A possibility is one pseudo loop connecting the reservoirs supplying pipes 1 and 5 through pipes 1, 6, 4, and 5; and the other connects the pump reservoir and the reservoir supplying pipe 1 through pipes 1, 6, and 7.

Continuity Equations
$$\begin{cases} -Q_1 + Q_2 + Q_6 = 0 \\ -Q_2 - Q_3 = -2.0 \\ Q_3 - Q_4 - Q_5 = 0 \\ Q_4 - Q_6 - Q_7 = 0 \end{cases}$$

Real Loop
$$\left\{ K_2 Q_2^{n_2} - K_3 Q_3^{n_3} - K_4 Q_4^{n_4} - K_6 Q_6^{n_6} = 0 \right.$$

Pseudo Loop

$$\begin{cases} K_1Q_1^{n_1} + K_6Q_6^{n_6} + K_4Q_4^{n_4} - K_5Q_5^{n_5} = -5 \\ K_1Q_1^{n_1} + K_6Q_6^{n_6} - K_7Q_7^{n_7} + 10.33\,G_1^2 \\ = -5 + 22.36 \end{cases}$$

Transformation Equation

$$\{ -Q_7 + G_1 = -0.137$$

2. **Solve the network of problem 1 giving the flow rates in each pipe, the head loss in each pipe, and the head and pressures at each junction, if the elevation of all junctions is at 80 ft.**

Solution:

Pipe No.	1	2	3	4	5	6	7
Flow rate (cfs)	0.533	0.662	1.338	0.699	0.639	-0.129	0.828
Head loss (ft)	1.306	17.384	15.563	2.201	8.127	0.380	13.474

Junction No.	1	2	3	4
Elev. HGL	98.7	81.3	96.9	99.1
Pressure (psi)	8.10	0.56	7.32	8.28

3. **Solve problem 1 if the pump is removed from pipeline 7 and the system is supplied by the three reservoirs.**

Solution:

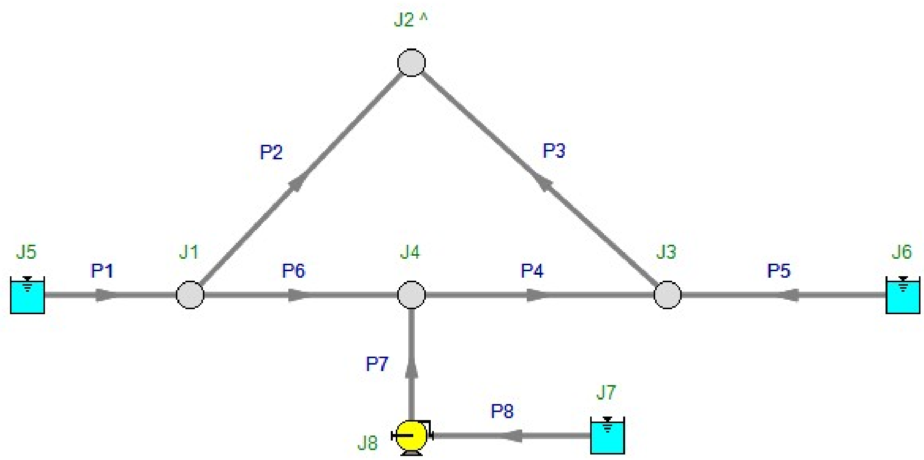
To solve this system there will only be seven unknowns instead of the eight as in problems 1 and 2, since the pump does not introduce an additional unknown G. The four continuity equations and the first two energy equations are identical to those given in the solution to problem 1. The final energy equation is,

$$K_1Q_1^{n_1} + K_6Q_6^{n_6} - K_7Q_7^{n_7} = -5$$

The solution by the linear theory method produces the following after three iterative solutions.

View Verification Case 63 Model

Verification Case 63



Verification Case 64

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify64.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 109-110

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	2.56	-0.32	2.44	0.73	0.88	1.12	1.83	3.17
AFT Fathom	2.53	-0.38	2.47	0.72	0.92	1.08	1.81	3.19

Pipe Head Loss (feet)	1	2	3	4	5	6	7	8
Jeppson	130.61	1.89	119.67	8.75	18.42	24.12	6.33	17.88
AFT Fathom	*128.35	**-2.66	122.19	8.50	*19.98	22.64	6.25	17.75

PRV EGL (feet)	Up	Down
Jeppson	53.8	50.0
AFT Fathom	49.9	49.9

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump and PRV data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps and PRVs at boundaries between pipes. Pumps and PRVs are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump or PRV is split into two equivalent pipes in AFT Fathom. In the case of the pump, where the split is made will have no impact on the results. If the PRV control pressure is specified in terms of head, the elevation of the PRV becomes important. In such cases, Jeppson specifies the elevation and AFT Fathom incorporates this.

Because there is one pump and one PRV in the example, there are two additional pipes in the AFT Fathom model. AFT Fathom pipes 1 and 10 together represent Jeppson pipe 1. Similarly, AFT Fathom pipes 5 and 9 represent Jeppson pipe 5.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson's results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the pipe head loss formula used by Jeppson differs from AFT Fathom. Jeppson's Hazen-Williams formula is given in his book and does not agree exactly with the accepted formula as used in AFT Fathom. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

Results for AFT Fathom also vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom help file.

Because of the slight differences in calculations between AFT Fathom and Jeppson, there is slightly less pressure head available across the PRV than the 50 feet specified in the problem statement. Thus, there is a warning message generated in AFT Fathom that the control valve is unable to control, and has failed open. Results are displayed above for the failed open case. AFT Fathom shows warnings in the Warnings section at the top of the Output window. In addition, the Valve Summary at the top of the Output window shows the PRV status.

List of All Verification Models

Verification Case 64 Problem Statement

Verification Case 64

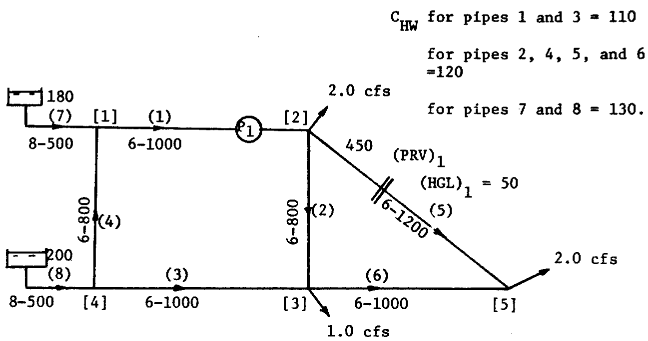
Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 109-110

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Junction	HGL (m)	Head (m)	Pressure (N/cm^2)
1	308.6	8.6	8.43
2	255.0	5.0	4.92
3	262.2	7.2	7.03
4	289.6	29.6	29.0
5	281.1	1.12	1.09
6	281.3	1.34	1.31
7	349.5	9.5	9.31
8	286.4	16.4	16.1
9	339.4	3.4	9.19
10	377.5	17.5	17.1
11	260.4	0.45	0.44
12	425.9	5.93	5.81
13	275.9	5.88	5.76
14	259.6	15.6	15.3
15	284.9	24.9	24.4
16	273.8	13.8	13.5
17	274.6	14.6	14.3
18	302.2	22.2	21.7
19	277.7	7.7	7.56
20	315.5	15.5	15.2
21	270.3	20.3	19.9
22	268.4	8.4	8.25
23	210.4	10.4	10.2
24	174.2	14.2	14.0
25	171.3	11.3	11.0
26	88.3	18.3	17.9
27	81.3	11.3	11.0
28	45.1	5.1	5.0
29	45.2	25.2	24.7
30	284.8	24.8	24.4
31	230.1	30.1	29.5
32	-37.5	12.5	12.2
33	-51.8	8.2	8.0

11. Solve the 8-pipe network shown in the accompanying sketch which is supplied by two reservoirs with water surfaces at 180 and 200 ft, respectively. The PRV is located 450 ft downstream from the upstream junction of pipe 5 and has a pressure setting which maintains the elevation of the HGL at 50 ft downstream from the valve.



Pump Characteristics

Q_{P_1} = 1.0 cfs, h_{P_1} = 40.', Q_{P_2} = 1.5 cfs, h_{P_2} = 35.', Q_{P_3} = 2.0 cfs, h_{P_3} = 26.'

Solution:

Pipe No.	1	2	3	4
Flow rate (cfs) Head loss (ft)	2.56 130.61	-0.32 1.89	2.44 119.67	0.73 8.75

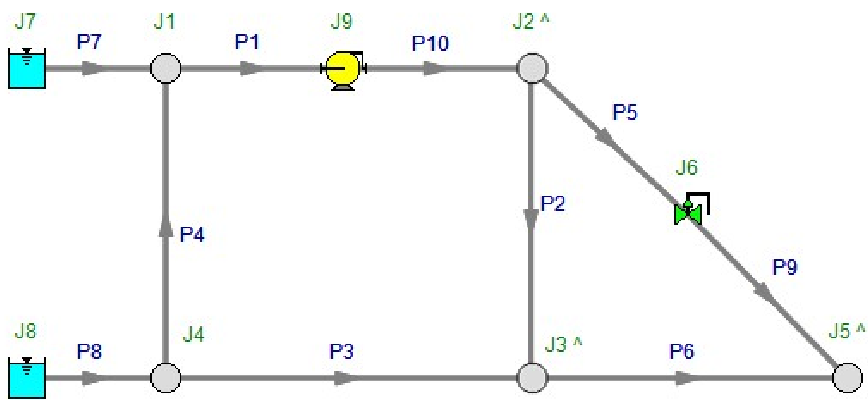
Pipe No.	5	6	7	8
Flow rate (cfs) Head loss (ft)	0.88 18.42	1.12 24.12	1.83 6.33	3.17 17.88

HGL upstream from PRV = 53.80 ft
HGL downstream from PRV = 50.00 ft

12. **Obtain the flow rates in each pipe, and the pressure at each junction of the network in the accompanying sketch. Water is flowing (in SI units the kinematic viscosity $\nu = 0.00000113$ N^2/s = 0.0133 Stokes, and the specific weight $\gamma = 9800$ N/m^3).**

View Verification Case 64 Model

Verification Case 64



Verification Case 65

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify65.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 110-111

FLUID: Water

ASSUMPTIONS: Assume water at 15 deg. C.

RESULTS:

Pipe Flow Rate (m3/sec)	1	2	3	4	5	6	7	8
Jeppson	0.6402	0.2857	0.1857	-0.0857	0.2344	-0.2402	0	0
AFT Fathom	0.651	0.282	0.182	-0.082	0.249	-0.251	0.000	0.000

Pipe Flow Rate (m3/sec)	9	10
Jeppson	0.12	-0.1902
AFT Fathom	0.120	-0.201

Pipe Head Loss (m)	1	2	3	4	5	6	7	8
Jeppson	6	16.21	12.32	1.79	30.32	11.77	0	0
AFT Fathom	6.132	15.664	11.798	-1.662**	29.124	-12.640**	0.000*	0.000*

Pipe Head Loss (m)	9	10
Jeppson	56.47	1.91
AFT Fathom	55.766	-2.104**

Node EGL (meters)	1	2	3	4	5	6
Jeppson	294	277.79	265.48	263.68	251.91	237.54
AFT Fathom	293.9	278.2	266.4	264.7	252.1	238.1

PRV EGL (meters)	1 Up	1 Down	2 Up	2 Down
Jeppson	251.91	237.54	265.48	237.54
AFT Fathom	252.1	238.1	266.4	238.1

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying PRV data is to lump it into a pipe, whereas AFT Fathom's method is to place PRVs at boundaries between pipes. PRVs are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the PRV is split into two equivalent pipes in AFT Fathom. If the PRV control pressure is specified in terms of head, the elevation of the PRV becomes important. In such cases, Jeppson specifies the elevation and AFT Fathom incorporates this.

Because there are two PRVs in the example, there are two additional pipes in the AFT Fathom model. AFT Fathom pipes 7 and 11 together represent Jeppson pipe 7. Similarly, AFT Fathom pipes 8 and 12 represent Jeppson pipe 8.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

In both Jeppson's solution and AFT Fathom's, the two PRVs cannot control to their set pressure head, and fail closed. Results are displayed above for the failed closed case. AFT Fathom shows warnings in the Warnings section at the top of the Output window. In addition, the Valve Summary at the top of the Output window shows the PRV status.

Results differ slightly between AFT Fathom and Jeppson. The head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 65 Problem Statement

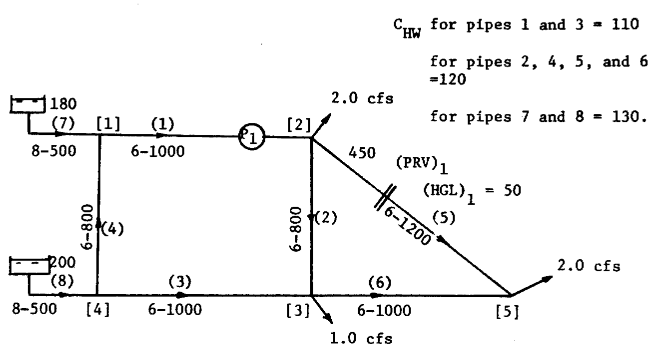
Verification Case 65

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 110-111

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

110 **PIPE NETWORK ANALYSES**



Pump Characteristics

$Q_{P_1} = 1.0$ cfs, $h_{P_1} = 40.'$, $Q_{P_2} = 1.5$ cfs, $h_{P_2} = 35.'$, $Q_{P_3} = 2.0$ cfs, $h_{P_3} = 26.'$

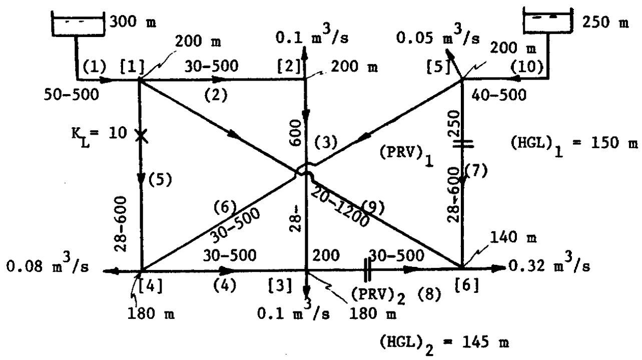
Solution:

Pipe No.	1	2	3	4
Flow rate (cfs) Head loss (ft)	2.56 130.61	-0.32 1.89	2.44 119.67	0.73 8.75

Pipe No.	5	6	7	8
Flow rate (cfs) Head loss (ft)	0.88 18.42	1.12 24.12	1.83 6.33	3.17 17.88

HGL upstream from PRV = 53.80 ft
HGL downstream from PRV = 50.00 ft

12. Obtain the flow rates in each pipe, and the pressure at each junction of the network in the accompanying sketch. Water is flowing (in SI units the kinematic viscosity $\nu = 0.00000113$ N²/s = 0.0133 Stokes, and the specific weight $\gamma = 9800$ N/m³).



Solution:

Pipe No.	1	2	3	4	5
Flow rate m^3/s Head loss (m)	0.6402 6.00	0.2857 16.21	0.1857 12.32	-0.0857 1.79	0.2344 30.32

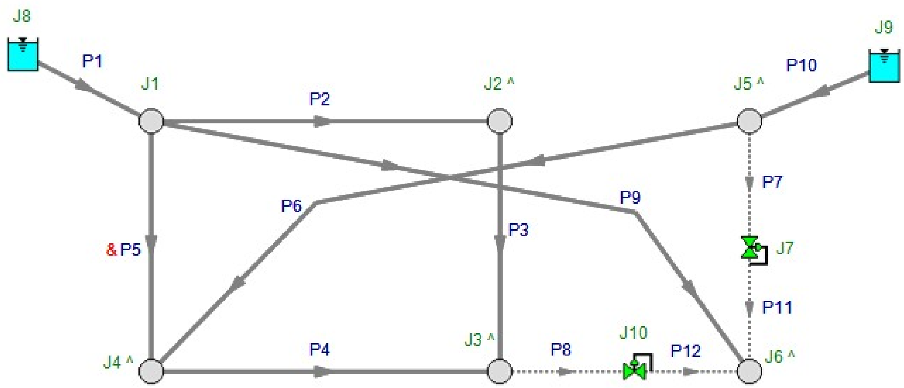
Pipe No.	6	7	8	9	10
Flow rate m^3/s Head loss (m)	-0.2402 11.77	0.0 0.0	0.0 0.0	0.12 56.47	-0.1902 1.91

Junction No.	1	2	3	4	5	6
Elev. of HGL Pressure (kPa)	294.0	277.79	265.48	263.68	251.91	237.54

PRV No.	1	2
Upstream, HGL (m) Downstream, HGL (m)	251.91 237.54	265.48 237.54

View Verification Case 65 Model

Verification Case 65



Verification Case 66

View Model Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify66.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 88-89

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	0.5	0.37	2.16	3.16	0	1	0.25	0.25
AFT Fathom	0.500	0.374	2.168	3.168*	0.000*	1.000	0.246	0.254

Pipe Flow Rate (ft3/sec)	9	10	11	12	13	14
Jeppson	0.05	1	0.79	0.71	0.13	1.84
AFT Fathom	0.046	1.000	0.794	0.706	0.126	1.832

Pipe Head Loss (feet)	1	2	3	4	5	6	7	8
Jeppson	2.44	0.63	19.27	6.4	0	6.4	1.01	1.06
AFT Fathom	2.409	0.636	19.217	6.375*	0.000*	6.329	0.989	1.045

Pipe Head Loss (feet)	9	10	11	12	13	14
Jeppson	0.06	2.56	2.73	2.22	0.09	2.21
AFT Fathom	0.056	2.532	2.699	2.148	0.085	2.179

PRV EGL (feet)	Up	Down
Jeppson	417.6	393.04
AFT Fathom	417.6	393.1

* AFT Fathom results combine two pipes, as discussed below

** The problem statement has a typo on pipe length. It says 1000 ft., and it should only be 500 ft.

DISCUSSION:

Jeppson's method of applying pump and PRV data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps and PRVs at boundaries between pipes. Pumps and PRVs are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump or PRV is split into two equivalent pipes in AFT Fathom. In the case of the pump, where the split is made will have no impact on the results. If the PRV control pressure is specified in terms of head, the elevation of the PRV becomes important. In such cases, Jeppson specifies the elevation and AFT Fathom incorporates this.

Because there is one pump and one PRV in the example, there are two additional pipes in the AFT Fathom model. AFT Fathom pipes 4 and 16 together represent Jeppson pipe 4. Similarly, AFT Fathom pipes 5 and 15 represent Jeppson pipe 5.

In both Jeppson's solution and AFT Fathom's, the PRV cannot control to its set pressure head, and it fails closed. Results are displayed above for the failed closed case. AFT Fathom shows warnings in the Warnings section at the top of the Output window. In addition, the Valve Summary at the top of the Output window shows the PRV status.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 66 Problem Statement

Verification Case 66

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 88-89

Jeppson Title Page

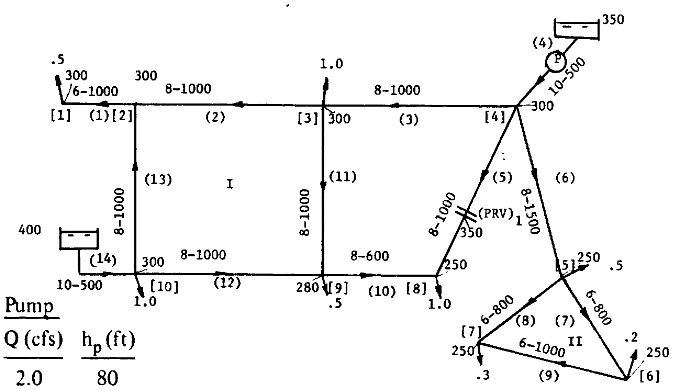
Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

The pressure upstream from the PRV equals 117.19 ft. and downstream equals 55 ft. Consequently, the assumption used in writing the final loop equation above is correct. Had the solution given a negative flow rate in pipe 6, this assumption would be incorrect, since the PRV would then have acted as a check valve and allowed the elevation of the HGL downstream from the PRV to rise above 55 ft. Should this have occurred, then the flow rate in pipe 6 would no longer be unknown, but equal to zero. Under these conditions caused by the PRV shutting flow off, the final equation above would be replaced by:

$$-(HGL)_1 - K_5 Q_5^{n_5} - K_4 Q_4^{n_4} = -100$$

From the above example, it should be evident how networks containing PRV's might be analyzed by the linear theory method. The assumption is initially made that the pressure (or the elevation of the HGL) immediately downstream from the PRV is constant and equal to the valve setting. The junction continuity equations are written as if no PRV's existed. To write the loop equations, pipes containing PRV's are disconnected from their upstream junctions and the PRV's are replaced by reservoirs. The real and pseudo loops are established for this modified network. After each iteration a check of the flow Q in each pipe containing a PRV is made. Should any of these flows be negative, then the pseudo loop equation which includes terms for that pipe is altered, replacing that Q as an unknown with the HGL immediately downstream from that PRV.

