AFT xStream™

Verification Cases

AFT xStream Version 3

Gas Transient Modeling in Piping Systems

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Applied Flow Technology

Dynamic solutions for a fluid world ™

Contents

AFT xStream Verification Overview

There are a number of aspects to the verification process employed by Applied Flow Technology to ensure that AFT xStream provides accurate solutions to gas transient and steamhammer problems in pipe flow systems. These are discussed in Verification Methodology. A listing of all of the verified models is given in Summary of Verification Models. The verification models are taken from numerous References.

Verification References

1. Sod, Gary. *A Survey of Several Finite Difference Methods for Systems of Nonlinear Hyperbolic Conservation Laws*. Journal of Computational Physics, Elsevier, 1978, 27 (1). ff10.1016/0021-9991(78)90023-2. hal-01635155 (https://hal.archives-ouvertes.fr/hal-01635155/document)
2. Moody, Frederick J., *Introduction to Unsteady Thermofluid Mechanics*, John Wiley & Sons, Inc., New York, NY, 1990.

Verification Methodology

The *AFT xStream* software is a gas transient and steamhammer analysis product intended for use by trained engineers. As a technical software package, issues of quality and reliability of the technical data generated by the software are important. The following description summarizes the steps taken by Applied Flow Technology to ensure the high quality of the technical data.

1. Comparison with open literature examples

There are not many published examples for gas transient systems available for comparison with AFT xStream. However, AFT xStream has been compared to several published systems with known analytical solutions, including Sod's shock tube example and a pipe rupture case from Moody.

2. Transient solver checks for artificial transient to ensure true steady initial conditions

Before running the transient solution, AFT xStream always runs the MOC Steady solution with no transient boundary conditions in effect. It then compares the initial conditions to the MOC Steady calculation to see if significant differences exist. If so, a warning is generated.

3. Steady-state and transient solution at time zero are self checking

AFT xStream has two Solvers – one for the steady-state and one for the transient. They use two entirely different solution algorithms. First the AFT Arrow Steady Solver is run, and then the results are used to initialize the MOC Transient Solver. Before the transient solver is actually run, the MOC Steady solution is run. If the Arrow Steady and MOC Steady solutions disagree, a warning is generated. Thus if there were fundamental calculation errors in either method then an artificial transient would be generated and the user warned. This does not ensure all transient calculations afterward are correct, but does ensure that the fundamental transient equations are being properly represented.

Summary of Verification Models

Comparison of AFT xStream predictions to the published calculation results is included herein for two cases from two sources.

Below is a summary of the cases.