Example No. 2 As an example of a network in which a PRV acts as a check valve, consider the 14-pipe network in the sketch below. In this network, pipe 5 contains a PRV with a pressure setting which results in



Pump

Q (cfs)	h_p (ft)
2.0	80
3.0	75
4.0	68

the elevation of the HGL downstream from the valve equaling 350 ft. For this network 10 linear junction continuity equations are available, and two energy equations around the real loops formed by pipes 2, 11, 12, and 13, and 8, 9, and 7, respectively. One additional energy equation is available from a pseudo loop connecting the reservoir to the source pump through a series of pipes such as 14, 13, 2, 3, and 4. The tenth and final equation is obtained by connecting the artificial reservoir created by the PRV to the real reservoir or source pump by a series of pipes such as 5, 10, 12, and 14. This final equation is,

$$K_5 Q_5^{n_5} - K_{10} Q_{10}^{n_{10}} - K_{12} Q_{12}^{n_{12}} - K_{14} Q_{14}^{n_{14}} = (HGL)_1 - 400$$

Initially, $(HGL)_1$ might be assumed known and equal to 350 ft. If this is done, however, the linear theory method immediately produces a negative flow rate, Q_5, from the first iteration. Consequently $Q_5 = 0$ and the above equation should be rewritten with $(HGL)_1$ unknown as,

$$-(HGL)_1 - K_{10} Q_{10}^{n_{10}} - K_{12} Q_{12}^{n_{12}} - K_{14} Q_{14}^{n_{14}} = -400$$

The solution obtained using this equation as the fourteenth equation of the system of equations is

Pipe No.	1	2	3	4	5	6	7
Flow rate (cfs)	0.50	0.37	2.16	3.16	0.0	1.00	0.25
Head loss (ft)	2.44	0.63	19.27	6.40	0.0	6.40	1.01

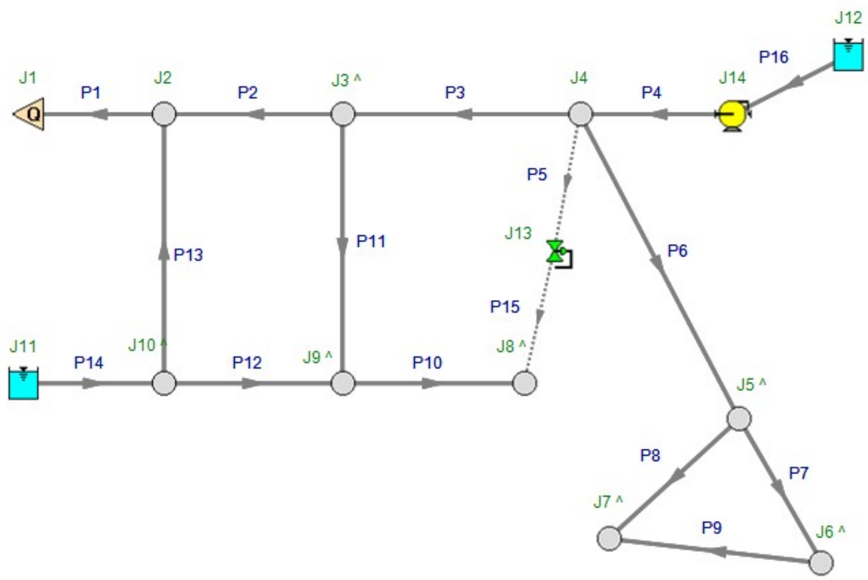
Pipe No.	8	9	10	11	12	13	14
Flow rate (cfs)	0.25	0.05	1.00	0.79	0.71	0.13	1.84
Head loss (ft)	1.06	0.06	2.56	2.73	2.22	0.09	2.21

The elevations of the HGL upstream and downstream from the PRV are 417.60 ft and $(HGL)_1 = 393.04$ ft, respectively.

While it is possible to obtain a solution to a network such as this 14-pipe network in a single analysis, it might be analyzed piecemeal more efficiently. Since all the demands at junctions 5, 6, and 7 must come through pipe 6, its flow rate can be determined readily as the sum of these demands or $Q_6 = 1.0$ cfs. Now the subnetwork consisting of pipes 7, 8, and 9 can be disconnected from the remainder of the network and each subnetwork analyzed separately. When large distribution systems are involved, considerable reduction in storage requirements as well as computational efficiency can result by disconnecting the network into subnetworks where possible.

View Verification Case 66 Model

Verification Case 66



Verification Case 67

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify67.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 95-98

FLUID: Water

ASSUMPTIONS: Assume water at 60 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	2.94	-1.76	-0.54	1.74	0.88	-2.55	-3.35	2.17
AFT Fathom	2.94	-1.76	-0.54	1.74	0.88	-2.56	-3.36	2.18

Pipe Flow Rate (ft3/sec)	9	10	11	12	13	14	15	16
Jeppson	3.07	-0.44	-0.58	0.64	0.73	1.32	1.18	0.80
AFT Fathom	3.07	-0.44	-0.58	0.64	0.73	1.32	-1.19	0.80

Pipe Flow Rate (ft3/sec)	17	18	19	20	21	22	23	24
Jeppson	-2.29	-0.17	0.09	3.27	2.45	-0.04	1.15	-0.41
AFT Fathom	-2.29	-0.17	0.09	3.28	2.46	-0.04	1.15	-0.42

Pipe Flow Rate (ft3/sec)	25	26	27	28
Jeppson	6.84	6.01	3.35	-2.39
AFT Fathom	6.85	6.02	3.34	-2.40

Pipe Head Loss (feet)	1	2	3	4	5	6	7	8
Jeppson	6.41	11.85	0.62	9.10	10.76	30.20	45.50	51.10
AFT Fathom	6.38	-11.80	-0.61	9.02	10.62	-30.11	-45.32	50.87

Pipe Head Loss (feet)	9	10	11	12	13	14	15	16
Jeppson	43.90	7.92	9.90	8.20	9.38	59.00	29.60	23.40
AFT Fathom	43.73	-7.87	-9.86	8.15	9.29	58.74	-29.47	23.28

Pipe Head Loss (feet)	17	18	19	20	21	22	23	24
Jeppson	7.86	1.37	0.28	37.60	30.70	0.01	6.90	6.91
AFT Fathom	-7.82	-1.32	0.27	37.46	30.56	-0.01	6.89	-6.90

Pipe Head Loss (feet)	25	26	27	28
Jeppson	30.6	0.91	0.31	8.43
AFT Fathom	*30.517	*0.90	*0.304	-8.437

Node EGL (feet)	1	2	3	4	5	6	7	8
Jeppson	1365	1359	1347	1348	1357	1346	1316	1270
AFT Fathom	1365.2	1358.8	1347.0	1347.7	1356.7	1346.1	1315.9	1270.6

Node EGL (feet)	9	10	11	12	13	14	15	16
Jeppson	1321	1329	1339	1347	1392	1354	1361	1354
AFT Fathom	1321.5	1329.4	1339.2	1347.4	1392.0	1354.5	1361.4	1354.5

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there are three pumps in the example, there are three additional pipes in the AFT Fathom model. AFT Fathom pipes 25 and 29 together represent Jeppson pipe 25. Similarly, AFT Fathom pipes 26 and 31 represent Jeppson pipe 26, and AFT Fathom pipes 27 and 30 represent Jeppson pipe 27.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

Results for AFT Fathom vary somewhat from previous versions of AFT Fathom (prior to version 7) because the equation used to convert the Hazen-Williams factor to the Darcy-Weisbach friction factor was modified to use the traditional formula, as given in the AFT Fathom help file.

List of All Verification Models

Verification Case 67 Problem Statement

Verification Case 67

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 95-98

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Pipe No.	1	2	3	4	5	6	7
Q (cfs) h_f (ft)	0.979 4.23	0.680 18.32	1.320 15.16	0.527 1.28	0.793 12.39	0.300 1.88	0.227 1.11

4. Solve the single real loop pipe network supplied by the three pumps described just before these example problems by the Hazen-Williams equation assuming all pipes have a coefficient $C_{HW} = 120$.

Solution:

The equations are the same as previously given with the exception that the n's = 1.852, and the K's are constant and given by: $K_1 = 4.85$, $K_2 = 39.3$, $K_3 = 9.69$, $K_4 = 4.85$, $K_5 = 19.7$, $K_6 = 19.7$, $K_7 = 19.7$. The solution is

Pipe No.	1	2	3	4	5	6	7
Q (cfs) h_f (ft)	0.784 3.09	0.681 19.3	1.313 16.1	0.564 1.68	0.751 11.6	0.257 1.59	0.368 3.09

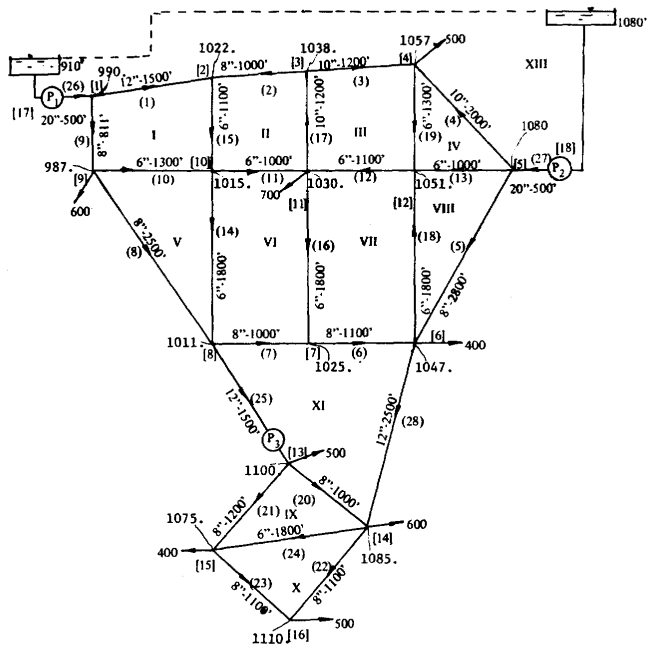
5. Solve problem 3 using the Hazen-Williams formula with all $C_{HW} = 120$.

Solution:

Pipe No.	1	2	3	4	5	6	7
Q (cfs) h_f (ft)	0.965 4.53	0.684 19.45	1.316 16.13	0.522 1.45	0.795 12.86	0.281 1.87	0.241 1.41

6. Obtain a solution to the 28 pipe network below by the linear theory method through use of a program such as the one whose input is described in Appendix C.

96 PIPE NETWORK ANALYSES



Pipe No.	C_{HW}	Pipe No.	C_{HW}
1	130	15	120
2	130	16	120
3	120	17	130
4	120	18	120
5	120	19	120
6	120	20	130
7	120	21	120
8	120	22	120
9	100	23	120
10	100	24	120
11	100	25	130
12	130	26	110
13	130	27	110
14	120	28	120

Pump Characteristics

Pump No. 1 $h_p = -0.417\,Q_p^2 + 5.14\,Q_p + 440.3$

Pump No. 2 $h_p = -0.378\,Q_p^2 - 5.09\,Q_p + 298.2$

Pump No. 3 $h_p = -2.51\,Q_p^2 + 16.7\,Q_p + 155.3$

Data for program:

```
    O PROBLEM 6, CHAPTER  5
    $SPECIF NPIPE=28,NRES=0,NPUMP=3,NBPUMP=1,NFLOW=1,COEFRO-120.,
     NPLENG=1 $END
    27
    1080.
26  27  25
5.568  455.97  7.0156  455.81  7.795  455.  910.
 2.004  280.5  2.3385  284.25  3.341  277.  1080.
4.009  182.  4.454  180.  4.9  177.  0.
1  12.   1.5   130.  1  2  1022.
2  8.  1.  130.  3  2
3  10.   1.2   3  4  500.  1057
4  10.   2.  5  4
5  8.  2.8   5  6  400.  1047.
6  8.  1.1   7  6
7  8.  1.  8  7
8  8.  2.5   9  8  1011.
9  8.  .811  100.  1  9  600.  987.
10 6.  1.3   100.  9  10 1015.
11 6.  1.  100.  10 11 700.  1030.
12 6.  1.1   130.  12 11
13 6.  1.  130.  5  12 1051.
14 6.  1.8   10 8
15 6.  1.1   2  10
16 6.  1.8   11 7  1025.
17 10.   1.2   130.  11 3  1038.
18 6.  1.8   6  12
19 6.  1.3   4  12
20 8.  1.  130.  13 14 600.  1085.
21 8.  1.2   13 15 400.  1075.
22 8.  1.1   14 16 500.  1110.
23 8.  1.1   15 16
24 6.  1.8   14 15
25 12. 1.5 130. 8  13  500. 1100
26 20.   .5  110.  1  990.
27 20.   .5  110.  5  1080.
28 12.   2.5    6  14
END
```

Solution:

One pseudo loop must be defined which connects the reservoirs from which pumps 1 and 2 obtain their supplies, giving 16 junction continuity equations and 12 energy equations. In addition three linear transformation equations are added for the three pumps in the network. The data required by the program in Appendix C follows. The solution is provided below.

Pipe No.	1	2	3	4	5	6	7
Q (cfs)	2.94	-1.76	-0.54	1.74	0.88	-2.55	-3.35
h_f (ft)	6.41	11.85	0.62	9.10	10.76	30.2	45.5

Pipe No.	8	9	10	11	12	13	14
Q (cfs)	2.17	3.07	-0.44	-0.58	0.64	0.73	1.32
h_f (ft)	51.1	43.9	7.92	9.90	8.20	9.38	59.0

Pipe No.	15	16	17	18	19	20	21
Q (cfs)	1.18	0.80	-2.29	-0.17	0.09	3.27	2.45
h_f (ft)	29.6	23.4	7.86	1.37	0.28	37.6	30.7

Pipe No.	22	23	24	25	26	27	28
Q (cfs)	-0.04	1.15	-0.41	6.84	6.01	3.35	-2.39
h_f (ft)	0.01	6.90	6.91	30.6	0.91	0.31	8.43

Junction No.	1	2	3	4	5	6	7	8
Elev. HGL (ft)	1365	1359	1347	1348	1357	1346	1316	1270

Junction No.	9	10	11	12	13	14	15	16
Elev. HGL (ft)	1321	1329	1339	1347	1392	1354	1361	1354

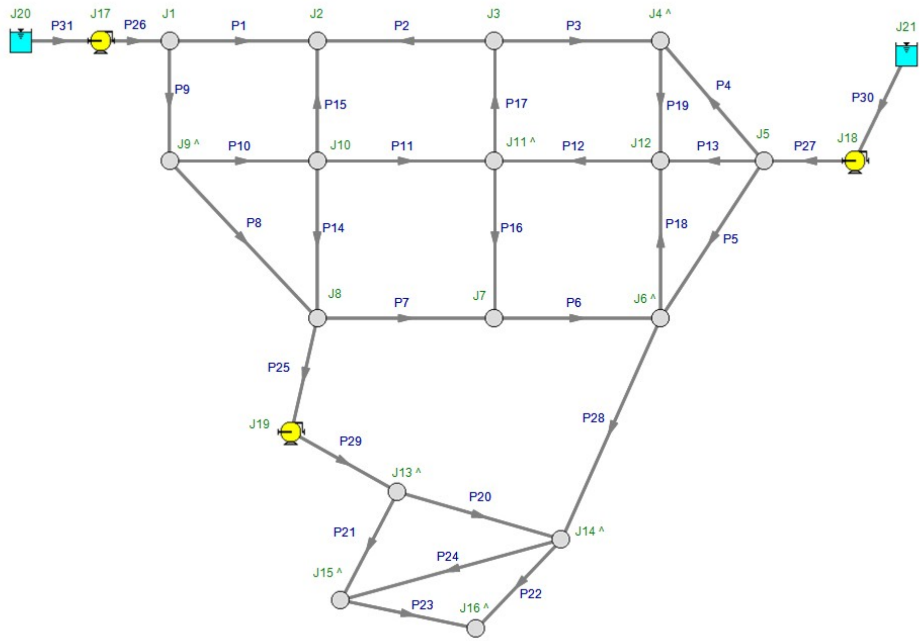
7. **What are the flow rates and head losses, as well as the elevation of the HGL in the network of problem 6 if pipeline 28 is taken out of operation. If the elevation at junctions are as given below in one of the tables of solutions what is the pressure at each junction. The solution should be obtained by the computer program such as those whose input is described in Appendix C.**

Solution:

With pipeline 28 removed a single line connects two separate networks, each of which might be solved as separate problems. Obviously, if the analysis were being done by hand two separate analyses would be used and subsequently tied together. With the computer, however, it is more convenient to consider only one network containing 27 pipes, 16 junctions, and 10 real loops. The additional energy equation comes from

View Verification Case 67 Model

Verification Case 67



Verification Case 68

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify68.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 105-109

FLUID: Water

ASSUMPTIONS: Assume water at 22 deg. C.

RESULTS:

Pipe Flow Rate (m3/sec)	1	2	3	4	5	6	7	8
Jeppson	0.0869	0.0208	0.0132	0.0153	0.0092	0.0274	0.0254	0.0148
AFT Fathom	0.087	0.021	0.013	0.015	0.009	0.027	0.026	0.015

Pipe Flow Rate (m3/sec)	9	10	11	12	13	14	15	16
Jeppson	-0.0027	0.0299	0.0212	0.0081	0.0089	0.0164	0.0273	0.0107
AFT Fathom	-0.002	0.031	0.022	0.008	0.007	0.019	-0.026	0.011

Pipe Flow Rate (m3/sec)	17	18	19	20	21	22	23	24
Jeppson	0.0118	0.0078	0.0243	0.0791	0.0351	-0.0035	0.0106	0.0166
AFT Fathom	0.012	0.007	0.025	0.079	0.035	-0.003	-0.010	0.016

Pipe Flow Rate (m3/sec)	25	26	27	28	29	30	31	32
Jeppson	0.0119	0.0277	0.0805	-0.0043	0.0061	-0.0065	0.0148	0.0071
AFT Fathom	0.012	0.028	0.081	-0.004	0.006	-0.006	0.015	0.007

Pipe Flow Rate (m3/sec)	33	34	35	36	37	38	39	40
Jeppson	0.0046	0.0274	0.0094	0.0219	0.1306	-0.0214	-0.0299	-0.0238
AFT Fathom	0.005	0.027	0.009	0.022	0.130	-0.021	-0.030	-0.024

Pipe Flow Rate (m3/sec)	41	42	43	44	45	46	47	48
Jeppson	0.0239	0.0287	0.045	0.0139	-0.0002	0.0056	0.0051	0.0167
AFT Fathom	0.024	0.029	0.045	0.014	0.000	0.006	0.005	0.017

Pipe Flow Rate (m3/sec)	49	50	51	52	53	54	55	56
Jeppson	0.0137	-0.0078	0.0222	0.0547	0.021	0.0165	0.0135	-0.002
AFT Fathom	0.014	-0.008	0.022	0.055	0.021	0.016	0.013	-0.002

Pipe Flow Rate (m3/sec)	57	58	59	60	61	62	63
Jeppson	0.0116	0.0109	0.0031	0.0072	0.0078	0.0003	0.1276
AFT Fathom	0.012	0.011	0.003	0.007	0.008	0.000	0.127

Pipe Head Loss (meters)	1	2	3	4	5	6	7	8
Jeppson	201.1	53.6	19.1	27.4	7.16	27.5	27.3	8.3
AFT Fathom	*201.95	53.59	19.23	27.33	7.02	27.42	27.31	8.08

Pipe Head Loss (meters)	9	10	11	12	13	14	15	16
Jeppson	0.22	68.4	63.1	5.11	10.1	38.1	91	78.9
AFT Fathom	-0.11	71.74	67.04	4.60	6.94	22.29	-82.39	83.02

Pipe Head Loss (meters)	17	18	19	20	21	22	23	24
Jeppson	165.5	86.5	48.4	166.9	150	16.3	117.7	45.8
AFT Fathom	156.70	73.67	51.38	*164.52	146.50	-14.69	-109.81	44.37

Pipe Head Loss (meters)	25	26	27	28	29	30	31	32
Jeppson	29.7	25.3	287.6	1.37	12.52	3.74	11.16	6.62
AFT Fathom	29.68	26.17	*289.26	-1.25	12.24	-3.35	10.99	6.82

Pipe Head Loss (meters)	33	34	35	36	37	38	39	40
Jeppson	0.8	27.6	3.5	31.8	297.8	12.7	24.4	15.7
AFT Fathom	0.82	27.31	3.47	31.60	299.65	-12.41	-24.29	-15.45

Pipe Head Loss (meters)	41	42	43	44	45	46	47	48
Jeppson	37.8	45.2	54.6	11.1	0.11	55	54.9	267.5
AFT Fathom	37.41	44.72	*54.40	10.98	-0.01	55.85	55.84	266.16

Pipe Head Loss (meters)	49	50	51	52	53	54	55	56
Jeppson	322.4	14.3	336.7	240.7	94.2	58.1	39.1	2.97
AFT Fathom	322.00	-14.12	336.10	*242.34	93.82	57.84	38.90	-2.92

Pipe Head Loss (meters)	57	58	59	60	61	62	63
Jeppson	93	82.9	7.07	36.2	43.2	0.12	284.5
AFT Fathom	92.57	82.65	7.01	35.99	42.89	0.11	286.52

Node EGL (meters)	1	2	3	4	5	6	7	8
Jeppson	308.6	255	262.2	289.6	281.1	281.3	349.5	286.4
AFT Fathom	307.17	253.58	260.61	287.94	279.75	279.86	351.50	284.46

Node EGL (meters)	9	10	11	12	13	14	15	16
Jeppson	339.4	377.5	260.4	425.9	275.9	259.6	284.9	273.8
AFT Fathom	344.56	366.85	261.54	418.23	271.73	257.04	283.21	272.22

Node EGL (meters)	17	18	19	20	21	22	23	24
Jeppson	274.6	302.2	277.7	315.5	270.3	268.4	210.4	174.2
AFT Fathom	273.04	300.35	276.06	313.48	268.75	266.57	208.73	172.75

Node EGL (meters)	25	26	27	28	29	30	31	32
Jeppson	171.3	88.3	81.3	45.1	45.2	284.8	230.1	-37.5
AFT Fathom	169.83	87.18	80.18	44.18	44.29	283.20	227.36	-38.80

Node EGL (meters)	33
Jeppson	-51.8
AFT Fathom	-52.92

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there are five pumps in the example, there are five additional pipes in the AFT Fathom model. AFT Fathom pipes 1 and 64 together represent Jeppson pipe 1. Similarly, AFT Fathom pipes 20 and 65 represent Jeppson pipe 20, AFT Fathom pipes 27 and 66 represent Jeppson pipe 27, AFT Fathom pipes 43 and 68 represent Jeppson pipe 43, and AFT Fathom pipes 52 and 67 represent Jeppson pipe 52.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree. Third, this system is highly networked which may cause some individual pipes, especially those with lower flow rates, to differ quite a bit from AFT Fathom. Looking at the system as a whole, the agreement is quite good between Jeppson and AFT Fathom.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 68 Problem Statement

Verification Case 68

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 105-109

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Junction No.	1	2	3	4	5	6
HGL (ft) Pressure (psi)	390.2 104.1	376.0 97.9	366.7 89.6	363.9 92.7	366.6 87.3	363.1 88.0

Junction No.	7	8	9	10	11	12
HGL (ft) Pressure (psi)	374.2 84.2	372.3 79.0	376.1 76.3	369.1 81.9	368.9 77.5	363.2 75.0

Junction No.	13	14	15	16	17	18
HGL (ft) Pressure (psi)	345.0 71.5	341.8 78.8	362.0 91.9	333.8 79.6	340.9 87.1	343.3 83.8

Junction No.	19	20	21	22	23	24
HGL (ft) Pressure (psi)	348.1 77.2	364.9 80.1	351.5 78.65	347.3 81.2	342.4 83.4	340.8 82.7

Junction No.	25	26	27	28	29
HGL (ft) Pressure (psi)	319.5 77.8	316.1 80.7	342.3 92.0	346.1 93.6	347.3 96.3

Using the linear theory program described in Appendix C, the solution was obtained in six iterations with a sum of changes in flow rate equal to 0.019. The solution requires 23.3 seconds of execution time on a UNIVAC 1108.