Case	Fluid	Reference
Case 1	Air	Sod
Case 2	Air	Moody

Verification Case 1

Problem Statement

PRODUCT: AFT xStream

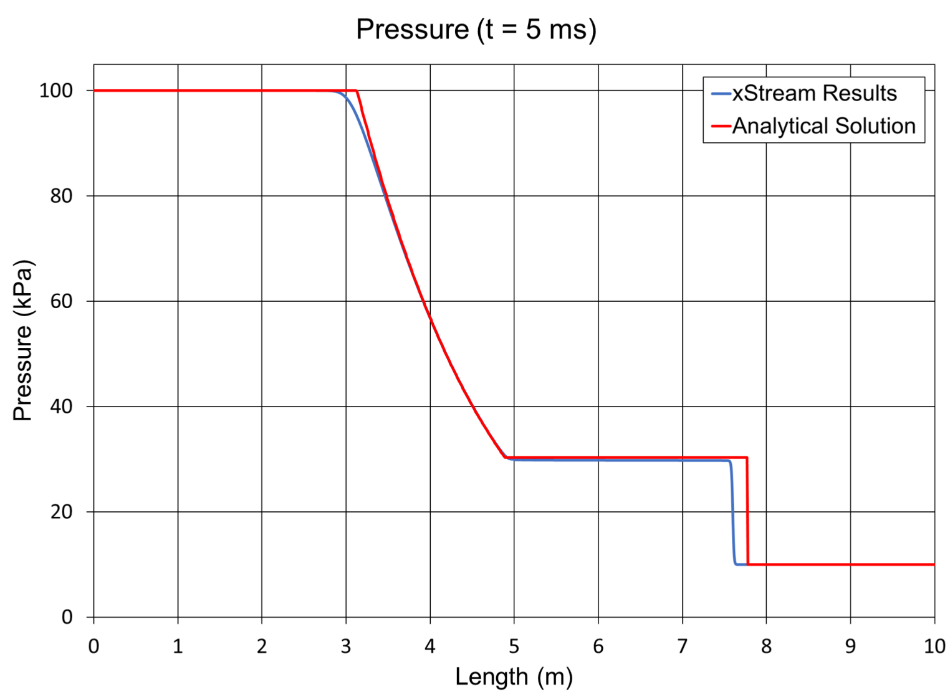
TITLE: XtrVerify1.xtr

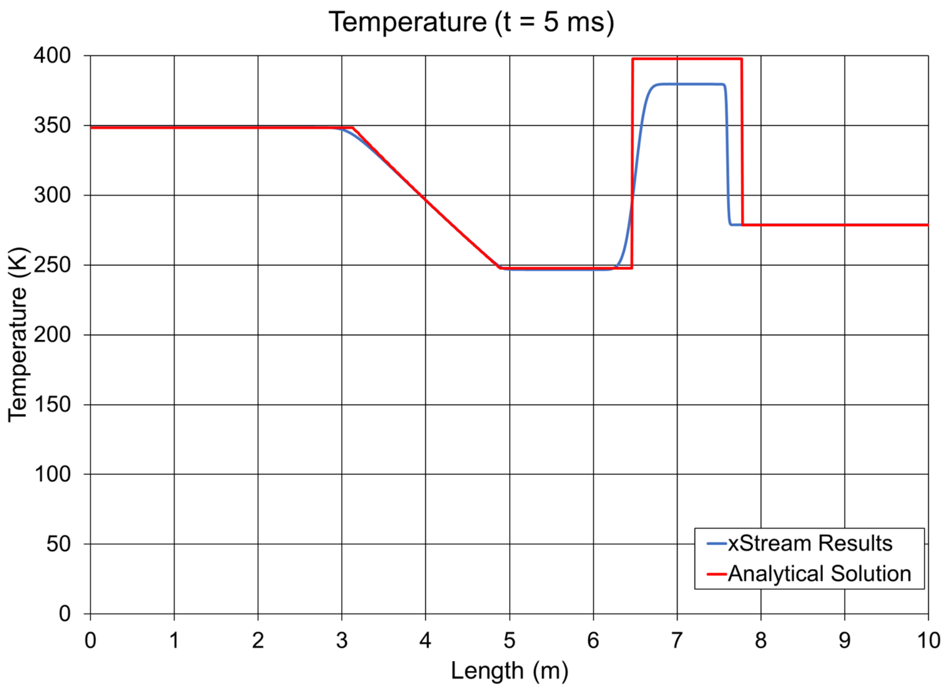
REFERENCE: Gary Sod. *A Survey of Several Finite Difference Methods for Systems of Nonlinear Hyperbolic Conservation Laws*. Journal of Computational Physics, Elsevier, 1978, 27 (1), pp.1-31, Fig. 4. ff10.1016/0021- 9991(78)90023-2. hal-01635155 (https://hal.archives-ouvertes.fr/hal-01635155/document)