10. Determine the flow rates and head losses of each of the 63 pipes of the network shown below. The diameter of each pipe is given in centimeters and the length of each pipe is given in meters. Thus pipe number 1 is 16 cm in diameter and has a length of 1500 m. All 63 pipes of the network have a wall roughness of 0.026 cm. The network is supplied by 2 reservoirs without pumps and 4 pumps obtaining their water from the reservoirs as shown in the sketch. A booster pump exists in pipe 43 also. The water surface elevations are as given on the sketch in meters and the pump characteristic curves are defined by the data for three points as given below.

Pump No.	Point 1		Point 2		Point 3	
	Q m^3/s	$h_p(m)$	Q m^3/s	$h_p(m)$	Q m^3/s	$h_p(m)$
1	0.03	84.	0.045	70.	0.06	52.
2	0.06	45.	0.105	39.	0.15	31.
3	0.06	45.	0.105	39.	0.15	31.
4	0.06	45.	0.105	39.	0.15	31.
5	0.03	9.0	0.045	7.5	0.06	5.5

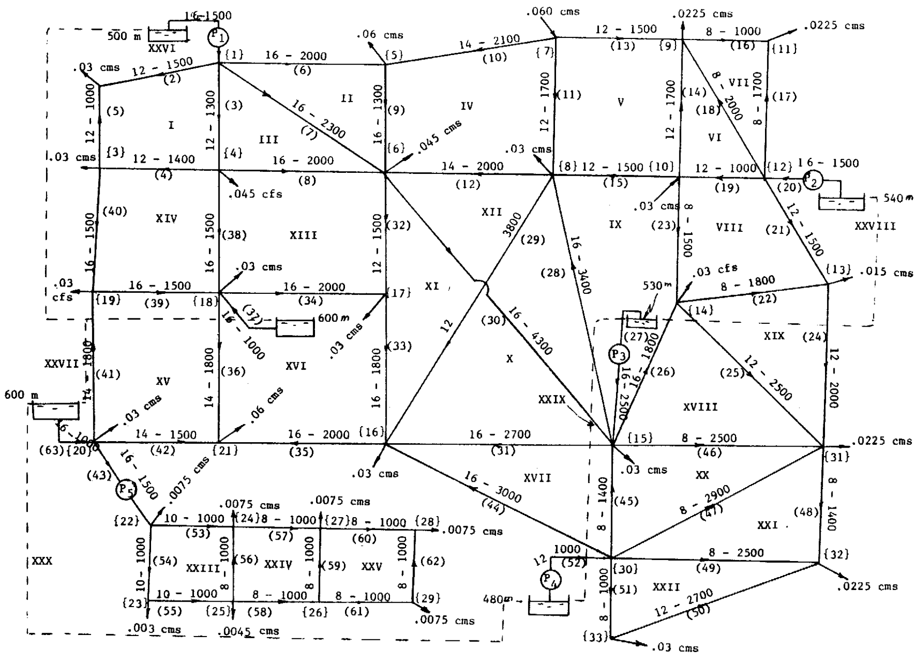
The elevation of the junctions are as given below. What is the elevation of the HGL at each junction, the head in meters, and the pressure?

Junction No.	1	2	3	4	5	6	7	8	9	10
Elev. (m)	300	250	255	260	280	200	340	270	330	360

Junction No.	11	12	13	14	15	16	17	18	19	20
Elev. (m)	260	420	270	240	260	260	260	280	270	300

Junction No.	21	22	23	24	25	26	27	28	29	30
Elev. (m)	250	260	200	160	160	70	70	40	20	260

Junction No.	31	32	33
Elev. (m)	200	-50	-60



Answer

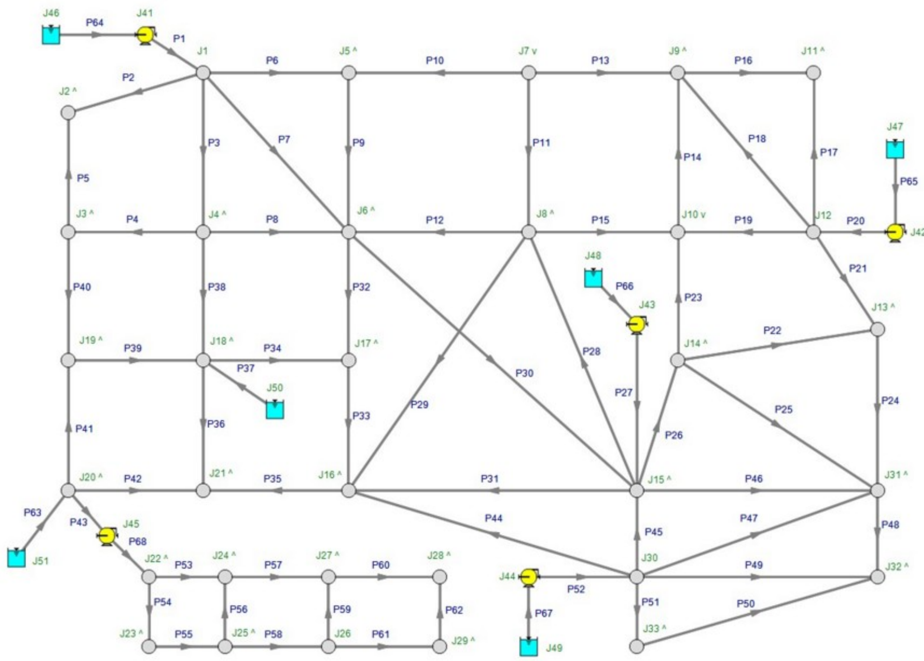
Pipe No.	Flow Rate (m^3/s)	Head Loss (m)	Pipe No.	Flow Rate (m^3/s)	Head Loss (m)
1	0.0869	201.1	33	0.0046	0.80
2	0.0208	53.6	34	0.0274	27.6
3	0.0132	19.1	35	0.0094	3.5
4	0.0153	27.4	36	0.0219	31.8
5	0.0092	7.16	37	0.1306	297.8
6	0.0274	27.5	38	-0.0214	12.7
7	0.0254	27.3	39	-0.0299	24.4
8	0.0148	8.3	40	-0.0238	15.7
9	-0.0027	0.22	41	0.0239	37.8
10	0.0299	68.4	42	0.0287	45.2
11	0.0212	63.1	43	0.0450	54.6
12	0.0081	5.11	44	0.0139	11.1
13	0.0089	10.1	45	-0.0002	0.11
14	0.0164	38.1	46	0.0056	55.0
15	0.0273	91.0	47	0.0051	54.9
16	0.0107	78.9	48	0.0167	267.5
17	0.0118	165.5	49	0.0137	322.4
18	0.0078	86.5	50	-0.0078	14.3
19	0.0243	48.4	51	0.0222	336.7
20	0.0791	166.9	52	0.0547	240.7
21	0.0351	150.0	53	0.0210	94.2
22	-0.0035	16.3	54	0.0165	58.1
23	0.0106	117.7	55	0.0135	39.1
24	0.0166	45.8	56	-0.0020	2.97
25	0.0119	29.7	57	0.0116	93.0
26	0.0277	25.3	58	0.0109	82.9
27	0.0805	287.6	59	0.0031	7.07
28	-0.0043	1.37	60	0.0072	36.2
29	0.0061	12.52	61	0.0078	43.2
30	-0.0065	3.74	62	0.0003	0.12
31	0.0148	11.16	63	0.1276	284.5
32	0.0071	6.62			

Junction	HGL (m)	Head (m)	Pressure (N/cm^2)
1	308.6	8.6	8.43
2	255.0	5.0	4.92
3	262.2	7.2	7.03
4	289.6	29.6	29.0
5	281.1	1.12	1.09
6	281.3	1.34	1.31
7	349.5	9.5	9.31
8	286.4	16.4	16.1
9	339.4	3.4	9.19
10	377.5	17.5	17.1
11	260.4	0.45	0.44
12	425.9	5.93	5.81
13	275.9	5.88	5.76
14	259.6	15.6	15.3
15	284.9	24.9	24.4
16	273.8	13.8	13.5
17	274.6	14.6	14.3
18	302.2	22.2	21.7
19	277.7	7.7	7.56
20	315.5	15.5	15.2
21	270.3	20.3	19.9
22	268.4	8.4	8.25
23	210.4	10.4	10.2
24	174.2	14.2	14.0
25	171.3	11.3	11.0
26	88.3	18.3	17.9
27	81.3	11.3	11.0
28	45.1	5.1	5.0
29	45.2	25.2	24.7
30	284.8	24.8	24.4
31	230.1	30.1	29.5
32	-37.5	12.5	12.2
33	-51.8	8.2	8.0

11. Solve the 8-pipe network shown in the accompanying sketch which is supplied by two reservoirs with water surfaces at 180 and 200 ft, respectively. The PRV is located 450 ft downstream from the upstream junction of pipe 5 and has a pressure setting which maintains the elevation of the HGL at 50 ft downstream from the valve.

View Verification Case 68 Model

Verification Case 68



Verification Case 69

View Model   Problem Statement

PRODUCT: AFT Fathom

TITLE: FthVerify69.fth

REFERENCE: Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 102-105

FLUID: Water

ASSUMPTIONS: Assume water at 70 deg. F.

RESULTS:

Pipe Flow Rate (ft3/sec)	1	2	3	4	5	6	7	8
Jeppson	11.61	3.18	2.18	1.5	2.22	-1.19	0.3	-0.68
AFT Fathom	11.506	3.175	2.175	1.498	2.201	-1.123	0.380	-0.670

Pipe Flow Rate (ft3/sec)	9	10	11	12	13	14	15	16
Jeppson	0.49	-1.43	2.63	2.06	10	0.93	1.49	1.89
AFT Fathom	0.570	-1.400	2.520	1.987	9.937	1.074	1.535	1.821

Pipe Flow Rate (ft3/sec)	17	18	19	20	21	22	23	24
Jeppson	1.54	1.47	2.75	2.71	-0.26	-1.45	-1.77	-1.71
AFT Fathom	1.322	1.396	2.480	2.461	-0.189	-1.271	-1.558	-1.543

Pipe Flow Rate (ft3/sec)	25	26	27	28	29	30	31	32
Jeppson	0.58	2.09	-2.1	0.07	1.18	2.73	1.14	-0.92
AFT Fathom	0.551	1.973	1.980	-0.065	1.055	2.561	1.089	-0.754

Pipe Flow Rate (ft3/sec)	33	34	35	36	37	38	39	40
Jeppson	4.72	3.51	0.71	-1.95	-2.72	0.88	-0.43	-3.72
AFT Fathom	4.632	3.298	0.642	-1.748	-2.439	0.815	-0.182	-3.344

Pipe Flow Rate (ft3/sec)	41	42	43	44	45	46	47	48
Jeppson	1.05	-1	-0.86	-0.5	-0.4	1.45	1.43	0.35
AFT Fathom	0.867	-0.893	-0.514	0.505	-0.444	1.307	1.307	0.361

Pipe Flow Rate (ft3/sec)	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**
Jeppson	2.55	1.45	4.25	9.03	1.24	-3.02	3.27	-1.21
AFT Fathom	2.462	1.387	4.125	8.950	1.582	-2.890	3.472	-1.600

Pipe Flow Rate (ft3/sec)	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**
Jeppson	-0.22	-1.22	0.92	-0.8	1.8	6.25	-5.14	2.8
AFT Fathom	-0.487	-1.141	0.903	-0.738	1.738	6.044	-3.950	2.667

Pipe Flow Rate (ft3/sec)	**65**
Jeppson	7
AFT Fathom	6.263

Pipe Head Loss (feet)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Jeppson	82.15	14.2	9.23	23.43	26.31	2.87	0.18	3.6
AFT Fathom	*80.43	14.15	9.18	23.32	25.86	**-2.54	0.27	**-3.51

Pipe Head Loss (feet)	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Jeppson	0.73	3.42	7.68	11.1	25.67	1.89	5.33	11.07
AFT Fathom	0.97	**-3.24	7.01	10.25	*26.74	2.48	5.58	10.27

Pipe Head Loss (feet)	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Jeppson	3.78	3.44	7.22	7.04	0.18	4.21	4.03	5.74
AFT Fathom	2.78	3.09	5.87	5.78	**-0.09	**-3.22	**-3.13	**-4.69

Pipe Head Loss (feet)	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
Jeppson	1.71	19.84	18.13	0.02	3.26	21.37	20.22	1.88
AFT Fathom	1.56	17.70	16.14	**-0.02	2.62	18.78	18.49	**-1.26

Pipe Head Loss (feet)	33	34	35	36	37	38	39	40
Jeppson	28.18	28.34	8.01	22.3	14.28	0.85	0.13	21.31
AFT Fathom	27.12	24.95	6.46	**-17.95	**-11.49	0.73	**-0.03	**-17.20

Pipe Head Loss (feet)	41	42	43	44	45	46	47	48
Jeppson	2.4	4.25	1.63	90	0.9	4.75	4.87	0.79
AFT Fathom	1.64	**-3.37	**-0.61	1.70	**-1.06	3.85	4.08	0.84

Pipe Head Loss (feet)	49	50	51	52	53	54	55	56
Jeppson	17.59	4.21	13.38	26.61	3.32	24.63	18.65	1.55
AFT Fathom	16.38	3.82	12.57	*26.06	5.25	**-22.45	*20.90	**-0.26

Pipe Head Loss (feet)	57	58	59	60	61	62	63	64
Jeppson	0.08	3.78	1.17	1.15	4.23	12.87	19.47	16.8
AFT Fathom	**-0.343	**-3.312	1.111	**-0.981	3.946	*12.006	**-29.681	15.549

Pipe Head Loss (feet)	65
Jeppson	23.88
AFT Fathom	23.043

Node EGL (feet)	1	2	3	4	5	6	7	8
Jeppson	390.2	376	366.7	363.9	366.6	363.1	374.2	372.3
AFT Fathom	393.20	379.10	369.90	367.40	369.60	366.40	376.70	374.20

Node EGL (feet)	9	10	11	12	13	14	15	16
Jeppson	376.1	369.1	368.9	363.2	345	341.8	362	333.8
AFT Fathom	377.00	371.20	371.10	366.40	350.20	347.60	366.10	341.20

Node EGL (feet)	17	18	19	20	21	22	23	24
Jeppson	340.9	343.3	348.1	364.9	351.5	347.3	342.4	340.8
AFT Fathom	346.90	348.50	352.40	367.90	355.40	351.60	347.50	346.90

Node EGL (feet)	25	26	27	28	29
Jeppson	319.5	316.1	342.3	346.1	347.3
AFT Fathom	329.70	324.40	347.10	350.50	351.40

* AFT Fathom results combine two pipes, as discussed below

** Note that AFT Fathom represents head loss on pipes with reverse flow as a negative. Jeppson represents it as positive regardless of the direction.

DISCUSSION:

Jeppson's method of applying pump data is to lump it into a pipe, whereas AFT Fathom's method is to place pumps at boundaries between pipes. Pumps are therefore a specific node (or junction) in AFT Fathom. To accommodate Jeppson's method, the pipe which contains the pump is split into two equivalent pipes in AFT Fathom. Where the split is made will have no impact on the results.

Because there are five pumps in the example, there are five additional pipes in the AFT Fathom model. AFT Fathom pipes 1 and 71 together represent Jeppson pipe 1. Similarly, AFT Fathom pipes 13 and 72 represent Jeppson pipe 13, AFT Fathom pipes 52 and 73 represent Jeppson pipe 52, AFT Fathom pipes 55 and 75 represent Jeppson pipe 55, and AFT Fathom pipes 62 and 74 represent Jeppson pipe 62.

Jeppson presents results in terms of HGL. However, Jeppson's method assumes EGL and HGL are essentially the same because of minimal velocity. Therefore, Jeppson results are presented in the results shown above as EGL.

Results differ slightly between AFT Fathom and Jeppson for a few reasons. First, Jeppson represents pump curves differently than AFT Fathom. Jeppson typically uses an exponential formula (see page 82), while AFT Fathom uses a polynomial based on a least squares curve fit. Second, the head loss formula used by Jeppson differs from AFT Fathom. Jeppson's formula is more common to the water industry, and assumes the head loss is proportional to flow rate to some power near but less than 2. AFT Fathom assumes it always proportional to flow rate to the power of 2. These differences affect the results to some degree. Third, this system is highly networked which may cause some individual pipes, especially those with lower flow rates, to differ quite a bit from AFT Fathom. Looking at the system as a whole, the agreement is quite good between Jeppson and AFT Fathom.

Slight differences in property and calculation constants that were assumed, as well as potential differences from Jeppson's solution tolerances, which are not known, may also contribute to differences in the solution results. Examples are the specific value of water density and gravitational constant.

List of All Verification Models

Verification Case 69 Problem Statement

Verification Case 69

Roland Jeppson, Analysis of Flow in Pipe Networks, 1976, Publisher Ann Arbor Science, Page 102-105

Jeppson Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

This solution took seven iterations to meet the error requirement that the sum of flow rate changes be less than 0.01 and required 11.7 seconds of execution time on a UNIVAC 1108 computer.

9. Solve the 65 pipe network below for the flow rates in all pipes, the head losses in all pipes, the elevation of the HGL at all junctions, and the pressure at all junctions. All pipes have e = 0.0102 inch. Three values of flow rate and head loss for the five pumps in the network are:

Pump No.	Point 1		Point 2		Point 3	
	Q_p	h_p	Q_p	h_p	Q_p	h_p
1	8.	200.	11.	180.	16.	80.
2	5.	180.	7.5	150.	10.	50.
3	4.	200.	6.	180.	10.	80.
4	4.	250.	6.	210.	8.	150.
5	2.	50.	4.	40.	6.	20.

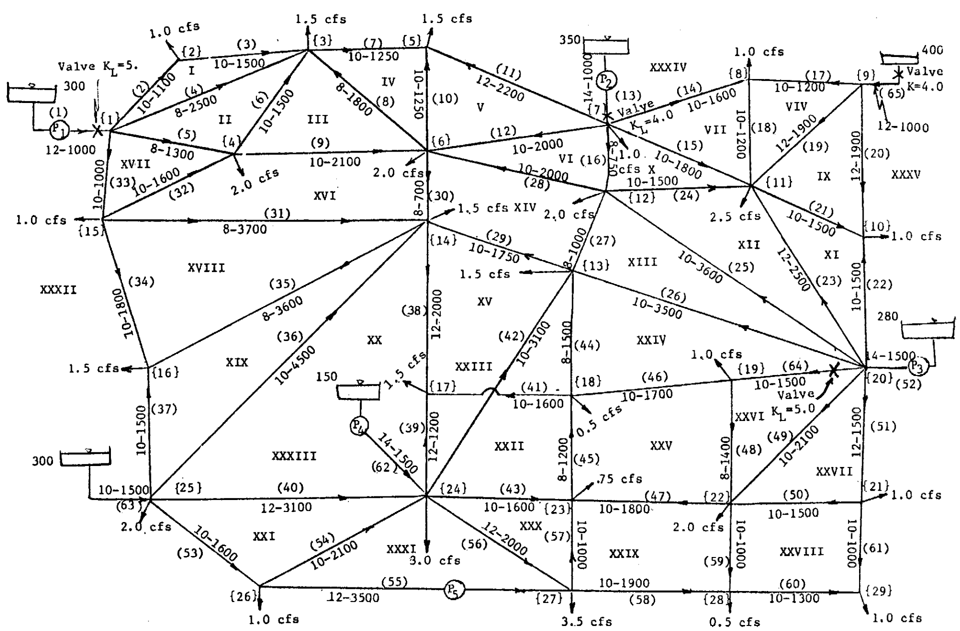
The elevations of the junctions are:

Junction No.	Elev. ft	Junction No.	Elev. ft	Junction No.	Elev. ft
1	150	11	190	21	170
2	150	12	190	22	160
3	160	13	180	23	150
4	150	14	160	24	150
5	165	15	150	25	140
6	160	16	150	26	130
7	180	17	140	27	130
8	190	18	150	28	130
9	200	19	170	29	125
10	180	20	180		

Solution:

Pipe No.	1	2	3	4	5	6	7
Q (cfs)	11.61	3.18	2.18	1.50	2.22	-1.19	0.30
h_f (ft)	82.15	14.20	9.23	23.43	26.31	2.87	0.18

Pipe No.	8	9	10	11	12	13	14
Q (cfs)	-0.68	0.49	-1.43	2.63	2.06	10.00	0.93
h_f (ft)	3.60	0.73	3.42	7.68	11.10	25.67	1.89


1.0 cfs
1.5 cfs
1.5 cfs
Valve K_L=5.
300
12-1000
10-1100
10-1500
8-2500
8-1800
10-1250
10-1250
12-2200
350
14-1000
Valve K_L=4.0
10-1600
XXXIV
1.0 cfs
10-1200
400
Valve K=4.0
12-1000
8-1300
10-2100
10-2000
10-2000
10-1800
10-1500
12-1900
12-1900
10-1200
2.0 cfs
2.0 cfs
2.0 cfs
2.5 cfs
10-1000
10-1600
1.0 cfs
8-3700
8-700
1.5 cfs
10-1750
1.5 cfs
8-1000
10-3600
12-2500
10-1500
1.0 cfs
XXXII
10-1800
8-3600
12-2000
10-3500
280
1.5 cfs
10-4500
150
1.5 cfs
10-3100
8-1500
1.0 cfs
10-1500
Valve K_L=5.0
10-1600
10-1700
0.5 cfs
300
10-1500
14-1500
12-1200
8-1200
8-1400
10-2100
12-1500
.75 cfs
10-1500
12-3100
10-1600
10-1800
2.0 cfs
10-1500
1.0 cfs
2.0 cfs
10-1600
10-2100
12-2000
10-1000
10-1000
10-1000
3.0 cfs
12-3500
10-1900
10-1300
1.0 cfs
3.5 cfs
0.5 cfs
1.0 cfs

Pipe No.	15	16	17	18	19	20	21
Q (cfs) h_f (ft)	1.49 5.33	1.89 11.07	1.54 3.78	1.47 3.44	2.75 7.22	2.71 7.04	-0.26 0.18

Pipe No.	22	23	24	25	26	27	28
Q (cfs) h_f (ft)	-1.45 4.21	-1.77 4.03	-1.71 5.74	0.58 1.71	2.09 19.84	-2.10 18.13	0.07 0.02

Pipe No.	29	30	31	32	33	34	35
Q (cfs) h_f (ft)	1.18 3.26	2.73 21.37	1.14 20.22	-0.92 1.88	4.72 28.18	3.51 28.34	0.71 8.01

Pipe No.	36	37	38	39	40	41	42
Q (cfs) h_f (ft)	-1.95 22.30	-2.72 14.28	0.88 0.85	-0.43 0.13	-3.72 21.31	1.05 2.40	-1.00 4.25

Pipe No.	43	44	45	46	47	48	49
Q (cfs) h_f (ft)	-0.86 1.63	-0.50 1.72	-0.40 0.90	1.45 4.75	1.43 4.87	0.35 0.79	2.55 17.59

Pipe No.	50	51	52	53	54	55	56
Q (cfs) h_f (ft)	1.45 4.21	4.25 13.38	9.03 26.61	1.24 3.32	-3.02 24.63	3.27 18.65	-1.21 1.55

Pipe No.	57	58	59	60	61	62	63
Q (cfs) h_f (ft)	-0.22 0.08	-1.22 3.78	0.92 1.17	-0.80 1.15	1.80 4.23	6.25 12.87	-5.14 19.47

Pipe No.	64	65
Q (cfs) h_f (ft)	2.80 16.80	7.00 23.88

Junction No.	1	2	3	4	5	6
HGL (ft) Pressure (psi)	390.2 104.1	376.0 97.9	366.7 89.6	363.9 92.7	366.6 87.3	363.1 88.0

Junction No.	7	8	9	10	11	12
HGL (ft) Pressure (psi)	374.2 84.2	372.3 79.0	376.1 76.3	369.1 81.9	368.9 77.5	363.2 75.0

Junction No.	13	14	15	16	17	18
HGL (ft) Pressure (psi)	345.0 71.5	341.8 78.8	362.0 91.9	333.8 79.6	340.9 87.1	343.3 83.8

Junction No.	19	20	21	22	23	24
HGL (ft) Pressure (psi)	348.1 77.2	364.9 80.1	351.5 78.65	347.3 81.2	342.4 83.4	340.8 82.7

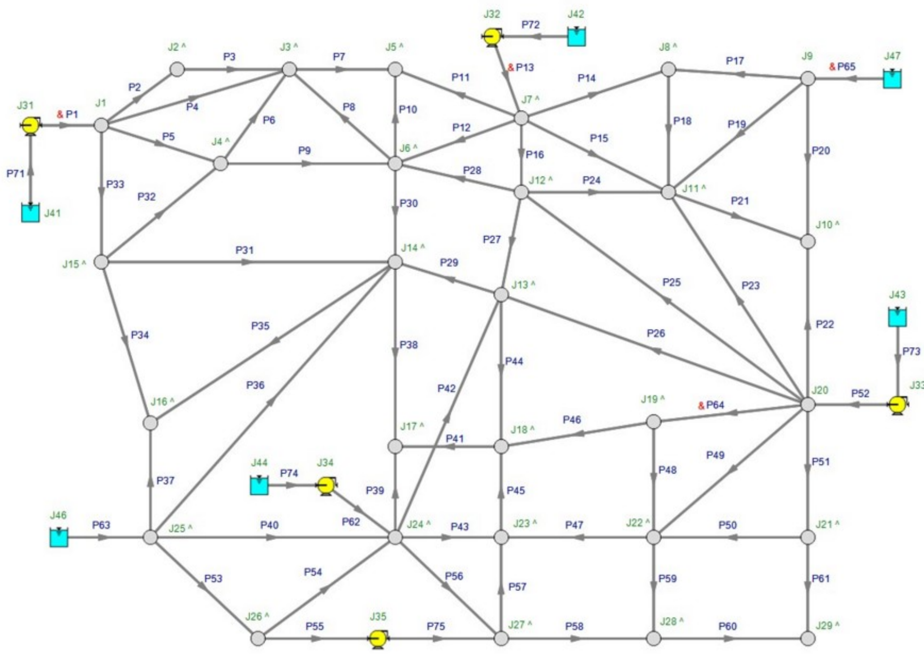
Junction No.	25	26	27	28	29
HGL (ft) Pressure (psi)	319.5 77.8	316.1 80.7	342.3 92.0	346.1 93.6	347.3 96.3

Using the linear theory program described in Appendix C, the solution was obtained in six iterations with a sum of changes in flow rate equal to 0.019. The solution requires 23.3 seconds of execution time on a UNIVAC 1108.

10. Determine the flow rates and head losses of each of the 63 pipes of the network shown below. The diameter of each pipe is given in centimeters and the length of each pipe is given in meters. Thus pipe number 1 is 16 cm in diameter and has a length of 1500 m. All 63 pipes of the network have a wall roughness of 0.026 cm. The network is supplied by 2 reservoirs without pumps and 4 pumps obtaining their water from the reservoirs as shown in the sketch. A booster pump exists in pipe 43 also. The water surface elevations are as given on the sketch in meters and the pump characteristic curves are defined by the data for three points as given below.

View Verification Case 69 Model

Verification Case 69



Verification Case 70

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify70.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Page 145-147, Case Study 6.1

FLUID: Water/Sand

ASSUMPTIONS: Water temperature is assumed to be 4 C to match the specified fluid specific gravity of 1. The pipe length and inlet pressure were not specified in the problem statement. They were assumed to be 100 m and 1 MPa, respectively. The choice of pipe length and inlet pressure does not affect the resulting calculations. It was also assumed that Vt/Vts(Xi) = 0.55 for coarse sand.

RESULTS:

Wilson, Addie, Sellgren & Clift Results:

Case	Velocity (meters/sec)	Cv (Decimal)	Im (m/m)	SEC (kW-hr/mton-km)
Cvd 0.221	5.70	0.221	0.0609	0.289
Cvd 0.2	6.30	0.200	0.0612	0.315
Cvd 0.183	6.90	0.183	0.0636	0.358

AFT Fathom Results:

Case	Velocity (meters/sec)	Cv (Decimal)	Im (m/m)	SEC (kW-hr/mton-km)
Cvd 0.221	5.72	0.221	0.0613	0.285
Cvd 0.2	6.32	0.200	0.0609	0.313
Cvd 0.183	6.91	0.183	0.0633	0.355

DISCUSSION:

There is close agreement between the results for AFT Fathom and the results for Wilson, Addie, Sellgren and Clift.

List of All Verification Models

Verification Case 70 Problem Statement

Verification Case 70

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 145-146, Case Study 6.1 a,b

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

substantially exceeds 0.4, the solids are almost uniformly distributed over the pipe section. This observation is consistent with the close particle spacing at high concentration which will reduce the effectiveness of turbulent suspension, so that the particles' submerged weight must be carried by granular contact. Further increases in C_v lead to larger hydraulic gradients and a pronounced rise in specific energy consumption.

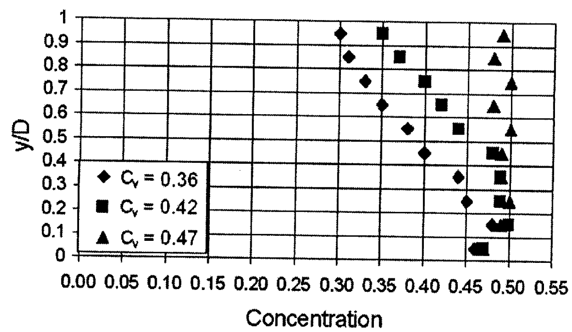


Figure 6.9. Concentration profiles for 180 μm sand tested by Gillies & Shook (1996)

The next step in describing slurry flows will be to discuss complex slurries – those with features such as very broad particle grading or non-Newtonian carrier fluids. Complex slurries are the subject of Chapter 7.

6.8 Case Studies

Case Study 6.1 - Effect of Particle Size and Grading on Sand Transport

In Case Study 5.1 a preliminary pipe selection was made for a system to transport 4000 tonnes/hour of coarse sand. One of the pipe sizes selected was D = 0.65 m, for which the combination V_m = 6.30 m/s and C_{vd} = 0.20 gives the required solids flow rate.

In that case study the mean size and grading of the particles was not specified; and thus the hydraulic gradient and the specific energy consumption could not be calculated. The present case study will investigate these points.

The first material to be considered is a clean sand (negligible fraction smaller than 40 μm) with S_s = 2.65, d_{50} = 0.70 mm and d_{85} = 1.00 mm. We will consider an aqueous slurry ($S_f = 1.00$) in the pipe with D = 0.65 m. The conditions found in Case Study 5.1 (C_{vd} = 0.20, V_m = 6.30 m/s) will be investigated initially, and the first requirement will be:

(a) Find the hydraulic gradient i_m and the specific energy consumption for this case. For these particles the associated velocity w can be read directly from Fig. 6.5, giving 0.127 m/s for the 0.70 mm particle and 0.148 m/s for the 1.00 mm one. When multiplied by cosh(60d/D), these become 0.1273 m/s and 0.1486 m/s, giving a ratio of 1.17. From Eq. 6.8, the associated value of σ_s is 0.069, and when this is substituted into Eq. 6.9, M is given as 1.79. As this is larger than 1.7, M = 1.7 will be used. (This result is not surprising, since d_{85}/d_{50} is less than 1.5, indicating a narrow particle grading).

With M evaluated, we can turn to V_{50}, equal to $w\sqrt{\frac{8}{f_w}}\cosh(60d/D)$, based on d_{50}. For f_w = 0.012 (found from the Stanton-Moody diagram for commercial steel pipe of this size and velocity range), the resulting value of V_{50} is 3.28 m/s. If the simplified approach of Section 6.5 had been used, M would remain at 1.7 and V_{50} would be 3.47 m/s. On the basis of mixture velocity V_m = 6.30 m/s, and $(S_m - S_f) = (1.65)\ 0.20 = 0.330$, Eq. 6.4 gives the solids effect $(i_m - i_f)$ as $(0.330)(0.22)[1.921]^{-1.7}$ or 0.0239 (m water/m pipe).

With no fines, the fluid is water and $i_f = i_w$, given by 0.012 $(6.3)^2/(19.62)(0.65)$, or 0.0373. The value of i_m is the sum of i_w and the solids effect or 0.0612 (m water/m pipe).

The specific energy consumption is proportional to i_m/S_sC_{vd}, which is 0.115, giving SEC of 0.315 kWh/tonne-km.

(b) Next, it is of interest to vary some of the quantities and as input, to find the effect on i_m and SEC. The initial case to be investigated is a variation of V_m only, with C_{vd} kept constant. The second case involves adjusting C_{vd} as well as V_m, so that the solids throughput Q_s is maintained at a constant value.

The effect of varying V_m alone could be obtained by differentiation, but it is probably clearer simply to change V_m by about 10% in each direction, repeating the calculations for, say, V_m = 6.9 m/s and 5.7 m/s. As i_w varies with V_m^2 (ignoring any changes in f_w) and $(i_m - i_w)$ varies with $V_m^{1.7}$, the calculations are straightforward. For V_m = 6.9 m/s, i_m = 0.0653 and SEC = 0.336

kWh/tonne-km. For V_m = 5.7 m/s, i_m = 0.0589 and SEC = 0.304 kWh/tonne-km.

If the same values of V_m are employed, but C_{vd} is adjusted to maintain Q_s, the values shown in Table 6.1 are obtained.

Table 6.1.

V_m (m/s)	C_{vd}	i_m	SEC (kWh/tonne-km)
5.70	0.221	0.0619	0.289
6.30	0.200	0.0612	0.315
6.90	0.183	0.0636	0.358

As with the figures for the constant-C_{vd} case, this table indicates a significant increase in SEC when V_m is increased to 6.90 m/s. As before, the specific energy consumption is diminished when V_m is decreased to 5.70 m/s, but once again, the change is smaller for a decrease in velocity. Thus it would appear that the lower velocity of 5.7 m/s could be attractive. On the other hand, the lower velocity is less than 20 percent above the deposition limit (V_{sm} = 4.84 m/s by Eq. 5.11, see Case Study 5.1), and considerations of system stability may be involved. In order to resolve these questions, it will be necessary to know the pump characteristic as well as the system characteristic. Pump characteristics will be discussed in Chapter 9, and the intersection of pump and system characteristics in Chapter 13.

(c) It is also of interest to find the effect on i_m of changes in particle grading. As an initial example, suppose that the coarse sand specified above is replaced by a fine, clean, narrow-graded sand with d_{50} = 0.20 mm. From Fig. 6.5, w for this sand is 0.080 m/s. In the pipe of 0.65 m diameter the resulting value of V_{50} from Eq. 6.2 is 2.08 m/s. For V_m = 6.3 m/s, and C_{vd} = 0.20, the solids effect as calculated from Eq. 6.4 is 0.0110, giving i_m = 0.0483 m water/m pipe. (If the simplified approach had been used the value of i_m would be 0.0498 m water/m pipe). Clearly, i_m is considerably less than the value of 0.0612 found in part (a) for the coarse sand. It may be compared with the result of the 'equivalent fluid' model, by which i_m is estimated as the product of S_m and i_w. As S_m = 1.330, and i_w = 0.0373, the equivalent fluid model gives i_m = 0.0496, rather close to the result of the new model for this specific instance of fine-sand slurry flow.

View Verification Case 70 Model

Verification Case 70

Verification Case 71

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify71.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Page 148-149, Case Study 6.2

FLUID: Water/Coal

ASSUMPTIONS: The water temperature is not specified. It was assumed to be 20 deg. C because the viscosity ratio in equation 6.11 in the reference is based on the viscosity at 20 deg. C. The pipe length and inlet pressure were not given. They were assumed to be 100 m and 5 bar, respectively. The choice of pipe length and inlet pressure does not affect the resulting calculations. The calculation was run once to determine the mass flow rate of 1857 m-ton/hr at the minimum Im, then run again with the flow set to this value. In this way, the proper value of SEC was determined for the calculation. Also, while AFT Fathom calculates the expected value of 1.94 m/s for the Settling Velocity, the example in the reference uses a value of 1.90 m/s, so this value was used for the calculations in AFT Fathom as well. Assume M = 1.7.

RESULTS:

	V50 (meters/sec)	**Velocity at min Im (meters/sec)**	**Im (m/m)**	**SEC (kW-hr/mton-km)**
Wilson, Addie, Sellgren & Clift	2.64	3.1	0.0313	0.244
AFT Fathom	2.65	3.1	0.0313	0.243

List of All Verification Models

Verification Case 71 Problem Statement

Verification Case 71

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 148-149, Case Study 6.2

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

Consider next a coarse sand with d_{50} = 0.70 mm as before, but with a broader particle grading so that d_{85} = 1.50 mm. For the latter size, w = 0.18 m/s, and for the 0.65 m pipe cosh(60d/D) is 1.01. Substitution into Eq. 6.8 shows that the value of σ_s for this material is 0.145. When this is substituted into Eq. 6.9, M is found to be 1.38. (The simplified approach gives M = 1/ln(1.50/0.70) = 1.31.)

The value of V_{50} remains at 3.28 m/s and for V_m = 6.30 m/s and C_{vd} = 0.20, the solids effect becomes $(0.330)(0.22)[1.921]^{-1.38}$ or 0.0295. Adding i_w of 0.0373 gives i_m = 0.0668 m water/m pipe, which is about 9% higher than the value obtained previously for the narrow-graded particle distribution with the same median size. (A similar increase, about 11%, is obtained if the simplified approach is used throughout.)

Case Study 6.2 - Transport of Coal Slurry

The material for this study is coal, with S_s = 1.4, μ_s = 0.44, C_{vb} = 0.60. The coal is clean with few fines, and d_{50} = 2.0 mm and d_{85} = 2.8 mm (as the ratio of d_{85}/d_{50} is less than 1.5, the particle grading is narrow, and M will be taken as 1.7).

The pipe to be used has D = 0.440 m and f_w = 0.013. Solids concentration C_{vd} can be taken initially as 0.25.

The minimum in the curve of constant C_{vd} is to be investigated, comparing the velocity at this point with V_{sm} and V_{50}, and finding the minimum specific energy consumption and how this is influenced by changes in C_{vd}.

(a) the limit of deposition will be determined first. For d = 2.0 mm and D = 0.440 m, Fig. 5.3 shows that V_{sm} would be 4.1 m/s for material with S_s = 2.65, and for S_s = 1.4, the right-hand side of that figure gives V_{sm} = 1.90 m/s. Equation 5.11 gives a value of 1.94 m/s, but the smaller number should be taken, i.e. V_{sm} = 1.90 m/s. (Note that for these 2.0 mm coal particles the nomographic chart shows a smaller value than Eq. 5.10; the reverse of the condition found in Case Study 5.1 for 0.7 mm sand).

(b) To obtain V_{50} it is sufficient to use the simplified approach of Eq. 6.11 which gives 2.64 m/s. For this narrow- graded material M = 1.7, and with $(S_s - 1)C_{vd}$ = 0.4 (0.25) = 0.10, the expression for solids effect (Eq. 6.4) becomes:

$$i_m - i_w = 0.022\left(\frac{V_m}{2.64}\right)^{-1.7} = 0.115\,V_m^{-1.7}$$

Also, i_w is $\frac{0.013V_m^2}{(19.62)(0.44)} = 0.00151V_m^2$. The sum of these two terms is i_m, and differentiation gives the following relation for the velocity at minimum i_m:

$$1.7(0.115)V_m^{-2.7} = 2(0.00151)V_m$$

from which V_m equals 3.1 m/s. At this point i_m is calculated to be 0.0313 m water/m pipe and i_m/S_sC_{vd} is 0.089, giving SEC = 0.244 kWh/tonne-km. (This is the quantity which must be provided by the pumps to the slurry, the energy which must be supplied to the pumps will, of course, be larger).

(c) In this case, the value of V_m at the minimum point is comfortably above the deposition limit of 1.9 m/s, so no problem with deposition should arise. However, stability problems would occur near the minimum, as shown in Chapter 13, and thus the operating velocity should be larger than 3.1 m/s. It is also worth noting that in the present study, the flow is definitely heterogeneous near the minimum point. At the minimum point V_m/V_{50} equals 1.17 and the stratification ratio, which is given by 0.5 $(V_m/V_{50})^{-1.7}$ equals 0.38, implying that about 38% of the submerged weight of solids is carried by intergranular contact, with fluid support accounting for the rest.

(d) Finally, it is of interest to consider the effect of a change in concentration on the minimum point. As shown in the discussion following Eq. 6.5, the velocity at minimum i_m varies *cæteris paribus* with $C_{vd}^{0.27}$. Thus, if C_{vd} increases from 0.25 to 0.30 the value of V_m at the minimum point will increase from 3.1 m/s to 3.25 m/s; not a large change. It can readily be calculated that the equivalent value of i_m is 0.0345 m water/m pipe. This gives SEC = 0.224 kWh/tonne-km, or about 92% of the value obtained previously for C_{vd} = 0.25.

REFERENCES

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View Verification Case 71 Model

Verification Case 71

Verification Case 72

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify72.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Page 243, Case Study 10.1

FLUID: Water/Sand

ASSUMPTIONS: Water temperature is assumed to be 4 C. M is assumed to be 1.7. Also, the system configuration is not given (i.e. pressures, pipe lengths, pipe diameters, etc) and were assumed. These assumptions do not affect the final results of the problem. Use simplified slurry calculations. Use ANSI/HI Standard 12.1-12.6-2005 calculations to determine the pump corrections.