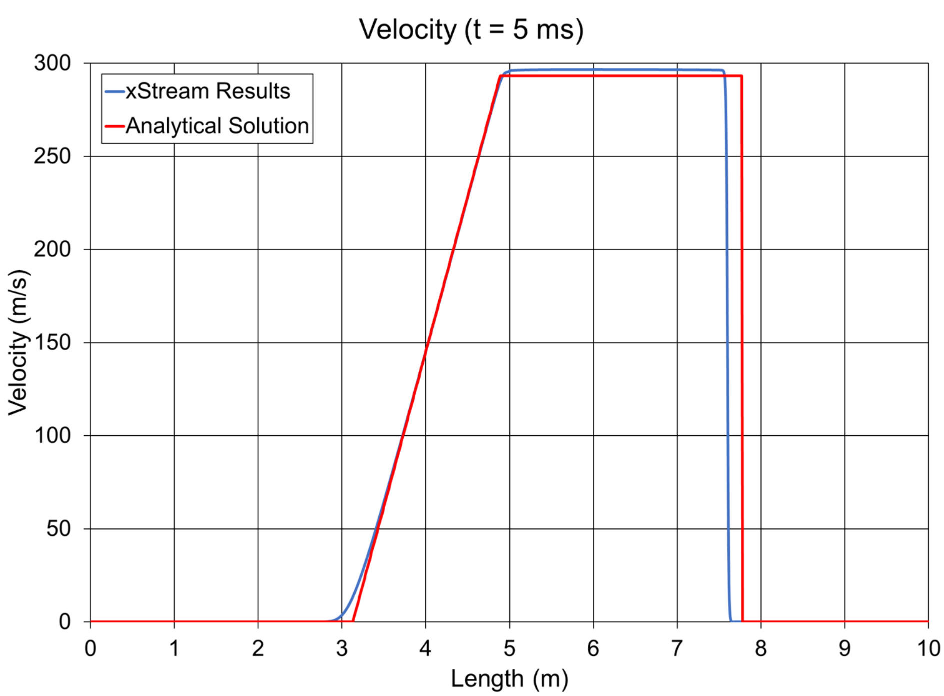
FLUID: Air

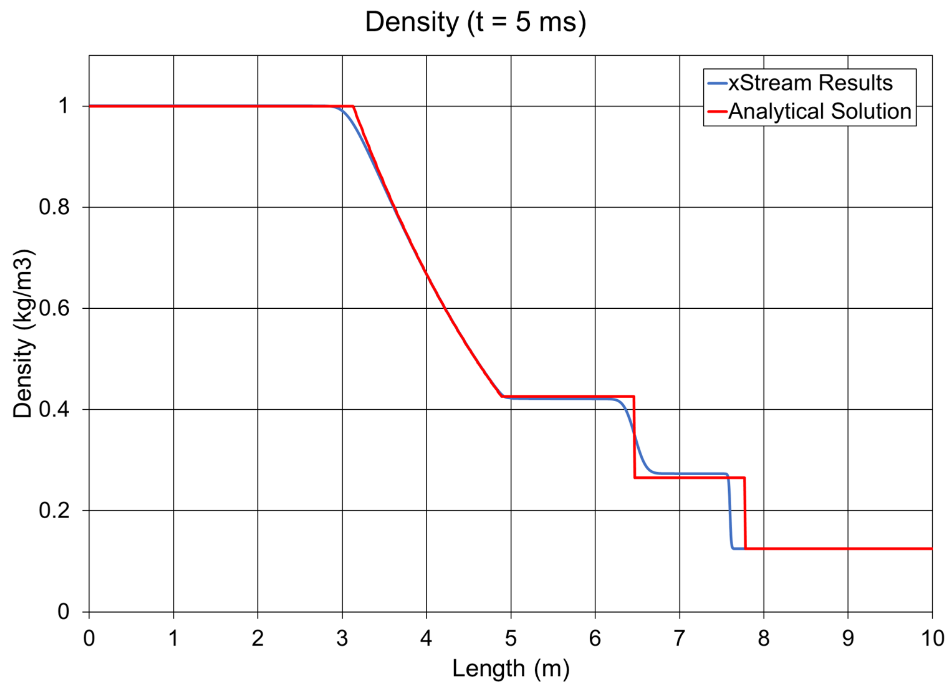
ASSUMPTIONS: Calorically perfect gas, ideal gas

RESULTS:




Temperature (t = 5 ms)
Temperature (K)
400
350
300
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
9
10
Length (m)
xStream Results
Analytical Solution


Velocity (t = 5 ms)
Velocity (m/s)
300
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
9
10
Length (m)
xStream Results
Analytical Solution



DISCUSSION:

The shock tube apparatus studied for this problem involves a high pressure and low pressure region initially separated by a diaphragm. The high pressure region (x = 0 to x = 5) is initially fully enclosed with a dead end at x = 0. The low pressure region has an open end at x = 10. Both regions are initially at rest. At the beginning of the transient the diaphragm is broken, resulting in a compression wave forming at the broken diaphragm and propagating out from that point. In xStream the initial conditions at the dead end for the high pressure region are modeled using an assigned pressure junction. Since the simulation ends before the wave reflects this is reasonable, but would not be an accurate way to model the dead end for a longer simulation. The diaphragm is represented by a valve which is closed at time 0, then opens instantaneously once the transient begins to represent the diaphragm bursting open.

An analytical solution to this problem can be found by making several simplifying assumptions, including that the pipes are adiabatic and frictionless, and that the gas is calorically perfect and ideal. This means that the fundamental compressible flow equations can be solved assuming flow is isentropic, except at the shock wave. The analytical solution was calculated using the basic equations as are described on page 3, along with the fundamental compressible flow equations.

The initial values for pressure and temperature were chosen in order to satisfy the pressure and density ratios specified by the reference for the steady state.

It can be seen that the relative magnitudes of the pressure, density, and velocity are reasonably close to the analytical solution, though the shock wave front is shifted slightly to the left. There is a more noticeable difference in the maximum temperature results for xStream and the analytical solution, though the margin of error is still relatively small. The lower temperature predicted by xStream will also cause a lower sonic velocity for the fluid, which explains the shift in the shock wave front.

Verification Case 1 Problem Statement

Verification Case 1

Gary Sod. A Survey of Several Finite Difference Methods for Systems of Nonlinear Hyperbolic Conservation Laws. Journal of Computational Physics, Elsevier, 1978, 27 (1), pp.1-31, Fig. 4.

switch. The one chosen was suggested by Hyman [13]. Replace $\phi^n_{i+\frac{1}{2}}$ in (45a) by $\beta\phi^n_{i+\frac{1}{2}}$ where

$$\beta = \tfrac{1}{3}, \quad \text{if } \alpha^n_{i+1} > \alpha_i^n + (\Delta x/3)$$
$$= 1, \quad \text{otherwise.}$$

This type of switch greatly reduces the smearing of the contact discontinuity as well as the shock wave. This switch is a type of artificial compression.

3. The Shock Tube Problem

Figure 2 represents the initial conditions in a shock tube. A diaphragm at x_0 separates two regions (regions 1 and 5) which have different densities and pressures. The two regions are in a constant state. The initial conditions are $p_1 > p_5$, $\rho_1 > \rho_5$, and $u_1 = u_5 = 0$; i.e., both fluids are initially at rest. At time $t > 0$ (see Fig. 3) the diaphragm is broken. Consider the case before any wave has reached the left or right boundary. Points x_1 and x_2 represent the location of the head and tail of the rarefaction wave (moving to the left). Although the solution is continuous in this region

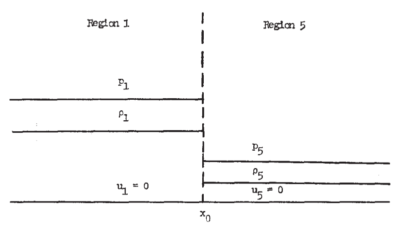


Fig. 2. Shock tube at $t = 0$.

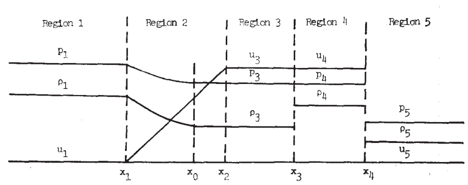


Fig. 3. Shock tube at $t > 0$.

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(region 2) some of the derivatives of the fluid quantities may not be continuous The point x_3 is the position that an element of fluid initially at x_0 has reached by time t. Point x_3 is called a contact discontinuity. It is seen that across a contact discontinuity the pressure and the normal component of velocity are continuous. However, the density and the specific energy are not continuous across a contact discontinuity. Point x_4 is the location of the shock wave (moving to the right). Across a shock all of the quantities (ρ, m, e, and p) will in general be discontinuous.

In the study of the above numerical methods the following test problem was considered: $\rho_1 = 1.0$, $p_1 = 1.0$, $u_1 = 0.0$, $\rho_5 = 0.125$, $p_5 = 0.1$, and $u_5 = 0$. The ratio of specific heats γ was chosen to be 1.4. In all of the calculations $\Delta x = 0.01$. For the Rusanov scheme the value of ω (see Table I) was taken to be 1.0. In the scheme of Boris and Book the parameter η was taken to be 0.125. For Hyman's scheme the value of δ was taken to be 0.8. The constant in the artificial viscosity term ν was taken to be 1.0 in all but one case. Also the value of σ (see Table I) was taken to be 0.9.