RESULTS:

	AFT Fathom	Wilson, Addie, Sellgren & Clift
De-Rating Correction:	4.7%	4.7%

DISCUSSION:

The correction factor calculated in AFT Fathom was CH = 95.3%. This is related to the correction given in Wilson, Addie, Sellgren & Clift by subtracting CH from 100%.

List of All Verification Models

Verification Case 72 Problem Statement

Verification Case 72

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 243, Case Study 10.1

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

concentrations, specifically with broad particle size distributions.

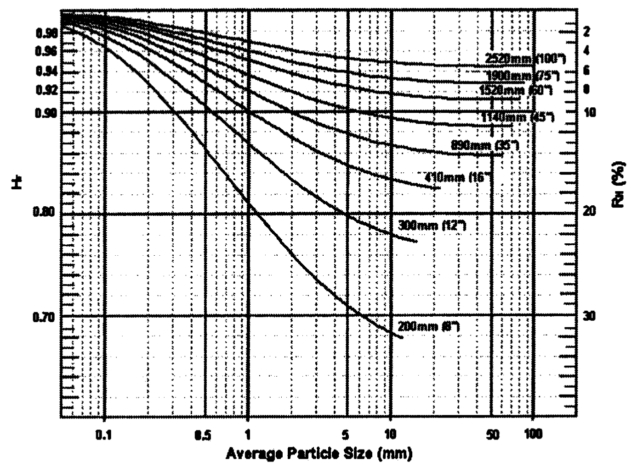

Figure 10.9. Generalised solids-effect diagram for pumps of various sizes (impeller diameters) based mainly on tests at The GIW Hydraulic Laboratory. For solids concentration by volume, C_{vd} = 15% with relative density of solids, S_s = 2.65 and a negligible amount of fine particles

Example 10.1

A sand slurry is to be pumped using a large pump with a 1320 mm impeller. Determine the reduction in head R_H if the solids concentration by volume C_{vd} = 0.20, the solids specific gravity S_s = 2.65 and the average particle size d_{50} = 400 μm.

It follows from the diagram that R_H = 3.5% for S_s = 2.65, d_{50} = 400 μm and C_{vd} = 15%. Correction for C_{vd} = 20% gives: 3.5 (20/15) = 4.7%.

Example 10.2

Determine R_H for a pump with an impeller diameter of 800 mm pumping an ore product (S_s = 4.0) with d_{50} = 500 μm, X_h = 0.28 and delivered volumetric concentration of 20%.

View Verification Case 72 Model

Verification Case 72

Verification Case 73

View Model Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify73.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Page 243-244, Case Study 10.2

FLUID: Water/Ore

ASSUMPTIONS: Water temperature is assumed to be 4 deg. C. M is assumed to be 1.7. Also, the system configuration is not given (i.e. pressures, pipe lengths, pipe diameters, etc) and were assumed. These assumptions do not affect the final results of the problem. Use simplified slurry calculations. Use ANSI/HI Standard 12.1-12.6-2005 calculations to determine the pump corrections.

RESULTS:

	AFT Fathom	Wilson, Addie, Sellgren & Clift
De-Rating Correction:	6.4%	6.2%

DISCUSSION:

The correction factor calculated in Fathom was CH = 93.6%. This is related to the correction given in Wilson, Addie, Sellgren & Clift by subtracting CH from 100%.

List of All Verification Models

Verification Case 73 Problem Statement

Verification Case 73

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 243-244, Case Study 10.2

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

concentrations, specifically with broad particle size distributions.

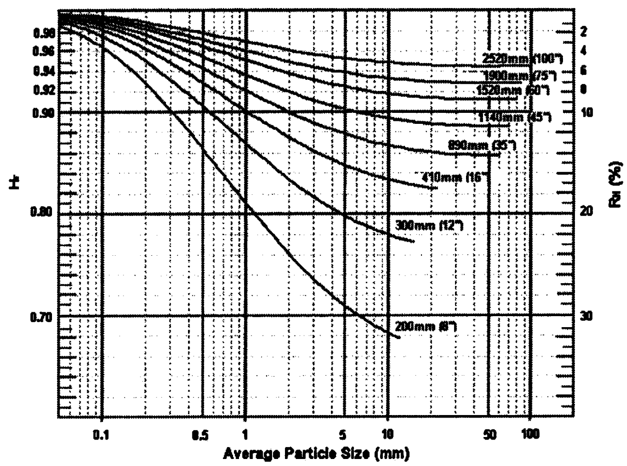

Figure 10.9. Generalised solids-effect diagram for pumps of various sizes (impeller diameters) based mainly on tests at The GIW Hydraulic Laboratory. For solids concentration by volume, C_{vd} = 15% with relative density of solids, S_s = 2.65 and a negligible amount of fine particles

Example 10.1

A sand slurry is to be pumped using a large pump with a 1320 mm impeller. Determine the reduction in head R_H if the solids concentration by volume C_{vd} = 0.20, the solids specific gravity S_s = 2.65 and the average particle size d_{50} = 400 μm.

It follows from the diagram that R_H = 3.5% for S_s = 2.65, d_{50} = 400 μm and C_{vd} = 15%. Correction for C_{vd} = 20% gives: 3.5 (20/15) = 4.7%.

Example 10.2

Determine R_H for a pump with an impeller diameter of 800 mm pumping an ore product (S_s = 4.0) with d_{50} = 500 μm, X_h = 0.28 and delivered volumetric concentration of 20%.

It follows from the diagram that $R_H = 6\%$ for $d_{50} = 500$ μm for $S_s = 2.65$ and $X_H = 0$. Correction factor for $C_{vd} = 20\%$, $S_s = 4$ and $X_h = 0.28$ are; respectively:

$$\frac{20}{15} = 1.33$$

$$\left[\frac{4-1}{1.65}\right]^{0.65} = 1.475$$

$$(1 - 0.28)^2 = 0.52$$

R_H is then $6 \cdot 1.33 \cdot 1.47 \cdot 0.52 = 6.2\%$.

Example 10.3

A smaller pump is to be used to pump a slurry of the same solids as in Example 10.1, but at a volumetric concentration of 10%. The pump has an impeller diameter of 380 mm.

Following Example 10.1 with a pump impeller diameter 380 mm then the diagram gives $R_H = 7\%$. Correction for $C_{vd} = 10\%$ means that $R_H = 7 \cdot (10/15) = 4.7\%$, say 5%. Assuming that R_η equal to R_H, it is also 5%. The head required for this application is 45m of slurry at a flow rater of 0.0625 m^3/s. The pump has a 100mm discharge branch, 150mm suction branch and an impeller diameter of 380 mm. Its clear-water performance characteristics are shown on Figure 10.10. The running speed and power requirement for pumping this slurry are to be determined.

The head ratio H_r was defined in section 10.1 as $1 - R_H$, and for the present example this equals 0.95, i.e.

$$H_r = \frac{H_m}{H_w} = 0.95.$$

In order to be able to produce the required 45m head of slurry, the pump must be capable of producing a head of water, H_w, given by

$$H_w = \frac{45.0}{0.95} = 47.4\text{m of water}$$

View Verification Case 73 Model

Verification Case 73

Verification Case 74

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify74.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Page 244-246, Case Study 10.3

FLUID: Water/Sand

ASSUMPTIONS: Water temperature is assumed to be 4 deg. C. M is assumed to be 1.7. The pipe length was assumed to be 144.5 m to achieve the specified flow of 62.5 l/s. The inlet and outlet pressures were assumed. These pressures do not affect the final results of the problem. Use simplified slurry calculations. Use ANSI/HI Standard 12.1-12.6-2005 calculations to determine the pump corrections.

RESULTS:

	AFT Fathom	Wilson, Addie, Sellgren & Clift
Corrected Efficiency	71.85%	71.3%
Corrected Power	48.51 kW	45.1 kW

DISCUSSION:

The answers in this case are close, but not exact. This is primarily due to some assumptions and rounding done in the example in the reference. For example, the flow rate of 62.5 l/s selected in the book assumed a speed of 1500 rpm. Examining figure 10.10, one can see that this is an approximation, so a slight difference in the pump head curve is introduced in this way. Also, the correction factor was rounded off from 4.7% to 5.0%. Based on these approximations, the results from AFT Fathom match very closely.

List of All Verification Models

Verification Case 74 Problem Statement

Verification Case 74

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 244-246, Case Study 10.3

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

It follows from the diagram that R_H = 6% for d_{50} = 500 µm for S_s = 2.65 and X_H = 0. Correction factor for C_{vd} = 20%, S_s = 4 and X_h = 0.28 are; respectively:

$$\frac{20}{15} = 1.33$$

$$\left[\frac{4-1}{1.65}\right]^{0.65} = 1.475$$

$$(1 - 0.28)^2 = 0.52$$

R_H is then $6 \cdot 1.33 \cdot 1.47 \cdot 0.52 = 6.2\%$.

Example 10.3

A smaller pump is to be used to pump a slurry of the same solids as in Example 10.1, but at a volumetric concentration of 10%. The pump has an impeller diameter of 380 mm.

Following Example 10.1 with a pump impeller diameter 380 mm then the diagram gives R_H = 7%. Correction for C_{vd} = 10% means that R_H = $7 \cdot (10/15)$ = 4.7%, say 5%. Assuming that R_η equal to R_H, it is also 5%. The head required for this application is 45m of slurry at a flow rater of 0.0625 m^3/s. The pump has a 100mm discharge branch, 150mm suction branch and an impeller diameter of 380 mm. Its clear-water performance characteristics are shown on Figure 10.10. The running speed and power requirement for pumping this slurry are to be determined.

The head ratio H_r was defined in section 10.1 as 1 – R_H, and for the present example this equals 0.95, i.e.

$$H_r = \frac{H_m}{H_w} = 0.95.$$

In order to be able to produce the required 45m head of slurry, the pump must be capable of producing a head of water, H_w, given by

$$H_w = \frac{45.0}{0.95} = 47.4\text{m of water}$$

For this water head, and the discharge of 62.5 L/s, the pump characteristics shown on Fig. 10.10 indicate that the pump must run at 1500 rpm, and that the clear water efficiency η_w will be 75%.

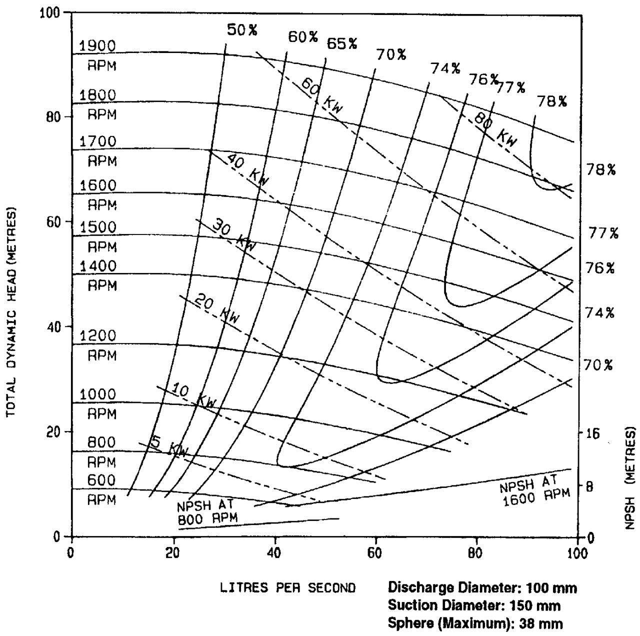

Figure 10.10. Pump characteristic curves for Example 10.4

With R_η assumed to be equal to R_H, then

$$\eta_r = 1 - R_\eta = 0.95 = \frac{\eta_m}{\eta_w}$$

Thus

$$\eta_m = \eta_r \eta w = 0.95\,(0.75) = 0.713 \text{ or } 71.3$$

The slurry specific gravity, S_m, is 1 + 0.10 (2.65 - 1) or 1.165, and by Eq. 10.1 P_m equals $S_m P_w$. For the calculated values of η_m and other quantities the power requirement is found to be

$$P_m = \frac{1.165(9.81)(45.0)(0.0625)}{0.713} = 45.1\,kW$$

REFERENCES

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View Verification Case 74 Model

Verification Case 74

Verification Case 75

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE:

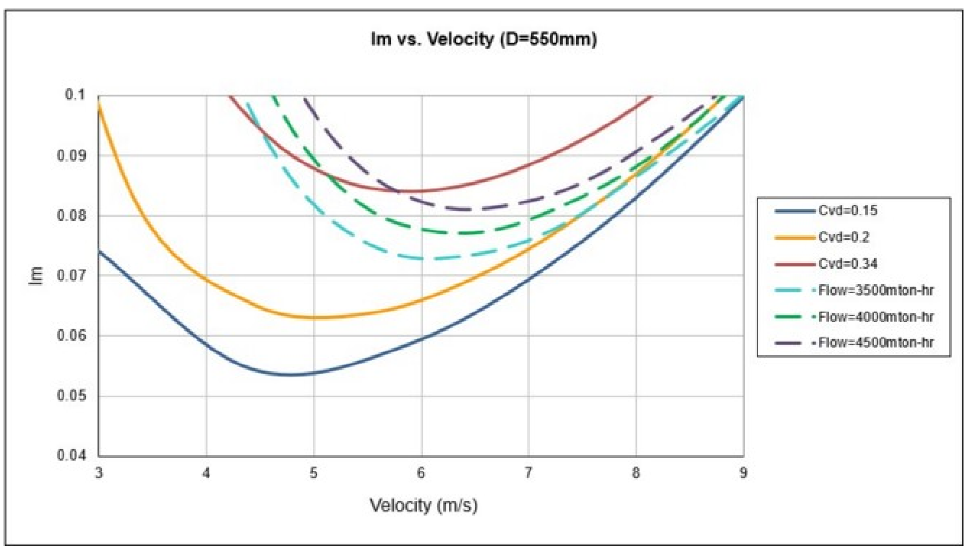
- FthVerify75 - 550 mm.fth
- FthVerify75 - 650 mm.fth
- FthVerify75 - 750 mm.fth

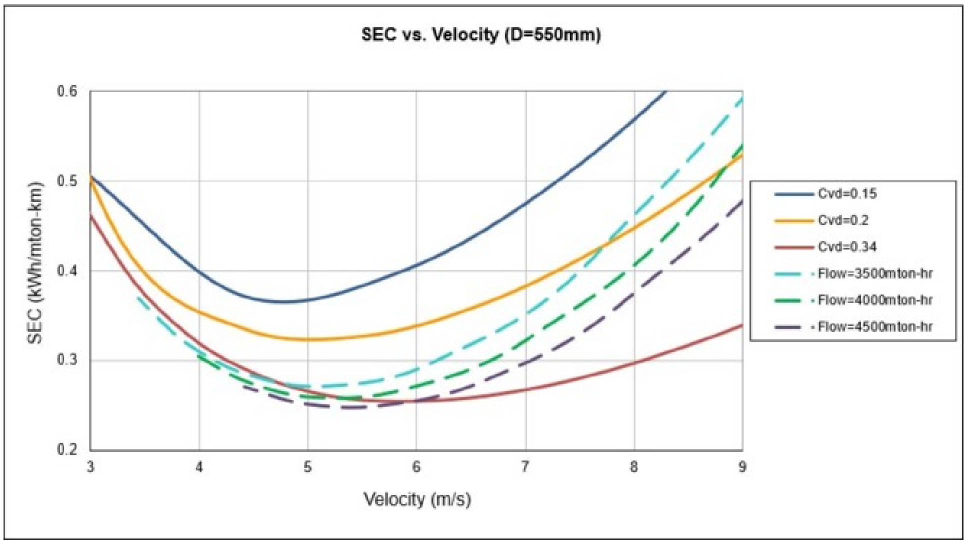
REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 329-337, Case Study 13.1

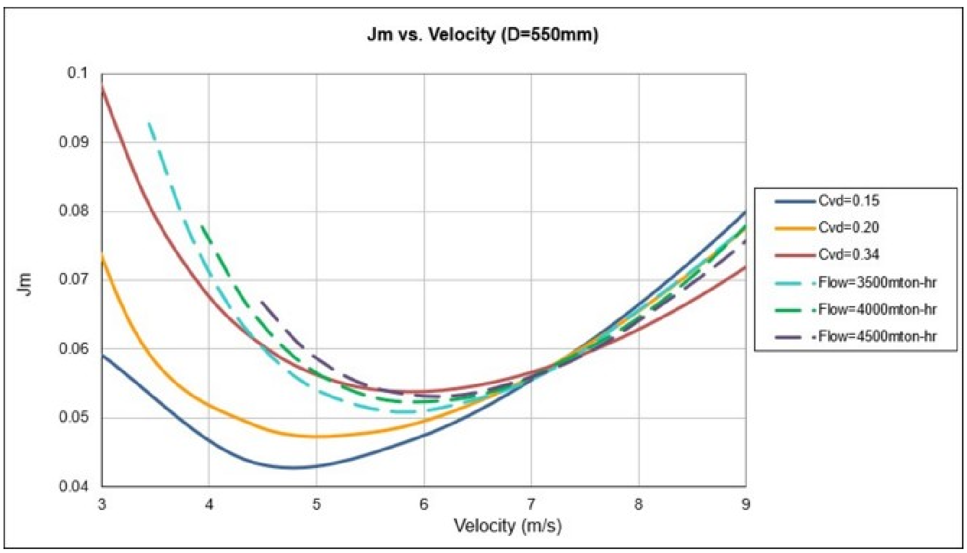
FLUID: Water/Sand

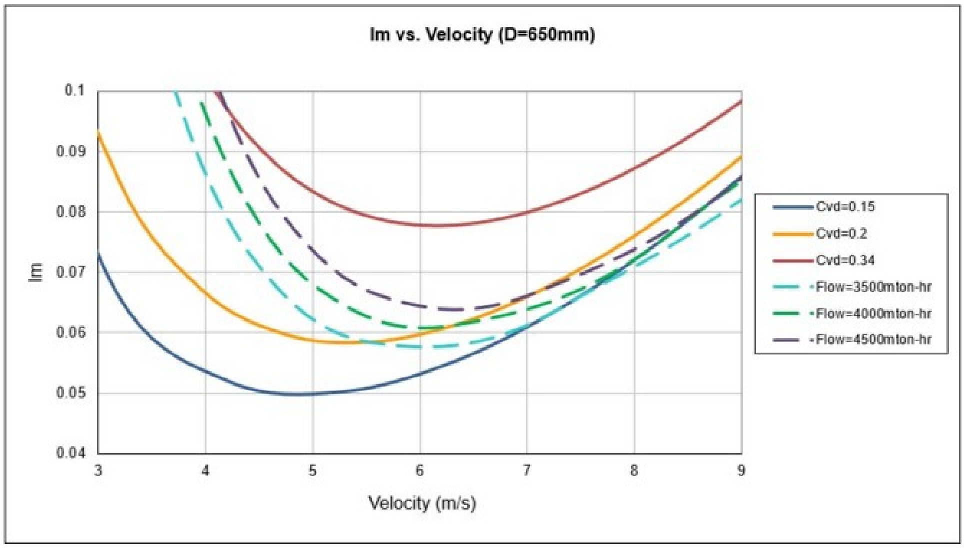
ASSUMPTIONS: Water temperature is assumed to be 4 deg. C. Other than the pipe diameter, the system configuration is not given (i.e. pressures, pipe length, etc) and were assumed. These assumptions do not affect the final results of the problem. Assume Vt/Vts = 0.56 for sand.

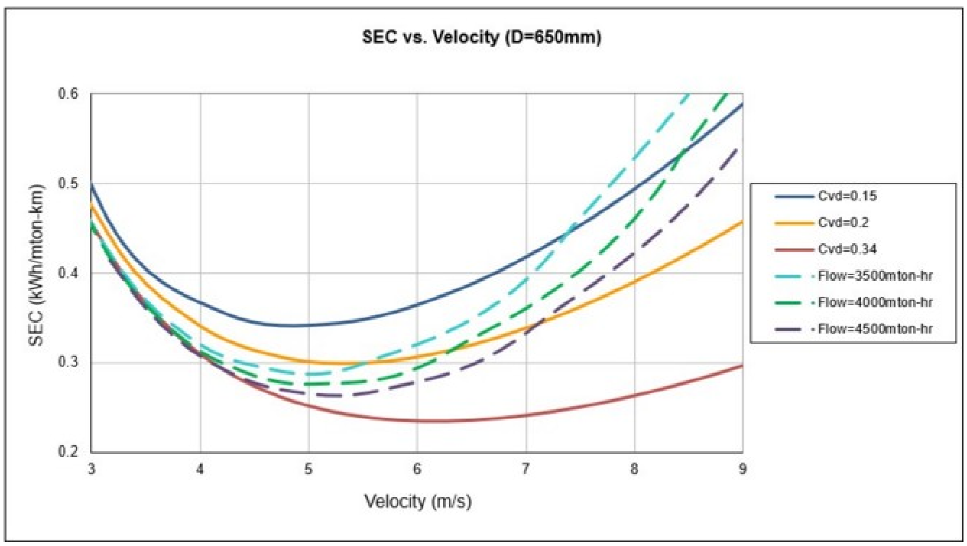
RESULTS:

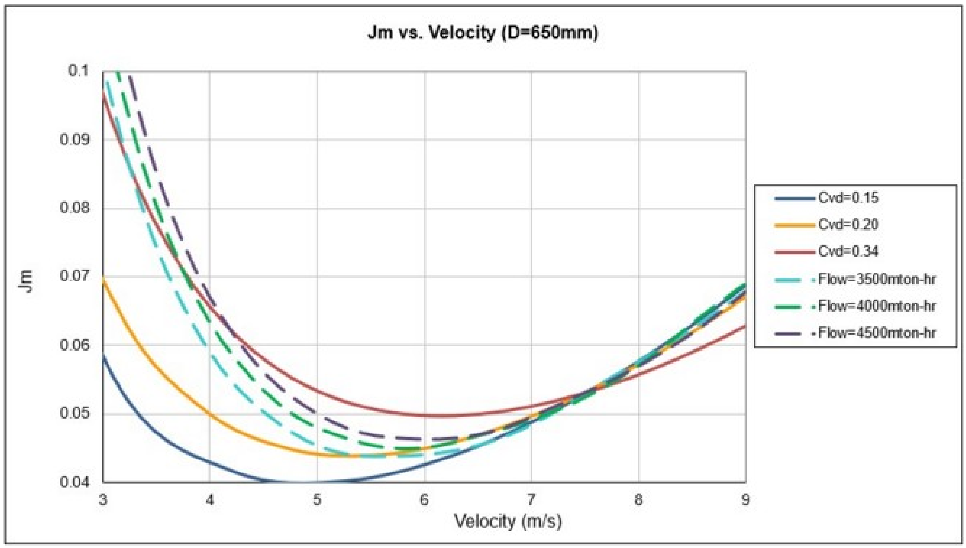


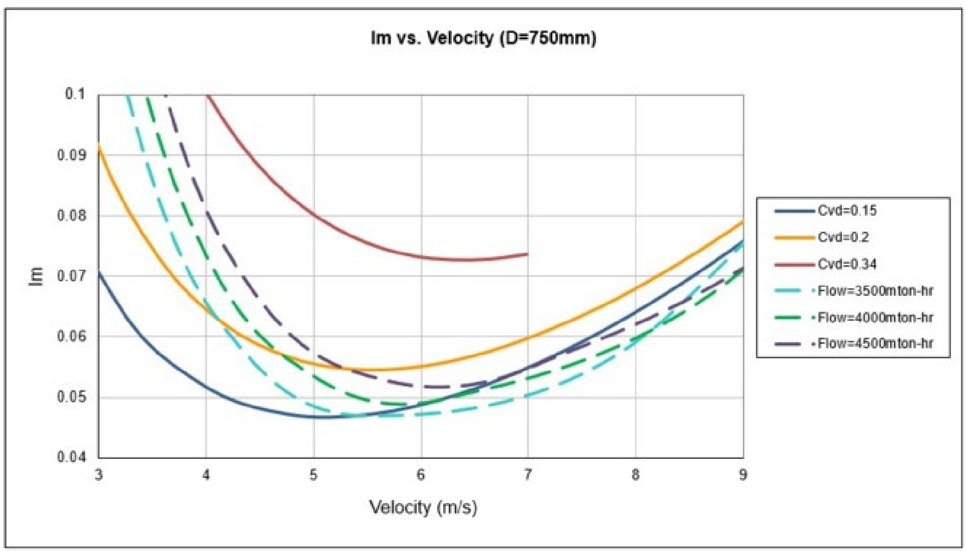

SEC vs. Velocity (D=550mm)
SEC (kWh/mton-km)
Velocity (m/s)
0.6
0.5
0.4
0.3
0.2
3
4
5
6
7
8
9
Cvd=0.15
Cvd=0.2
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr

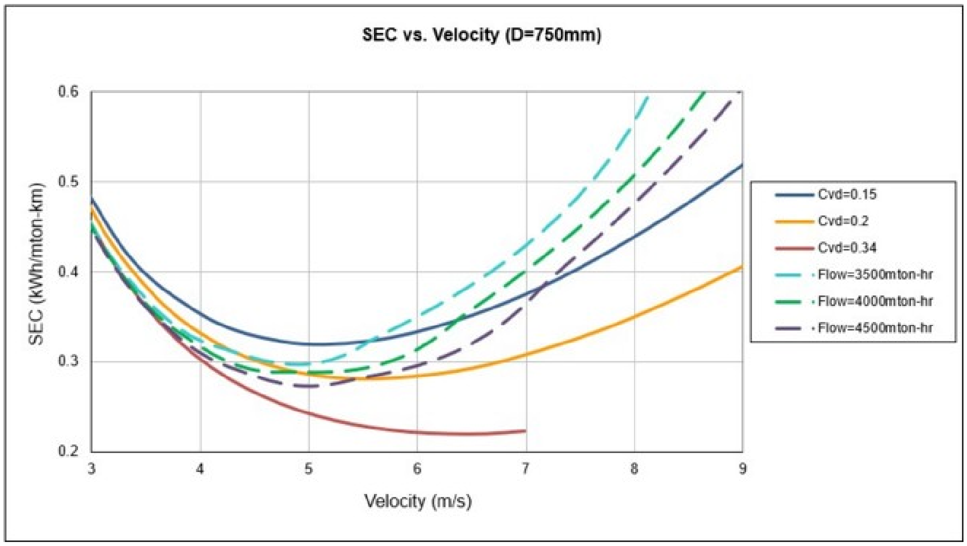

Jm vs. Velocity (D=550mm)
Jm
Velocity (m/s)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
3
4
5
6
7
8
9
Cvd=0.15
Cvd=0.20
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr

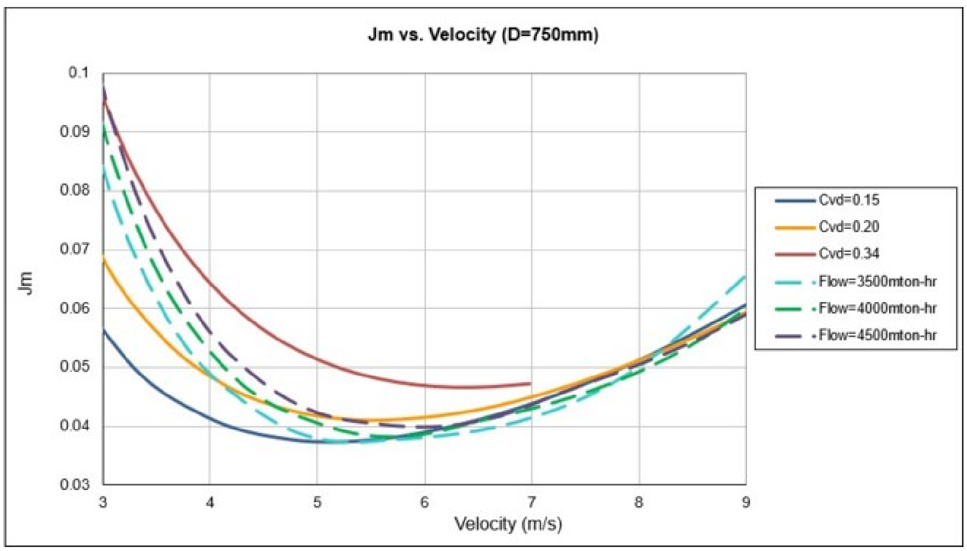

Im vs. Velocity (D=650mm)
Im
Velocity (m/s)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
3
4
5
6
7
8
9
Cvd=0.15
Cvd=0.2
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr


SEC vs. Velocity (D=650mm)
SEC (kWh/mton-km)
Velocity (m/s)
0.6
0.5
0.4
0.3
0.2
3
4
5
6
7
8
9
Cvd=0.15
Cvd=0.2
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr


Jm vs. Velocity (D=650mm)
Jm
Velocity (m/s)
Cvd=0.15
Cvd=0.20
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr


Im vs. Velocity (D=750mm)
Im
Velocity (m/s)
Cvd=0.15
Cvd=0.2
Cvd=0.34
Flow=3500mton-hr
Flow=4000mton-hr
Flow=4500mton-hr





List of All Verification Models

Verification Case 75 Problem Statement

Verification Case 75

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 329-337, Case Study 13.1

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

13.5 Case Study

Case Study 13.1

Case Studies 5.1 and 6.1 were concerned with the transport of 4000 tonnes per hour of coarse sand ($S_s = 2.65$) as a slurry in water, at delivered concentrations up to $C_{vd} = 0.24$. In this case study the design will be completed by examining system operability. Because the earlier studies all represent stages in the design, they will be summarised here.

Case Study 5.1 was the preliminary selection of pipe size, not needing information on the particle size beyond noting that the slurry has settling characteristics. Three pipes sizes were identified as suitable for more detailed study, keeping $V_m > V_{sm}$ to avoid deposition. The results are shown on Table 13.1.

Table 13.1

Table 13.1 Condition	D (m)	V_{sm} (m/s)	C_{vd}	V_m (m/s)
A	0.55	4.45	0.24	7.35
B	0.60	4.65	0.24	6.17
C	0.65	4.84	0.20	6.30

In Case Study 6.1 the particle size distribution was introduced, and used to calculate the friction gradient and specific energy consumption. It was shown that, for the 0.65 m pipe with 20% solids of the type specified (Condition C above) the friction gradient is 0.0612 m. water/m. pipe with specific energy consumption (SEC) of 0.315 kWh/tonne-km. It was also found that increasing C_{vd} and decreasing V_m reduced the specific energy consumption. A revised operating point (Condition D) was therefore chosen for further analysis.

D	=	0.65 m (V_{sm} = 4.84 m/s)
C_{vd}	=	0.221
V_m	=	5.70 m/s
i_m	=	0.0619 m.water/m.pipe
SEC	=	0.289 kWh/tonne-km

The feasibility of operating at this point was not investigated in Case Study 6.1. Note that for these conditions the 'standard velocity' V_x is 7.52 m/s (from Eq. 13.1) and the velocity at minimum point V_o is 5.48 m/s (from Eq. 13.3). Thus the proposed operating velocity V_m lies between V_o and V_x. It is presumed that the pumps have been selected to operate at V_m and that they have the usual falling characteristics: i.e. head developed decreases with discharge.

We now pursue this study further by examining system operability. Figures 13.9 to 13.14 show the system characteristics and SEC curves for three pipe sizes: 0.65, 0.75 and 0.55 m.

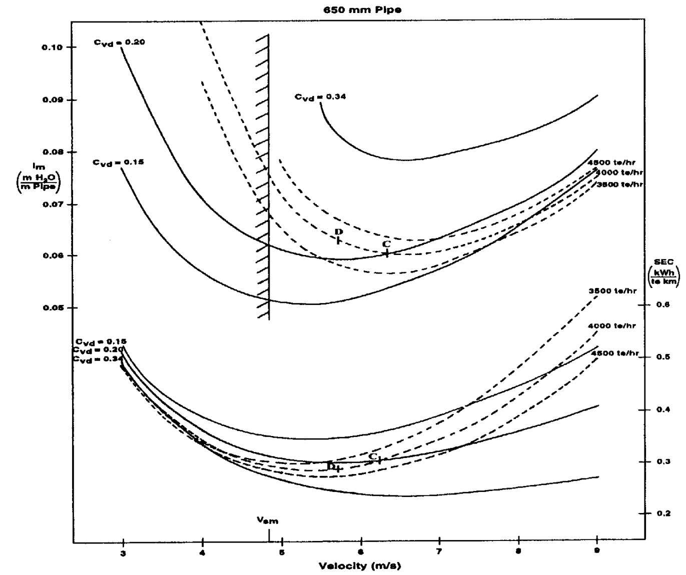


Figure 13.9. Friction gradient (i_m) and specific energy consumption (SEC) for transport of coarse sand in 0.65 m pipe

The curves are all calculated using the approach illustrated in Case Study 6.1, with values for the deposit velocity (V_{sm}) obtained from Eq. 5.10 as in Case Study 5.1. We consider first the pipe size which appeared to be attractive in the earlier case studies : D = 0.65 m. Figure 13.9 shows the system characteristics (expressed in terms of i_m) and SEC curves. The full curves are drawn for constant delivered concentration. The broken curves refer to constant throughput: throughput can be kept constant by increasing mixture velocity while decreasing delivered concentration. Conditions C and D considered in earlier case studies correspond to the points indicated on this figure.

Only now that the system characteristics are plotted is it evident that Condition D is too far back on the system curves to represent a feasible operating point for this material - the system characteristic shows i_m decreasing as V_m increases, so that operation will be unstable. Even Condition C is marginal. Given that the booster pump will have variable - speed diesel drive, operation at C might be feasible if solids consistency and throughput do not fluctuate too widely. However, if the variations are significant - for example, throughput variations from 3500 tonne/hr to 4500 tonne/hr (see Figure 13.9) - then operation will become unstable.

To achieve truly flexible operation, it is preferable to operate around the 'standard' velocity, where $i_w = j_m$. Figure 13.10 shows the same system characteristics, but now plotted in terms of j_m (i.e. i_m/S_m in m. slurry/m. pipe). The recommended operating velocity is therefore around point E, i.e. at V_m of 7.54 m/s, with C_{vd} = 0.168 and with SEC increased to 0.421 kWh/tonne-km. Thus, once operability is considered, it is seen that lower concentrations should be used, with correspondingly higher energy consumption.

We consider now whether going to a larger pipe would improve matters. Figures 13.11 and 13.12 show the curves of i_m, SEC and j_m for a pipe of 0.75 m diameter. To achieve operation at the standard velocity, it is now necessary to reduce the concentration further still: V_m = 7.83 m/s, with C_{vd} = 0.121. Again because of the cost of pumping the conveying water, the specific energy consumption is high: SEC is 0.511 kwh/tonne-km at a throughput of 4000 tonnes per hour. Furthermore, increasing the pipe size will increase the capital cost. Thus the 0.75 m pipe is not recommended.

A smaller pipe size should also be considered. Figures 13.13 and 13.14 show the curves of i_m, SEC and j_m calculated in the usual way for a pipe 0.55 m in diameter. Condition A from Case Study 5.1 is now seen to be quite close to the standard velocity. It therefore represents a flexible but stable operating point. Furthermore, because the delivered concentration is relatively high, at about 24% solids, the specific energy consumption is quite attractive, at 0.353 kWh/tonne-km.

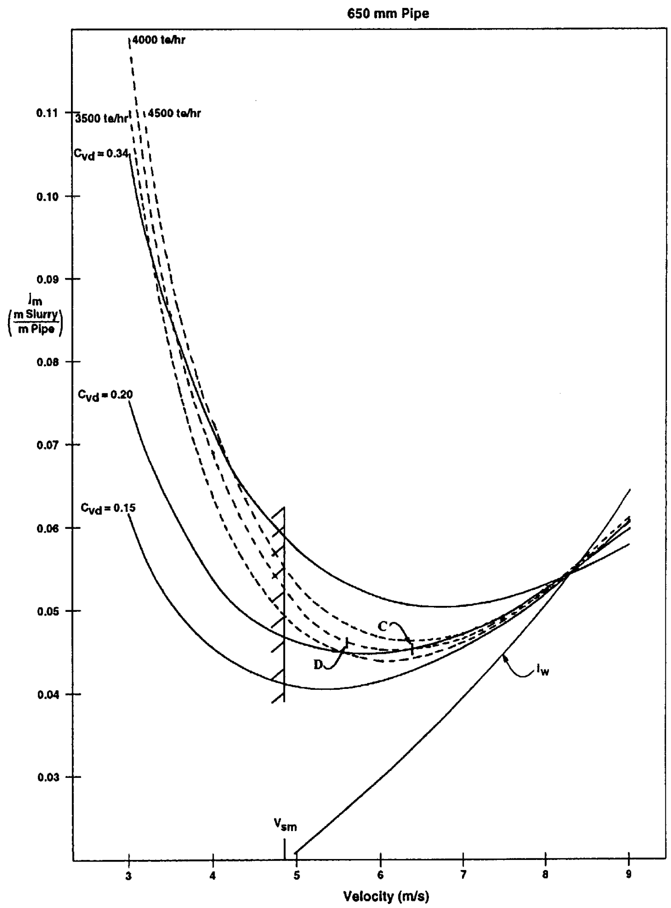


Figure 13.10. Friction gradient in terms of head of slurry (j_m) for coarse sand in 0.65 m pipe

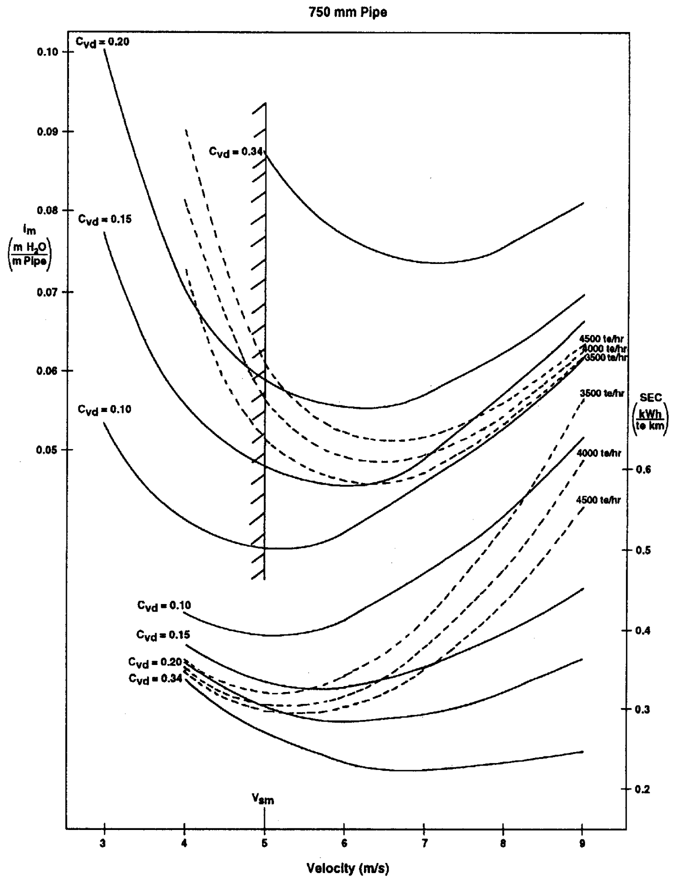


Figure 13.11. Friction gradient (i_m) and specific energy consumption (SEC) for transport of coarse sand in 0.75 m pipe

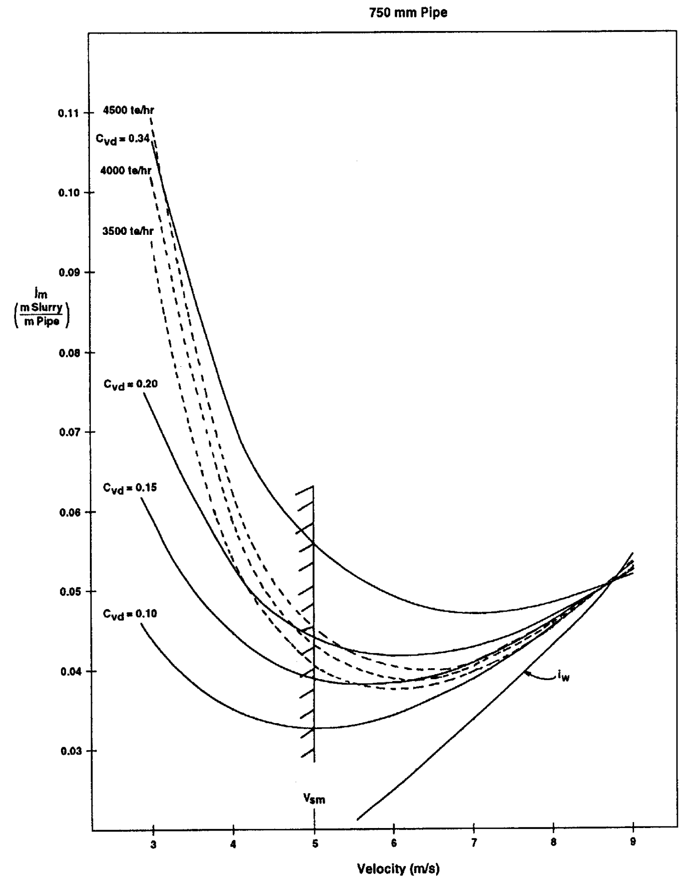


Figure 13.12. Friction gradient in terms of head of slurry (j_m) for coarse sand in 0.75 m pipe

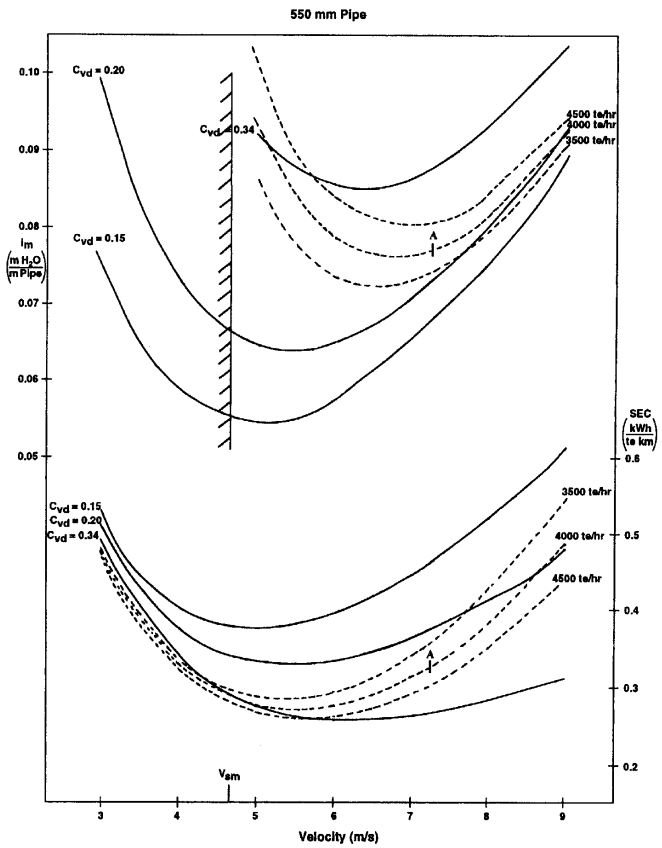


Figure 13.13. Friction gradient (i_m) and specific energy consumption (SEC) for transport of coarse sand in 0.55 m pipe

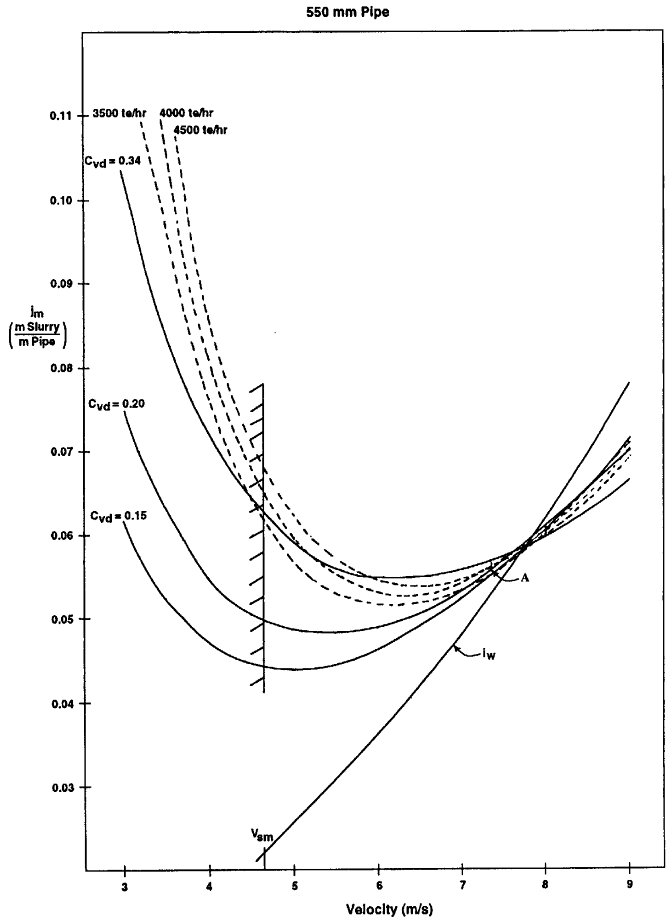


Figure 13.14. Friction gradient in terms of head of slurry (j_m) for coarse sand in 0.55 m pipe

Thus, of the three pipe sizes considered in this case study, the smallest would be preferred. In addition to its favorable specific energy consumption, it will have the least capital cost. It is left to the reader to carry out the comparison for the intermediate size, D = 0.60 m. A full design would now proceed to consider pump selection again, essentially repeating Case Study 12.1 for the revised pipe size.