In Glimm's original construction a new value of ξ was chosen for each grid point i and each time level n. The practical effect of such a choice with finite Δx is disastrous since our initial data is not close to constant (which was an assumption made by Glimm). In fact, if ξ is chosen for each i and n, it is possible that a state will propagate to the left and to the right and thus create a spurious state. An improvement due to

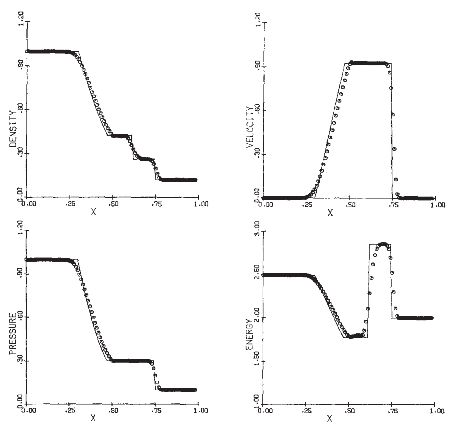

FIG. 4. Godunov's method.

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Verification Case 2

Problem Statement

PRODUCT: AFT xStream

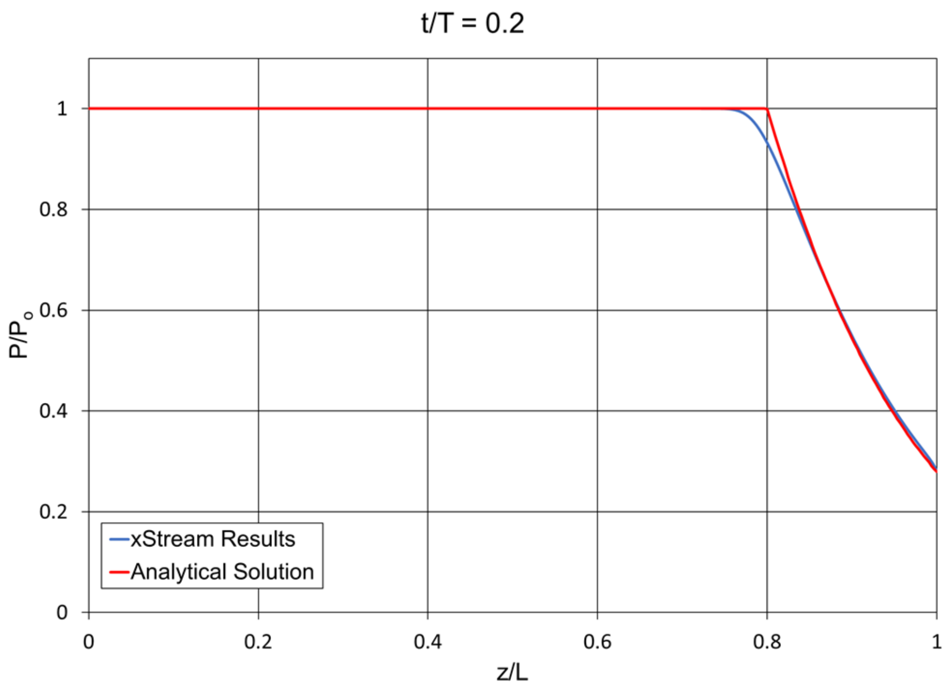
TITLE: XtrVerify2.xtr

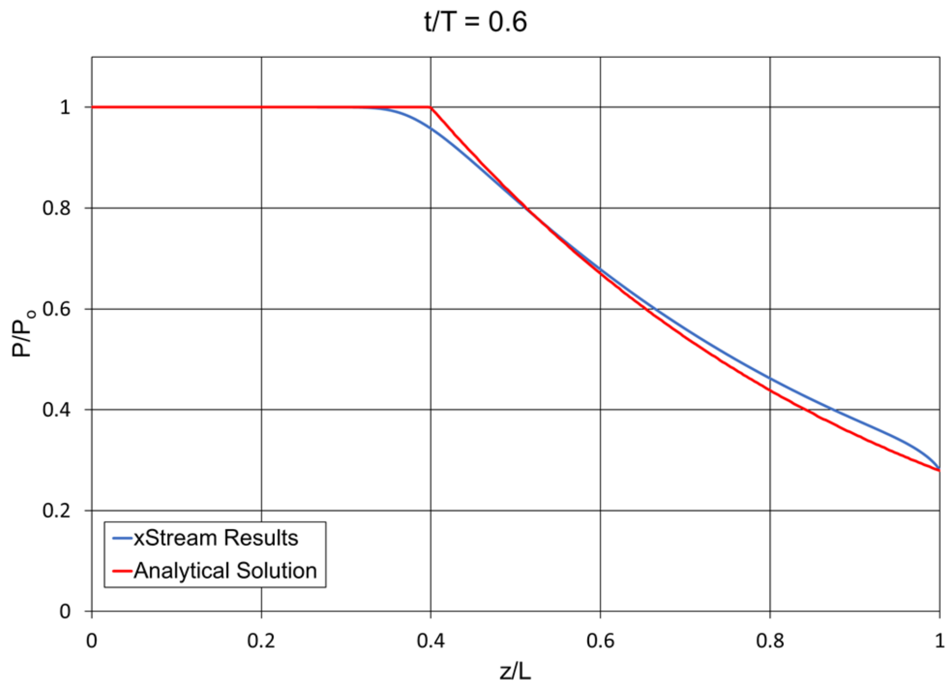
REFERENCE: Moody, Frederick J., *Introduction to Unsteady Thermofluid Mechanics*, John Wiley & Sons, Inc., New York, NY, 1990, Page 452-453, Example 8.5, Figures 8.16 & 8.17

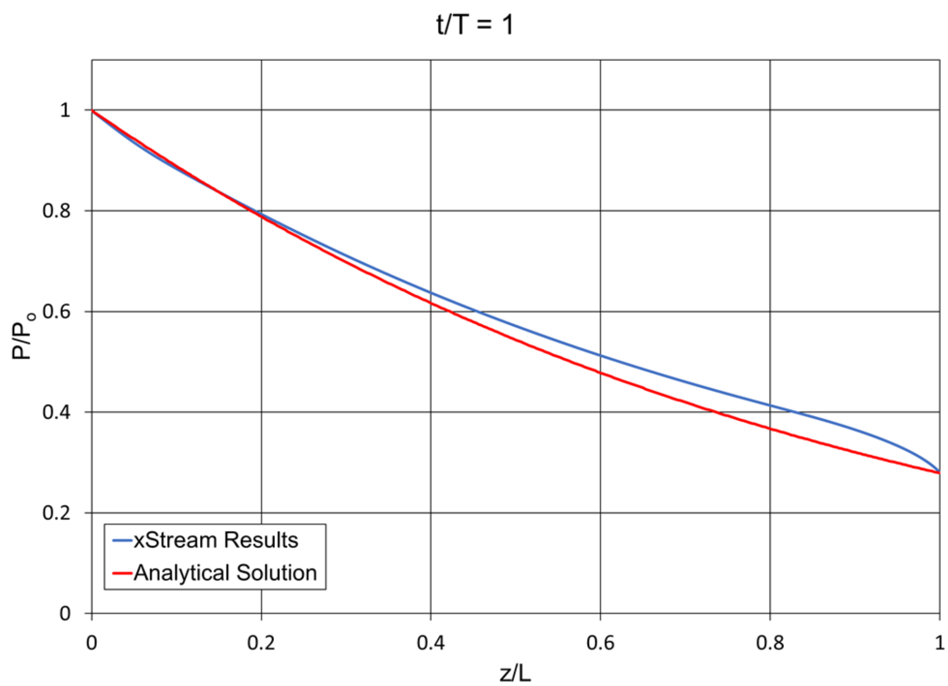
FLUID: Air

ASSUMPTIONS: Calorically perfect gas, friction is negligible

RESULTS:







DISCUSSION:

To find an analytical solution for this example Moody assumes the process is adiabatic, the pipes are frictionless, and that the fluid is ideal and calorically perfect. The assumptions made by Moody allow an isentropic solution to be found for the expansion wave.

The tube rupture is modeled as an instantaneous drop in static pressure based on the dimensionless static pressure drop ratio in Moody Figure 8.16, P/P_o.

Though Moody assumes that the pipes are frictionless, the pipes in xStream are instead defined as hydraulically smooth. This is done to prevent supersonic flow in the xStream model.

The difference in friction models and the fact that the fluid in xStream is not modeled as calorically perfect can account for the minor differences between the xStream results and the analytical solution.