The conclusions illustrated by this case study are general: for slurries with settling characteristics, considerations of energy consumption together with operational stability favour use of high solids concentration and small pipe diameter. Also, as indicated throughout this chapter, stable operation rather than deposition controls the operating velocity for this type of slurry.

On the negative side, the mixture velocity can be too high. For example, Condition A, as selected above, has a design mixture velocity of 7.35 m/s, and the pipe will experience substantial wear. As noted in Chapter 11, for some applications, e.g. conveying of phosphate matrix, wear is accommodated by rotating the pipe at intervals, usually through 120°, so as to expose new parts of the pipe wall to the stratified solids which cause the abrasion.

REFERENCES

Spells, K.E. (1955). Correlations for use in transport of aqueous suspensions of fine solids through pipes. *Trans. Instn. Chem. Engrs.*, Vol. 33, pp. 79-84.

Wilson, K.C., Clift, R. & Sellgren, A. (2002).Operating points for pipelines carrying concentrated heterogeneous slurries. *Powder Technology*, Vol. 123, pp. 19-24.

View Verification Case 75 Model

Verification Case 75

Verification Case 76

View Model   Problem Statement

PRODUCT: AFT Fathom (with SSL Module)

TITLE: FthVerify765.fth

REFERENCE: Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 117-118, Case Study 5.1

FLUID: Water/Sand

ASSUMPTIONS: Water temperature is assumed to be 20 deg. C. Other than the pipe diameter, the system configuration is not given (i.e. pressures, pipe length, etc) and were assumed. These assumptions do not affect the final results of the problem. Use the Minimal slurry calculation method. Assume the sliding friction coefficient is 0.4.

RESULTS:

Wilson, Addie, Sellgren & Clift Results:

Case	Velocity (meters/sec)	Settling Velocity (meters/sec)
D = 0.60 m	6.17	4.65
D = 0.55 m	7.35	4.45
D = 0.65 m	5.26	4.84

AFT Fathom Results:

Case	Velocity (meters/sec)	Settling Velocity (meters/sec)
D = 0.60 m	6.18	4.65
D = 0.55 m	7.36	4.45
D = 0.65 m	5.27	4.84

DISCUSSION:

The results from AFT Fathom match the results from Wilson, Addie, Sellgren & Clift very closely. It should be noted that the settling velocities for the 0.55 m and 0.65 m pipe diameter cases are assumed to be misprinted in the reference as 5.45 m/s and 5.84 m/s, because these cases should "bound" the 0.60 m case.

List of All Verification Models

Verification Case 76 Problem Statement

Verification Case 76

Wilson, Addie, Sellgren & Clift, Slurry Transport Using Centrifugal Pumps 3rd Edition, 2006, Publisher Springer, Pages 329-337, Case Study 13.1

Wilson, Addie, Sellgren & Clift's Title Page

Note: If you are having trouble reading the scanned image below, it is usually much easier to read when printed.

covered in the following chapter, will generally display smaller values of the solids effect.

Another type of stratified flow that may be encountered is flow above a stationary bed of solids. This type of operation is generally uneconomic, and thus is seldom a deliberate design choice, but it is sometimes found in existing pipelines. Flow over a stationary bed has been studied experimentally, and analysed by a computer model (Nnadi & Wilson, 1992; Pugh, 1995). For most cases of this type of flow encountered in pipelines a rough approximation to the computer output may be sufficient. In such instances, the following equation for estimating i_m may be useful. It is based on particles near the 'Murphian' size, which tend to be disproportionately represented in stationary deposits

$$i_m = 0.32\,(S_s - 1)^{1.05}\,C_{vd}^{0.6}\left(\frac{V_m}{(2gD)^{1/2}}\right)^{-0.1} \tag{5.14}$$

In some cases it will be necessary to incorporate the effects of pipe slope, which will be dealt with in Chapter 8. Before considering sloping flows, however, it will be necessary to investigate fluid suspension of particles, and show how suspended load combines with contact load in heterogeneous slurry flow. These are the subjects of Chapter 6.

5.6 Case Studies

Case Study 5.1 - Preliminary Pipe Sizing

A pipe is to be designed to convey solids at 4000 tonnes/hour (on a dry-weight basis). The material is described only as 'coarse sand', and it is estimated that the largest volumetric solids concentration which can be fed to the pumps is C_{vd} = 0.24. Preliminary estimates are required of suitable combinations of pipe size, C_{vd} and V_m. For convenience it will be assumed that pipes are available with inside diameter D in increments of 50 mm.

First, the tonnes per hour is converted to m^3/s of solids, Q_s, by dividing by the estimated sand density of 2.65 tonnes/m^3 and by 3600 s/hour, giving Q_s = 0.419 m^3/s. The mixture discharge Q_m equals Q_s/C_{vd} and hence the minimum Q_m, corresponding to the maximum C_{vd} of 0.24, equals 1.747 m^3/s.

The next step is to select a pipe size which gives a reasonable velocity for this value of Q_m. For example, a pipe of D = 0.60 m has a cross-sectional area of 0.283 m^2, and hence the velocity in this pipe must be at least 1.747/0.283 or 6.17 m/s in order to satisfy the maximum - C_{vd} condition.

This velocity seems to be in the right 'ball park', but must be compared with the deposition velocity V_{sm}. The particle size distribution of the coarse sand may not be known, but it is safe to assume that it contains grains of the 'Murphian' size, which is about 0.7 mm for this pipe diameter. For this particle and pipe size combination, the nomographic chart (Fig. 5.3) gives $V_{sm} \approx 5.8$ m/s, but it should be recalled that the value of V_{sm} estimated from the chart tends to be high. This point can be checked by means of Eq. 5.11, using a reasonable value of f_w, say 0.012. The result is a revised value of V_{sm} of only 4.65 m/s, so that the minimum operating velocity of 6.17 m/s provides a generous margin against deposition.

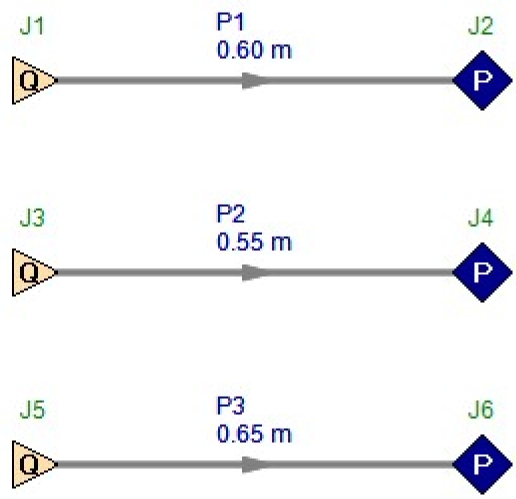
Other pipe sizes should also be investigated. The next smallest pipe (D = 0.55 m), is found to have a minimum operating velocity of 7.35 m/s, based on $C_{vd} = 0.24$. This can be compared to V_{sm} values of 5.4 m/s from Fig. 5.3 and 5.45 m/s from Eq. 5.11. Using the latter number as the upper limit for deposition, it is seen that the minimum operating velocity would be 65 percent in excess of the deposition limit, producing stable operation at the cost of high frictional losses. This pipe size could merit further investigation of the sort illustrated in case studies in subsequent chapters, but it is obvious that no smaller pipe would be suitable, and the 0.55 m pipe appears at present to be considerably less attractive than the 0.60 m size.

Turning to a larger pipe, with D = 0.65 m, it is found that for $C_{vd} = 0.24$ the minimum velocity is 5.26 m/s, while the V_{sm} values are 6.1 m/s from Fig. 5.3 and 5.84 m/s from Eq. 5.11. The minimum operating velocity of 5.26 m/s is less than 10 percent in excess of V_{sm} by Eq. 5.11, which is not a sufficient margin. A margin of 30 percent should be adequate, giving $V_m \approx$ 6.30 m/s. Increasing the velocity to this value requires a lower solids concentration, i.e. 0.20 instead of 0.24. This combination (D = 0.65 m, V_m = 6.3 m/s, $C_{vd} = 0.20$) merits further investigation, but any larger pipe size (which would involve higher operating velocity and lower delivered concentration), would probably not be suitable.

It is worth noting that this case study shows that a preliminary selection of appropriate pipe sizes can be made on the basis of criteria of continuity and deposition velocity, even though very little is known about the solids in the slurry. In the present instance, it is found that to transport 4000 tonnes of coarse sand per hour, only pipes of internal diameter 0.60 m and 0.65 m appear to be attractive.

View Verification Case 76 Model

Verification Case 76



Verification Case 77

View Model   Problem Statement

PRODUCT: AFT Fathom (with ANS Module)

TITLE: FthVerify77.fth

REFERENCE: William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Example 5.8, pages 179-180.

FLUID: Methanol

ASSUMPTIONS: N/A

RESULTS:

	Janna	AFT Fathom
Minimum Diameter	6.338 cm	6.337 cm
Nominal Size	2-1/2 in	2-1/2 in

DISCUSSION:

The AFT Fathom model used an assigned pressure upstream and a receiving reservoir at atmospheric pressure. The only design requirement for the system was a minimum flow rate of 0.8 m3/min.

The automated sizing was a discrete sizing to minimize flow volume.

List of All Verification Models

Verification Case 77 Problem Statement

Verification Case 77

William S. Janna, Introduction to Fluid Mechanics, 1983, PWS Publishers, Example 5.8, pages 179-180.

Janna's Title Page

Because V_2 and f are unknown, a trial and error solution is required. As a first trial assume $f = 0.03$; then

$$V_2 = 6.72 \text{ ft/s}$$

$$\left.\begin{aligned} Re &= \frac{\rho V D}{\mu g_c} = \frac{62.4(6.72)(0.1342)}{6 \times 10^{-4}} = 9.38 \times 10^4 \\ \frac{\epsilon}{D} &= \frac{0.00085}{0.1342} = 0.0063 \end{aligned}\right\} \text{ Moody diagram: } f = 0.0335$$

As a second trial assume $f = 0.0335$; then

$$V_2 = 6.51 \text{ ft/s}$$

$$\left.\begin{aligned} Re &= 9.1 \times 10^4 \\ \frac{\epsilon}{D} &= 0.0063 \end{aligned}\right\} \text{ Moody diagram } f = 0.034 \quad \text{(close enough)}$$

Using $f = 0.034$, $V_2 = 6.49$ ft/s, and the continuity equation, we find

$$Q = A_2 V_2 = 0.01414(6.49)$$

$$Q = 0.0918 \text{ ft}^3\text{/s} = 5.51 \text{ ft}^3\text{/min}$$

EXAMPLE 5.8

Methyl alcohol is used in a processing plant where a flow rate of 0.8 m^3/min of the liquid must be supplied. The available liquid pump can supply this flow rate only if the pressure drop in the supply line is less than 15 m head of water. The pipeline is made up of 50 m of soldered drawn copper tubing and follows the path shown in Figure 5.24. Determine the minimum size of tubing required.

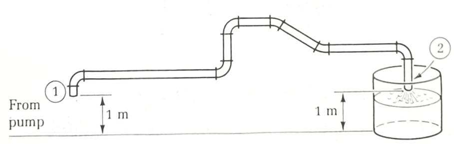


Figure 5.24. Sketch for Example 5.8.

SOLUTION

Write the Bernoulli equation from 1 to 2 as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D}\frac{V^2}{2g} + \Sigma K \frac{V^2}{2g}$$

The pressure drop is given in terms of meters of water:

$$\frac{p_1 - p_2}{\rho}\frac{g_c}{g} = 15 \text{ m of water}$$

where ρ is for water. From Appendix Table A-5 for methyl alcohol, $\rho = 789 \text{ kg/m}^3$ and $\mu = 0.56 \times 10^{-3} \text{ N-s/m}^2$. To convert the pressure drop to meters of alcohol, multiply by the ratio of densities:

$$\frac{p_1 - p_2}{\rho}\frac{g_c}{g} = \frac{15(1\,000)}{789} = 19.01 \text{ m of methyl alcohol}$$

The velocity at point 1 is the same as that at point 2 because the area at these points is the same. Moreover, $z_1 = z_2$. Thus the Bernoulli equation becomes

$$19.01 = \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right)$$

From continuity, we have $V = 4Q/\pi D^2$. From Table 5.4, we find the minor losses for four 90° elbows (assumed regular because nothing special was specified) and two 45° elbows; for lack of more specific data, take the minor loss coefficients to be the same as those for threaded fittings. Note that an exit fitting is not included because the exit kinetic energy is not zero. Therefore,

$$\Sigma K = 4(1.4) + 2(0.35) + 1.0 = 6.3$$

From Table 5.2 for drawn tubing, $\epsilon = 0.000\,15$ cm. By substitution into the Bernoulli equation, we get

$$19.01 = \frac{16Q^2}{2g\pi^2 D^4}\left[\frac{f(50)}{D} + 6.3\right]$$

With $Q = 0.8 \text{ m}^3/\text{min} = 0.013 \text{ m}^3/\text{s}$, the equation becomes

$$\frac{19.01(2)(9.81)\pi^2}{16(0.013)^2} = \frac{f(50)}{D^5} + \frac{6.3}{D^4}$$

or

$$3.67 \times 10^{-5} f = D^5 - 4.63 \times 10^{-6} D$$

A trial and error approach is required, but in this case it is easier to assume a diameter. As a first trial assume $D = 0.05$ m; then

$f = 0.001\,2$ (by equation above; 0.001 2 is unlikely—not on Moody diagram)

As a second trial assume $D = 0.06$ m; then

$f = 0.014$ (by equation)

$$A = \frac{\pi D^2}{4} = 0.002\,83 \text{ m}^2$$

$$V = \frac{Q}{A} = \frac{0.013}{0.002\,83} = 4.60 \text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{789(4.6)(0.06)}{0.56 \times 10^{-3}}$$

$$\left.\begin{aligned} &= 3.89 \times 10^5 \\ \frac{\epsilon}{D} &= \frac{0.000\,15 \text{ cm}}{6.0 \text{ cm}} = 0.000\,002\,5 \end{aligned}\right\} \quad \text{Moody diagram: } f = 0.014$$

The f value calculated by equation is slightly less than f from the Moody diagram. At this point, then, rather than using randomly chosen values of D, we use Appendix Table C-2 values for type M copper tubing (soldered fittings). For our next trial we therefore assume that $D = 6.338$ cm (slightly larger than the 6-cm diameter of the second trial), corresponding to the $2\frac{1}{2}$-standard type M. As a third trial assume $D = 0.06338$ m; then

$$A = 0.004\,017 \text{ m}^2 \qquad \text{(from Appendix Table C-2)}$$

$$f = 0.020 \qquad \text{(by equation)}$$

$$V = 3.236 \text{ m/s}$$

$$\left.\begin{aligned} \text{Re} &= 2.74 \times 10^5 \\ \frac{\epsilon}{D} &= 0.000\,024 \end{aligned}\right\} \quad \text{Moody diagram: } f = 0.015$$

Thus the calculated value of $f = 0.020$ is greater than that from the Moody diagram ($f = 0.015$) for the $2\frac{1}{2}$-standard type M copper tube. Anything smaller will reduce the volume flow rate; anything larger will work but incurs an unnecessary expense. Thus select

$$\underline{D = 6.338 \text{ cm} \qquad (2\tfrac{1}{2}\text{-standard type M copper tube})}$$

5.8 / Hydraulic and Energy Grade Lines

Hydraulic and energy grade lines give a graphic presentation of the flow quantities in a particular configuration. The **hydraulic grade line** is a plot of pressure versus distance; the **energy grade line** is a plot of the sum of pressure and kinetic energy as a function of distance. These concepts are best illustrated by example.

Consider two large reservoirs connected with a pipe as shown in Figure 5.25. Because one liquid reservoir surface is elevated above the other, a flow exists in the pipe. The total mechanical energy in the system at section 1 is

$$E_1 = \frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1$$

View Verification Case 77 Model

Verification Case 77

Verification Case 78

View Model   Problem Statement

PRODUCT: AFT Fathom (with ANS Module)

TITLE: FthVerify78.fth

REFERENCE: James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Example 6.6, page 179.

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

	John & Haberman	AFT Fathom Continuous	AFT Fathom Discrete
Minimum Diameter	0.0311 m	0.02848 m	0.03505 m
Nominal Size	-	-	1-1/4 in

DISCUSSION:

The AFT Fathom pipe material library only has ductile iron data from 3 inch to 54 inch. The minimized pipe size is less than 3 inch so steel pipe was substituted for cast iron. Because the roughness of steel pipe is less than cast iron, the minimized size for the continuous case was smaller than the reference value.

Both the continuous and discrete automated sizing scenarios were run to minimize flow volume.

List of All Verification Models

Verification Case 78 Problem Statement

Verification Case 78

James John, William Haberman, Introduction to Fluid Mechanics, 2nd Ed., 1980, Prentice-Hall, Example 6.6, page 179.

John & Haberman's Title Page

determined; a trial-and-error solution is called for. As a first trial, assume $f = 0.02$.

$$(9.81\ \text{m/s}^2)(20\ \text{m}) = \left[\frac{0.02 \times 250}{0.10} + (\underset{\substack{\text{square-}\\\text{edged}\\\text{inlet}}}{0.5} + \underset{\text{4 elbows}}{3.0} + \underset{\substack{\text{globe}\\\text{valve}}}{6.4}) + 1\right]\frac{V_e^2}{2}$$

$$\frac{V_e^2}{2} = \frac{9.81(20)}{50 + 9.9 + 1}\ \text{m}^2/\text{s}^2$$

or

$$V_e = 1.795\ \text{m/s}$$

For this velocity,

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.795\ \text{m/s})(0.10\ \text{m})}{1.00 \times 10^{-6}\ \text{m}^2/\text{s}} = 1.795 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{0.12 \times 10^{-3}\ \text{m}}{0.10\ \text{m}} = 1.2 \times 10^{-3}$$

From the Moody diagram, we obtain $f = 0.022$. Since this value does not agree with our initial trial, we shall now make a second trial, starting with $f = 0.022$.

$$\frac{fL}{D} = \frac{0.022 \times 250}{0.10} = 55$$

$$\frac{V_e^2}{2} = \frac{9.81(20)}{55 + 9.9 + 1}$$

or

$$V_e = 1.725\ \text{m/s}$$

For this velocity,

$$\text{Re} = \frac{VD}{\nu} = \frac{1.725 \times 0.10}{10^{-6}} = 1.725 \times 10^5$$

From the Moody diagram, we find $f = 0.022$. It can be seen that generally we are able to converge on the correct answer quite rapidly; no more than two trials are required, as a rule. The water flow rate is

$$\begin{aligned} Q &= AV \\ &= \left[\frac{\pi}{4}(0.10)^2\ \text{m}^2\right] 1.725\ \text{m/s} \\ &= 0.01355\ \text{m}^3/\text{s} \\ &= \underline{13.55\ \text{l/s}} \end{aligned}$$

EXAMPLE 6.6. A pump is to be used to supply 5 liters per second of water from a reservoir to a point 400 m from the reservoir at the same level as the reservoir surface. Determine the minimum-size cast iron pipe required. The water temperature is 15°C; assume that minor losses can be neglected. (See Figure 6.33.) The pump supplies 50 kW of power to the water flow.

Solution: From (6.8) we can express the pump work in terms of reservoir surface and outlet conditions:

$$-\frac{1}{\dot{m}}\frac{dW_p}{dt} = \frac{p_e - p_0}{\rho} + g(z_e - z_0) + \frac{fL}{D}\frac{V^2}{2} + \frac{V_e^2}{2}$$

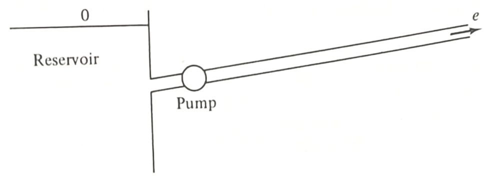


Figure 6.33

For our case,

$$\dot{m} = \rho Q = (999.2\ \text{kg/m}^3)(5 \times 10^{-3}\ \text{m}^3/\text{s}) = 4.996\ \text{kg/s}$$

$$-\frac{1}{\dot{m}}\frac{dW_p}{dt} = -\frac{1}{4.996\ \text{kg/s}}(-50 \times 10^3\ \text{N}\cdot\text{m/s}) = +10{,}010\ \text{m}^2/\text{s}^2$$

(*Note:* dW_p/dt is negative work in a thermodynamic sense, for it represents work done on the fluid.)

We now have

$$10{,}010 = \left(\frac{fL}{D} + 1\right)\frac{V^2}{2}$$

where

$$V = \frac{Q}{A} = \frac{5 \times 10^{-3}\ \text{m}^3/\text{s}}{(\pi/4)D^2\ \text{m}^2}$$

$$= \frac{6.366 \times 10^{-3}}{D^2}\ \text{m/s} \qquad \text{with } D \text{ in m}$$

$$10{,}010 = \left(\frac{400f}{D} + 1\right)\frac{4.053 \times 10^{-5}}{2D^4}$$

For this case, with D the unknown, the Reynolds number and f cannot be immediately determined. A trial-and-error procedure is required. As a first trial, assume that $f = 0.025$.

$$10{,}010 = \left[\frac{400(0.025)}{D} + 1\right]\frac{2.026 \times 10^{-5}}{D^4}$$

$$= \frac{20.26 \times 10^{-5}}{D^5} + \frac{2.026 \times 10^{-5}}{D^4}$$

The second term on the right is small compared to the first, so, to a first approximation,

$$10{,}010 \sim \frac{2.026 \times 10^{-4}}{D^5} \quad \text{and} \quad D = 0.0289\ \text{m}$$

For this first trial,

$$V = \frac{6.366 \times 10^{-3}}{(0.0289)^2} = 7.622\ \text{m/s}$$

$$\text{Re} = \frac{7.622 \times 0.0289}{1.15 \times 10^{-6}} = 1.915 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{0.26 \times 10^{-3}}{0.0289} = 0.0090$$

From the Moody diagram, we obtain $f = 0.036$. To start a second iteration, let $f = 0.036$. Therefore,

$$10{,}010 = \left[\frac{400(0.036)}{D} + 1\right]\frac{2.026 \times 10^{-5}}{D^4}$$

or

$$10{,}010 \sim \frac{0.0002917}{D^5} \quad \text{and} \quad D = 0.0311 \text{ m}$$

For this second trial,

$$V = \frac{6.366 \times 10^{-3}}{(0.0311)^2} = 6.582 \text{ m/s}$$

$$\text{Re} = \frac{6.582 \times 0.0311}{1.15 \times 10^{-6}} = 1.78 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{0.26 \times 10^{-3}}{0.0311} = 0.0084$$

From the Moody diagram, we obtain $f = 0.036$. The agreement with the assumed value is good enough so that $D = 0.0311$ m can be taken as the required diameter.

6.6 PIPES IN PARALLEL

The previous section has dealt with systems in which the resistances (friction and minor losses) were placed in series. In many cases, however, a system involves pipes in parallel, as shown in Figure 6.34. For example, it might be

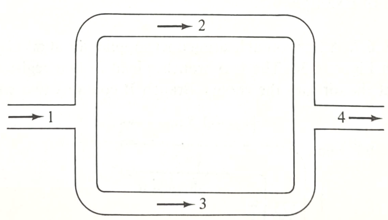


Figure 6.34

required to find the flows through 2 and 3 for given pipe diameters D_1, D_2, D_3, and D_4, given $p_1 - p_4$, and given flow at 1. In this case, the flow resistances are in parallel. There is a direct analog to a dc electrical circuit. In an electrical circuit, electrical current (amperes) flows through an electrical resistance measured in ohms. The driving force for such a system is voltage drop. In the circuit shown (Figure 6.35) resistances R_2 and R_3 are in parallel.

View Verification Case 78 Model

Verification Case 78

Verification Case 79

View Model   Problem Statement

PRODUCT: AFT Fathom (with ANS Module)

TITLE: FthVerify79.fth

REFERENCE: Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Example 8.8, page 377.

FLUID: Water

ASSUMPTIONS: N/A

RESULTS:

	Fox & McDonald	**AFT Fathom**
Minimum Diameter	6.065 in	6.065 in
Nominal Size	6 inch	6 inch

DISCUSSION:

The AFT Fathom pipe material library does not have drawn aluminum pipe so steel pipe size data was used. The roughness model was modified to use a relative roughness of 0.00001, which is representative of drawn tubing. With this setting, the results for nominal size matched exactly.

The automated sizing was discrete with the objective set to minimize flow volume.

List of All Verification Models

Verification Case 79 Problem Statement

Verification Case 79

Robert W. Fox and Alan T. McDonald, Introduction to Fluid Mechanics, 3rd Ed., 1985, John Wiley & Sons, Example 8.8, page 377.

Fox & McDonald's Title Page

quite old, choose $e/D = 0.005$. Then, from Fig. 8.14, guess $f \simeq 0.03$. Then a first approximation to $\bar{V}_2$ is

$$\bar{V}_2 = \left[2 \times \frac{32.2 \text{ ft}}{\text{sec}^2} \times 80 \text{ ft} \times \frac{1}{0.03(2040 + 8) + 1}\right]^{1/2} = 9.08 \text{ ft/sec}$$

Now check the value assumed for f.

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{9.08 \text{ ft}}{\text{sec}} \times \frac{\text{ft}}{3} \times \frac{\text{sec}}{1.2 \times 10^{-5} \text{ ft}^2} = 2.52 \times 10^5$$

For $e/D = 0.005$, $f = 0.031$ from Fig. 8.14. Using this value, we obtain

$$\bar{V}_2 = \left[2 \times \frac{32.2 \text{ ft}}{\text{sec}^2} \times 80 \text{ ft} \times \frac{1}{0.031(2040 + 8) + 1}\right]^{1/2} = 8.94 \text{ ft/sec}$$

Thus convergence is satisfactory. The volume flow rate is

$$Q = \bar{V}_2 A = \bar{V}_2 \frac{\pi D^2}{4} = \frac{8.94 \text{ ft}}{\text{sec}} \times \frac{\pi}{4}\left(\frac{1}{3}\right)^2 \text{ft}^2 \times \frac{7.48 \text{ gal}}{\text{ft}^3} \times \frac{60 \text{ sec}}{\text{min}}$$

$Q = 350$ gpm ← Q

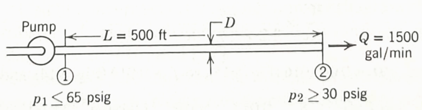
This problem illustrates the procedure for solving pipe flow problems in which the flow rate is unknown. Note that the velocity and, hence, the flow rate, is essentially proportional to $1/\sqrt{f}$. Doubling the value of e/D to account for aging reduced the flow rate by about 10 percent.

Example 8.8

Spray heads in an agricultural spraying system are to be supplied with water through 500 ft of drawn aluminum tubing from an engine-driven pump. In its most efficient operating range, the pump output is 1500 gpm at a discharge pressure not exceeding 65 psig. For satisfactory operation, the sprinklers must operate at 30 psig or higher pressure. Minor losses and elevation changes may be neglected. Determine the smallest standard pipe size that can be used.

EXAMPLE PROBLEM 8.8

GIVEN: Water supply system, as shown.



FIND: Smallest standard D.

SOLUTION:
Δp, L, and Q are known. D is unknown, so iteration will be required to determine the minimum standard diameter that satisfies the pressure drop constraint at the given flow rate.

The maximum allowable pressure drop is

$$\Delta p_{max} = p_{1\,max} - p_{2\,min} = (65 - 30)\text{ psi} = 35\text{ psi}$$

Computing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} \quad (8.28)$$

$$h_{l_T} = h_l + h_{l_m} = f\frac{L}{D}\frac{\bar{V}_2^2}{2} \qquad (h_{l_m} = 0(3))$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) $h_{l_T} = h_l$, i.e. $h_{l_m} = 0$
(4) $z_1 = z_2$
(5) $\bar{V}_1 = \bar{V}_2 = \bar{V}$; $\alpha_1 \simeq \alpha_2$

Then

$$\Delta p = p_1 - p_2 = f\frac{L}{D}\frac{\rho\bar{V}^2}{2}$$

Since trial values of D are to be assumed, it is convenient to substitute $\bar{V} = Q/A = 4Q/\pi D^2$ so that

$$\Delta p = f\frac{L}{D}\frac{\rho}{2}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{8fL\rho Q^2}{\pi^2 D^5} \quad (1)$$

The Reynolds number is needed to find f. In terms of Q,

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{\bar{V}D}{\nu} = \frac{4Q}{\pi D^2}\frac{D}{\nu} = \frac{4Q}{\pi\nu D}$$

Finally, Q must be converted to cubic feet per second.

$$Q = 1500\,\frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60\text{ sec}} \times \frac{\text{ft}^3}{7.48\text{ gal}} = 3.34\text{ ft}^3/\text{sec}$$

For an initial guess, take nominal 4 in. (4.026 in. i.d.) pipe:

$$Re = \frac{4Q}{\pi\nu D} = \frac{4}{\pi} \times \frac{3.34\text{ ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5}\text{ ft}^2} \times \frac{1}{4.026\text{ in.}} \times \frac{12\text{ in.}}{\text{ft}} = 1.06 \times 10^6$$

For drawn tubing, $e/D = 0.000{,}016$ (Fig. 8.15), so $f \simeq 0.012$ (Fig. 8.14), and

$$\Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = \frac{8}{\pi^2} \times 0.012 \times 500\text{ ft} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times (3.34)^2\,\frac{\text{ft}^6}{\text{sec}^2} \times \frac{1}{(4.026)^5\text{ in.}^5} \times 1728\,\frac{\text{in.}^3}{\text{ft}^3} \times \frac{\text{lbf}\cdot\text{sec}^2}{\text{slug}\cdot\text{ft}}$$

$$\Delta p = 172\text{ lbf/in.}^2 > \Delta p_{max}$$

Since this value is too large, try $D = 6$ in. (actually 6.065 in. i.d.):

$$Re = \frac{4}{\pi} \times \frac{3.34\ \text{ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5}\ \text{ft}^2} \times \frac{1}{6.065\ \text{in.}} \times \frac{12\ \text{in.}}{\text{ft}} = 7.01 \times 10^5$$

For drawn tubing, $e/D = 0.000{,}010$ (Fig. 8.15), so $f \simeq 0.013$ (Fig. 8.14), and

$$\Delta p = \frac{8}{\pi^2} \times 0.013 \times 500\ \text{ft} \times \frac{1.94\ \text{slug}}{\text{ft}^3} \times (3.34)^2 \frac{\text{ft}^6}{\text{sec}^2} \times \frac{1}{(6.065)^5\ \text{in.}^5} \times \frac{(12)^3\ \text{in.}^3}{\text{ft}^3} \times \frac{\text{lbf} \cdot \text{sec}^2}{\text{slug} \cdot \text{ft}}$$

$$\Delta p = 24.0\ \text{lbf/in.}^2 < \Delta p_{\max}$$

Since this value is less than the allowable pressure drop, we should check a 5 in. (nominal) pipe. With an actual i.d. of 5.047 in.,

$$Re = \frac{4}{\pi} \times \frac{3.34\ \text{ft}^3}{\text{sec}} \times \frac{\text{sec}}{1.2 \times 10^{-5}\ \text{ft}^2} \times \frac{1}{5.047\ \text{in.}} \times \frac{12\ \text{in.}}{\text{ft}} = 8.43 \times 10^5$$

For drawn tubing, $e/D = 0.000{,}012$ (Fig. 8.15), so $f \simeq 0.012$ (Fig. 8.14), and

$$\Delta p = \frac{8}{\pi^2} \times 0.012 \times 500\ \text{ft} \times \frac{1.94\ \text{slug}}{\text{ft}^3} \times (3.34)^2 \frac{\text{ft}^6}{\text{sec}^2} \times \frac{1}{(5.047)^5\ \text{in.}^5} \times \frac{(12)^3\ \text{in.}^3}{\text{ft}^3} \times \frac{\text{lbf} \cdot \text{sec}^2}{\text{slug} \cdot \text{ft}}$$

$$\Delta p = 55.5\ \text{lbf/in.}^2 > \Delta p_{\max}$$

Thus the criterion for pressure drop is satisfied for a minimum nominal diameter of 6 in. pipe. D

This problem illustrates the procedure for solving pipe flow problems when the diameter is unknown. Note from Eq. 1 that the pressure drop in turbulent pipe flow is proportional to f/D^5. The variation of f is small, so Δp at constant flow rate is approximately proportional to $1/D^5$.

We have solved each of Example Problems 8.7 and 8.8 by direct iteration. Several specialized forms of friction factor versus Reynolds number diagrams have been introduced to solve problems of this type without the need for iteration. For examples of these specialized diagrams, see [16] and [17].

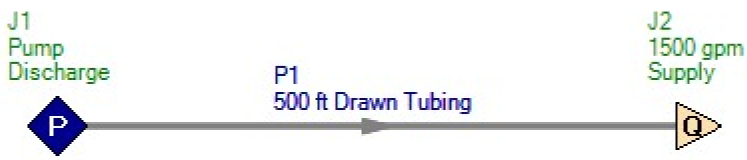
Example Problems 8.9 and 8.10 illustrate the evaluation of minor loss coefficients and the application of a diffuser to reduce exit kinetic energy from a flow system.

Example 8.9

Reference 18 reports results of measurements made to determine entrance losses for flow from a reservoir to a pipe with various degrees of entrance rounding. A copper pipe 10 ft long with 1.5 in. i.d. was used for the tests. The pipe discharged to atmosphere.

View Verification Case 79 Model

Verification Case 79



Verification Case 80

View Model   Problem Statement

PRODUCT: AFT Fathom (with ANS Module)

TITLE: FthVerify80.fth

REFERENCE: Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Example 5.10, page 5-12.

FLUID: Air

ASSUMPTIONS: N/A

RESULTS:

Duct Diameter (inches)	Lindeburg	AFT Fathom
Duct A-B	14	14
Duct C	12	12
Duct D-E	10	10
Duct F	8	8

DISCUSSION:

This duct sizing example used the static regain method to size ducts. The objective of this method is to equalize the pressures along the duct and at each register. This example also placed a maximum velocity limit of 1500 ft/min on the ducts.

To model this in AFT Fathom, two design requirements were created, one with a maximum pressure of 0.4 in H2O std gauge, and the other with a maximum velocity of 1500 ft/min. The velocity design requirement was applied to all ducts and the pressure design requirement was applied to all ducts that were connected between the registers. The first duct attached to the fan was allowed to exceed the maximum pressure design requirement.

A special material model was created of circular ducts from 4 inches to 36 inches with the inner diameter equal to the nominal size.

The automated sizing was discrete with the objective to minimize flow volume.

List of All Verification Models

Verification Case 80 Problem Statement

Verification Case 80

Michael R. Lindeburg, P.E., Mechanical Engineering Review Manual, 7th Ed., Professional Publications, Example 5.10, page 5-12.

Lindeburg's Title Page

step 3: Assume a main run velocity, v_m, from table 5.5.

step 4: Size the main run from $A_m = \frac{Q_m}{v_m}$.

step 5: Find the equivalent length of duct from the fan to the first take-off. It may be necessary to make some assumptions about the bend radii.

step 6: From figure 5.4, find the friction loss in the main run up to the first take-off. This is Δp_{main}.

step 7: The design pressure at the nearest grille outlet is p_d. Therefore, the pressure at the start of the main run is

$$p_{s,\,start} = p_d + \Delta p_{main} \qquad 5.42$$

step 8: If the fan outlet and the main velocities are different, calculate the regain.

$$\Delta p_{fan} = R\left(\left(\frac{v_{fan}}{4005}\right)^2 - \left(\frac{v_m}{4005}\right)^2\right) \qquad 5.43$$

If v_m is larger than v_{fan}, use $R = 1.1$. Otherwise, use $R = .75$.

step 9: Find the required fan outlet pressure.

$$p_{s,\,fan} = p_{s,\,start} - \Delta p_{fan} \qquad 5.44$$

If $p_{s,\,fan}$ is too high, choose a lower v_m and go to step 4.

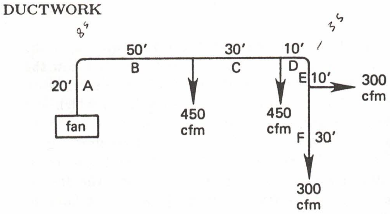
step 10: Find the equivalent length between grilles and take-offs. It may be necessary to make assumptions about sizes and radii. Proportion p_{var} along the grille run, giving the higher losses to longer runs and to runs with fittings.

step 11: Knowing L_e and Q_2 for each section, find v_2 from figure 5.6.

step 12: Solve for the duct size from $A = \frac{Q}{v}$.

Example 5.10

Use the static-regain method to size the duct system shown. The fan outlet is 1500 cfm at 1700 fpm. The equivalent length of the bend is 15 feet.



step 1: Assume $p_{var} = .10''$ w.g.

step 2: Assume an average $p_d = .25''$ w.g.

step 3: Assume $v_m = 1500$ fpm.

step 4: $A_m = \frac{1500}{1500} = 1$ sq. ft. For a round duct, the diameter will be approximately 14″.

step 5: The equivalent length of sections A and B, including the bend, is

$$20 + 15 + 50 = 85'$$

step 6: From figure 5.4, the friction loss per 100 feet is .20″ w.g. Thus, the loss in sections A and B is

$$\left(\frac{85}{100}\right).20 = .17''\text{ w.g.}$$

step 7: The pressure at outlet 1 is .25″ w.g. The pressure at the start of run A is .25 + .17 = .42″ w.g.

step 8: The fan regain is

$$.75\left[\left(\frac{1700}{4005}\right)^2 - \left(\frac{1500}{4005}\right)^2\right] = .03''\text{ w.g.}$$

step 9: The required fan pressure is .42 − .03 = .39″ w.g.

step 10: The equivalent lengths of the remaining sections are found:

section	equivalent length	Q
C	30	1050
D/E	10 + 10 + 15 = 35	600
F	30	300
	115′ total	

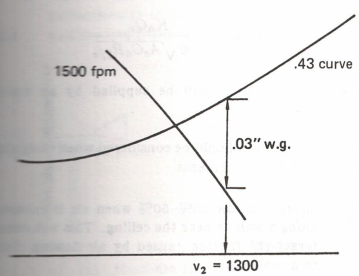
p_{var} is distributed along CDEF in proportion to the equivalent length. Thus, .03 is distributed to sections C and F, and $\frac{D}{E}$ is given as .04″ w.g.

step 11: $L_e/(Q^{.61})$ is calculated for each section:

section	$L_e/Q^{.61}$
C	0.43
D/E	0.71
F	0.92

v_2 is found from figure 5.6, allowing for a loss in the static pressure between the v_1 and $L_e/Q^{.61}$ curves equal to the proportion of p_{var} calculated in step 10.

$$v_{2,C} = 1275 \text{ fpm}$$
$$v_{2,D/E} = 1050 \text{ fpm}$$
$$v_{2,F} = 850 \text{ fpm}$$



step 12: The required duct areas and diameters are:

$$A_C = \frac{1050}{1275} = 0.82 \quad \text{or } 12'' \text{ diameter}$$
$$A_{D/E} = \frac{600}{1050} = 0.57 \quad \text{or } 10'' \text{ diameter}$$
$$A_F = \frac{300}{850} = 0.35 \quad \text{or } 8'' \text{ diameter}$$

5 DAMPERS

Dampers are included in all runs to balance the system. *Balancing* is the act of closing down dampers to equalize the friction losses in all ducts. Without dampering, the majority of the air flowing would escape out the closest grilles.

It is generally a good idea to install dampers even when more sophisticated design methods are used. Such dampers can be used for fine-tuning the installation.

Dampers can be motorized or operated manually. Figure 5.7 illustrates several common types of dampers.

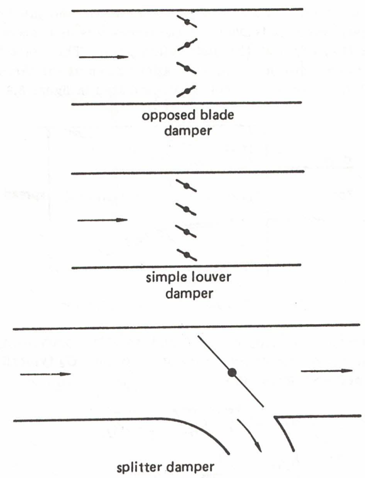


Figure 5.7 Types of Dampers

It is possible to design the outlet grilles to act as a damper. Such registers may be designed as a plate with multiple small holes through which the air must escape. The disadvantage of combining the tasks of air distribution and friction generation is that the noise created by the friction generation is projected directly into the room. It is better to have dampers installed close to the main supply and as far away from the grille as possible.

6 AIR DISTRIBUTION

An *outlet* is the general term used to describe any opening through which air enters the treated space. Although they are not strictly adhered to, the following definitions can be used to distinguish among outlets with different functions.

- grille: a decorative covering for an outlet
- diffuser: a functional grille guiding air direction
- register: a grille with an internal damper

The *gross area* or *core area* of the grille is its total cross sectional area. The total area of the openings in the grille constitutes the *free area* or *daylight area*.

View Verification Case 80 Model

Verification Case 80