Verification Case 2

Moody, Frederick J., Introduction to Unsteady Thermofluid Mechanics, John Wiley & Sons, Inc., New York, NY, 1990, Page 452-453, Example 8.5, Figures 8.16 & 8.17

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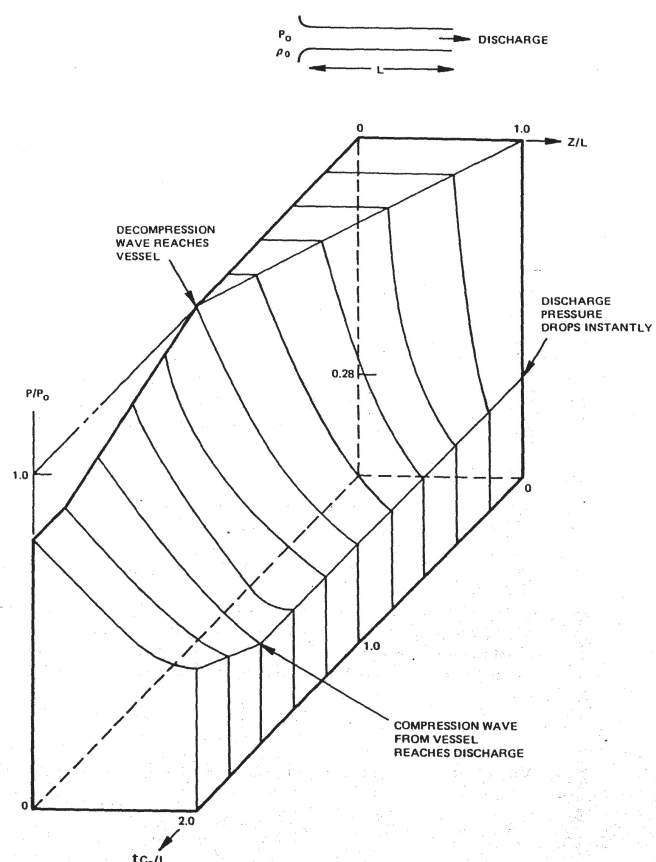

Figure 8.16 Pressure-Space-Time Surface, Pipe Rupture

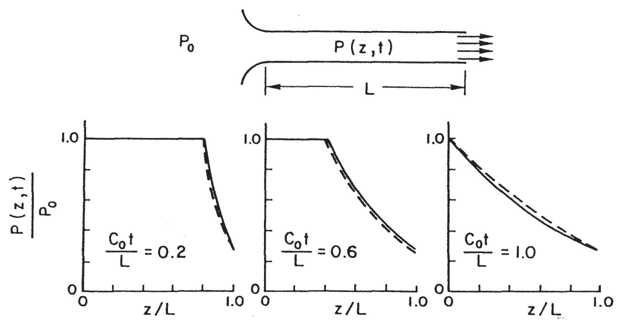

Figure 8.17 Comparison, Simple Wave Theory and Method of Characteristics. —— Simple Wave Theory; – – – Method of Characteristics

Pressure, time, and space variables were normalized so that the results apply to pipes of any length and any vessel stagnation conditions for a gas with $k \approx 1.4$.

Example 8.5: Simple Wave Theory and the MOC

Consider the pipe rupture and gas discharge of Fig. 8.16. Show that simple wave theory can be employed to predict the time-dependent pressure profile during the time $t = L/C_0$ while the initial decompression travels from the ruptured end to the reservoir.

Equation (8.7) is applied to the right end $z = L$, where the boundary condition for critical discharge is $V = C$. Since disturbance propagation is to the left, the negative sign is used, which gives $f(V) = L$ for all time. It follows from Eq. (8.7), the sound speed C of Eq. (8.11), and the initial condition $V_i = 0$, that

$$\frac{V}{C_i} = \frac{2}{k+1}\left(1 - \frac{L-z}{C_i t}\right)$$

The simple wave pressure profile obtained from Eq. (8.10) becomes

$$\frac{P}{P_i} = \left(\frac{2}{k+1} + \frac{k-1}{k+1}\frac{L-z}{C_i t}\right)^{2k/(k-1)}$$

The comparison given in Fig. 8.17 shows that the simple wave and MOC solutions give the same result. When the disturbance arrives at the reservoir, a return wave is propagated to the right, and simple wave theory is not valid for simultaneous right and left traveling disturbances.